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Damping Factor Function in AC Electrical Arc Models Part 1: Heat Process Relaxation Phenomena, their Approximations and Measurement

Introduction

Temporary changes of the column cross-section radius and the distribution of energy along and across an arc significantly affect the operation of electrotechnological devices and electric appliances. They are decisive for the possibility of breaking and re-igniting the arc. A quantity characterising such possibilities in the most complete manner is a function for the factor of damping energetic processes in an electric arc plasma. A coarse, and yet very comfortable approximation of this function is the time constant. The knowledge of the arc time constant is necessary for the following:

- selecting economical operating conditions of electrotechnological devices (welding, electrothermal etc.);
- optimum influence on the arc in switching devices (breakers, switches, contactors, relays);
- maximally intense influence on the arc in overcurrent and overvoltage protective devices (fuses, lightning protectors).

The arc may stop during voltage reduction, excessive stretching of the column, excessive current reduction, cooling of the gas area (sometimes also of the electrode) or as a result of contracting the column with a diaphragm. The above phenomena trigger deionisation in the plasma column and cooling of the active areas in the electrodes. Depending on the intensity and duration of these processes, re-ignition of the arc may be difficult or even impossible. For this reason, the time constant should be the following:

- low in electric appliances so that the arc can stop relatively quickly due to disturbances;
- high in electrotechnological equipment, in order to prevent undesired terminating of the arc due to disturbances.

Heat process relaxation phenomena in electric arc

Arc columns have a heat capacity, and yet they constitute certain resistance to thermal current. For this reason they have finite times of response to forced changes of thermal states. The amount of internal energy, accumulated in the arc, depends on many factors:

- plasma volume (radius, length and shape of the column);
- temperature distribution in the column;
- pressure of the plasma-forming gas;
- type of plasma-forming gas, degree of plasma-forming gas ionisation etc.

The enthalpy of the arc Q changes exponentially in time in accordance with the time constant [1]

$$\theta = \frac{1}{\dfrac{dP_0}{dQ} - \dfrac{dR}{dQ} I^2} \tag{1}$$

where P_0 – dissipated power; R – arc resistance. Hence one can see that θ depends on current I. In the case of alternating current, the factor θ depends on the momentary value of current.

As the electric current is characterised by thermal inertia, the changes of the thermal state and geometrical dimensions of the col-

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umn during current rushes or changes in the column length are not immediate, but proceed with a certain time constant. That is why arc resistance during the changes of current $r(i)$ alters exponentially from the stabilised value $r(I_0)$ to the value corresponding to new current $r(I_1)$. The time constant of the arc in constant pre-set heat transfer conditions is a time after which the arc column changes its resistance *e*–times after energy is no longer supplied to the column.

The time constant of the arc is defined by the column cooling conditions. In plasmatrons it is shorter by 2÷3 orders ($10^{-6} \div 10^{-7}$ s) if compared with ordinary free arcs. Even at a frequency of 200 kHz the arcs of AC plasmatrons have a hysteresis, which reflects their low time constant. The higher the flow rate of the gas flowing around the column or the rate of the arc motion in gas, the lower the time constant is. In the case of high gas flow rates, the arc time constant does not depend on the gas chemical composition or the type of electrodes [2].

When the arc is ignited or terminated the energy accumulated in the plasma volume unit is greater than the energy dissipated from this volume. For this reason, in furnaces with a high temperature of the atmosphere it is easy to observe the thermal breakdown of the inter-electrode gap with relatively low voltage. In turn, if the temperature of the atmosphere is low, the reproducing strength of the inter-electrode gap rises quickly to a certain initial value according to a certain exponential dependence. In order to be able to reliably re-ignite the arc, the rate of power source voltage reproduction should be higher than the critical value [1]. This principle is the basis for testing and assessing the quality of dynamic characteristics of welding sources. The lower limit of the reproducing voltage is defined by the arc ignition voltage. The rate of increasing reproducing strength of the arc column is defined by the inertia of heat processes, and especially by the arc time constant θ. In turn, the dynamic properties of the source are affected by the design and settings of the control system as well as by the passive conservative elements (inductance and capacitance) of its circuits.

The gas suppression ability is defined not only by its time constant but also by its electric strength. The reproduction of the electric strength of the inter-electrode section strongly depends on the falling rate of temperature T of the plasma left after the arc column. This can be roughly determined using the following dependence [1]:

$$T = T_{ot} - \left(T_0 - T_{ot}\right) \cdot e^{-t/\theta} \qquad (2)$$

where T_{ot} – ambient temperature; T_0 – temperature on the arc axis at the beginning of the process; θ - arc time constant. The electric strength of the inter-electrode gap is inversely proportional to temperature [1]

$$E_T = \frac{T_{ot}}{T} E_{ot} \qquad (3)$$

where E_T – electric strength at heightened temperature T; E_{ot} – electric strength at ambient temperature T_{ot}.

Approximating the factor of energetic process damping in electric arc

In the majority of simplified mathematical models, very roughly approximating the physical properties of the electric arc, the damping factor value of transitory processes (thermal and electric) is adopted as a constant quantity (the so-called time constant). It is the proportion of two quantities; the numerator is the heat capacity of the plasma channel, whereas the denominator is made up of parameters specifying the properties of energy dissipation [3]. More accurate models (e.g. Cassie-Mason or Lowke's) bind proportionally the time constant value with the cross-sectional area of the plasma column [4, 5]

$$\theta \propto p\, c_p \frac{r_a^2}{\lambda} \tag{4}$$

where p - gas density; c_p – specific heat of gas of pressure p; λ - gas heat conduction factor; r_a – arc column radius.

The confirmation of this assumption was also attempted using the approximation of experimental data [6], where one should bear in mind that the column cross-sectional area depends not only on electric current intensity but also on the type and pressure of plasma-forming gas, the temperature of the gas in the discharge area, the rate and direction of gas flow, the diameter of the discharge duct, the amplitude and frequency of the magnetic field etc. In some other models of the arc the time constant is adopted as being proportional to the column diameter (e.g. the model by A.A. Woronin [7-10]).

In relation to relatively long arcs the short conical part at the cathode is negligible and the shape of the whole plasma column can be assumed as cylindrical. Theoretical deliberations and experimentation are significantly simplified if one determines the geometrical dimensions of the DC arc. Results obtained in this way can be adopted for low-frequency AC arcs, assuming the necessity of maintaining within them the same plasma equilibrium.

In welding arcs the diameter of the arc column is, first of all, the function of current $d_a = f(I^{2/3})$ [11, 12]. This function is very close to the empirical formula [1]

$$d_a \cong k \times I^n, \text{ cm} \tag{5}$$

where $n = 0.6 \div 0.7$, obtained in the case of gas flowing around the arc in a longitudinal manner. If the arc is in the air, $k = 0.27$ cm×A^{-n}.

If one considers a strong-current arc, e.g. in a steelmaking arc furnace with a graphite cathode, burning in air, the measured column diameter amounts to approximately [13]:

$$d_a = 2r_K \cdot \left(0{,}864 - 0{,}253\frac{z}{r_K}\right)^{0{,}5} \tag{6}$$

In most operation regimes the gas atmosphere of the DC arc furnace is made up by carbon oxides. Another formula for the diameter was provided by R.T. Jones and Q.G. Reynolds [14]:

$$d_a = 2r_K \cdot \left[3{,}2 - 2{,}2\exp\left(-\frac{z}{5r_K}\right)\right] \tag{7}$$

where the cathode spot radius is

$$r_K = \sqrt{\frac{I}{\pi j_K}}, \text{ cm} \tag{8}$$

and $j_K = 3500$ A/cm^2 – current density in the cathode spot.

However, the function of the column diameter $d_a(i)$ is not monotonic in the range of weak currents. Plasma does not disappear along with the momentary reduction of current to zero. Yet, the weakening of the pinch effect may cause its expansion, accompanied however by some cooling and deterioration of electric conductance. This, in turn, depends on the conditions of heat exchange in the environment and the rate of current changes during polarisation alteration. Such behaviour is confirmed by the experimental tests of the arc time constant, which increases significantly in the range of weak currents, below approximately 18-30 A [15, 16]. The flow of current through such terminated arc plasma is possible after applying voltage from an additional source [17]. If currents are weak, the arc time constants are not only high but also strongly dependent on the type of gas (e.g. in elgas, SF_6, $\theta = 1 \div 2$ μs, in the air $\theta = 100 \div 200$ μs). In the range of strong currents the tendencies of time constant changes are reverse to the changes of column diameter (cross-sectional area). An increase in current intensity as well as an increase (with very weak saturation) in the column diameter function (formulas (5)-(7)) are accompanied by a decrease in the time constant, which stabilises at the lowest level. Time constants approaching one another and constituting approximately 10^{-4} s correspond to the values of current counted in hundreds and thousands of amperes, flowing through arcs in various gases. Therefore, the direct binding of the damping factor function by the proportionality formula (4) with the cross-sectional area $\theta(S(i))$ or the diameter $\theta(d_a(i))$ of the arc was not confirmed in practice (Fig. 1). Such an outcome can reflect the significant influence of the variability of other plasma parameters e.g. $\theta(d_a(i), c_p(i), p(i), \lambda(i))$, although in analytical deliberations it is not openly expressed [18].

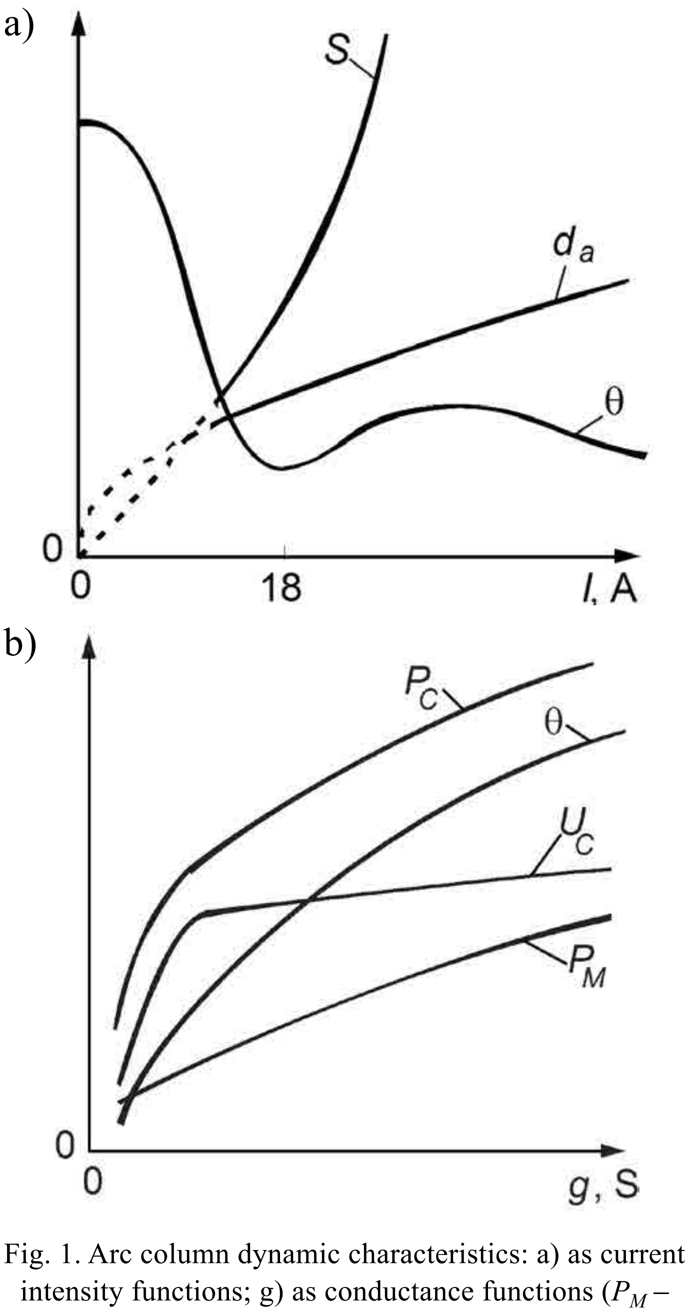

Fig. 1. Arc column dynamic characteristics: a) as current intensity functions; g) as conductance functions (P_M – power of the Mayr-Schwarz model, U_C, P_C – voltage and power of the Cassie-Schwarz model)

The data obtained in experimentation [15] reveal that the column structure can be strongly heterogeneous. In such a case the core of the column is composed of plasma characterised by a very high temperature (significantly over 8000 K), very low viscosity and low time constant θ_f. In turn, the plasma subsurface layer of a lower temperature (6000-8000 K) has the highest viscosity and a higher time constant θ_s. In calculations it is usually assumed that $\theta_f < \theta_s = \theta$. Depending on the type of gas, arcs burning in the areas where the atmosphere temperature is high take on a diffusive form and have one time constant.

As there is no specific universal analytical expression describing dynamic current-voltage characteristics of the arc, there is no resultant specific expression for the time constant either. In such a situation, probably the most appropriate approximation is a function dependent on current in a non-linear manner e.g. [19]:

$$\theta(|i|) = \theta_0 + \theta_1 exp(-\alpha|i|) \approx \begin{cases} \theta_1, \text{if } |i| \text{ small} \\ \theta_0, \text{if } |i| \text{ large} \end{cases} \tag{9}$$

where $\alpha > 0$, $\theta_1 >> \theta_0$ – constant approximation factors. The necessity of taking into consideration the non-linear damping factor function is especially visible in the hybrid models of the arc [19, 20] as they more precisely reproduce dynamic characteristics in wide ranges of current intensity. Another popular solution is to make the time constant dependent on arc conductance (Fig. 1b). Usually used for such purposes are the Schwarz-Avdonin models [8], in which the dependence has the following form:

$$\theta = \theta_0 g^{\alpha} \tag{10}$$

where θ_0, α - approximation factors.

If the basic factor facilitating the termination of AC arc is an insufficiently high momentary value of the current intensity module, at such a moment this value matches the maximum damping factor value. The said value can be determined by means of appropriate measurement methods [21] utilising the delay and increase in the re-ignition voltage. From the point of view of ensuring the arc burning stability and the continuity of electrotechnological device operation it is justified to experimentally determine such highest value θ. Due to the range of weak current, this constant can be used in the Mayr model. However, the time constant determined in the weak-current range is sometimes also used in the Cassie model [22], which may lead to discrepancies of experimentation and calculation results, especially in the range of strong currents. Therefore, if it is necessary to precisely reproduce the courses in circuits by means of the universal model of strong-current arc (e.g. hybrid TWV), the whole damping factor function $\theta(i)$ should be expressed by means of dependence.

Experimental methods for determining AC arc dynamic parameters

A characteristic feature of experimental methods for determining arc dynamic parameters is to take into account the whole range of physical phenomena taking place in the column, near-electrode areas and in electrodes themselves. The separation of individual components of energy processes is very difficult but sometimes possible by means of analysis [11, 23, 24]. Depending on the design and the principle of operation of electrotechnological arc or plasma-arc devices, applied technologies and operation modes, one can observe various levels of disturbances both as to the amplitude and the range of frequency, which can even exceed the values allowed by related standards. Such disturbances may originate from sources which are difficult to identify or eliminate and which disturb processes taking place in the arc column, electrodes, and even in the circuits of measurement systems. As the quantities $u(t)$ and $i(t)$ are registered along with random disturbances, calculating the values of conductance $g(t)$ on their basis comes down to solving a badly conditioned task. An improvement in the quality if input data can be obtained using appropriate methods for filtering and smoothing time courses [25]. However, prior to undertaking such actions it is necessary to solve the issue of recognising types of disturbances in order to weaken only the impact of natural disturbances and leave disturbances triggered on purpose.

Methods for determining the dynamic parameters of the AC arc can be divided into several groups:

1) methods using natural periodic courses of current and voltage;
2) methods introducing additional disturbances to periodic courses, using additional current sources;
3) methods introducing disturbances of the arc column length (voltage);
4) methods introducing disturbances of conditions of energy dissipation from the column [7, 21].

In the case of the methods utilising natural electric courses it is assumed that there is an unequivocal functional relationship between arc parameters and current, resistance or conductance. It means that, in specified conditions of heating and cooling the column, one set of model parameters corresponds to one value of current or arc conductance. Using properly processed (filtered and/or smoothed) data, one can apply one of the analytical or analytical-graphic methods (known as Amsinck, Ruppe, Asturian, Rijanto, Zuckler, Tajew, generalised etc.) in order to determine the simple parameters of the Mayr or Cassie models [22]. More complex models require the use of numerical methods.

There are also possibilities of the direct determination of the arc time constant, not requiring the calculations of the remaining parameters of specific mathematical models.

In the low-voltage arcs of sinusoidal alternating current and in the conditions of relatively low cooling intensity, before and after the passage of current through zero, it is possible to observe moments at which the first voltage derivative, in relation to time, equals zero. Using the measurement of time t0 from the moment at which sinusoidal current passes through zero until the moment of arc ignition or termination it is possible to determine the whole time constant using the following formula [21]:

$$\theta = \frac{t_0}{\sqrt{2}} \tag{11}$$

Another simple method uses the harmonic analysis of arc voltage [26]

$$\theta = \frac{1}{4\omega}\left(\frac{1}{\chi} - \chi\right) \tag{12}$$

where $\chi = A_{2n+1} / A_{2n-1} < 1$ amplitudes of the closest harmonic odds of voltage (usu. $\chi = A_3 / A_1$).

A special method for testing the AC arc consists in "placing" a properly selected high-frequency current (as to the amplitude and phase) on the current flowing through the arc [7]. For the purpose of testing air switches this frequency usually amounts to 20 kHz. In the case of switches with elgas the frequency is much higher and equals 70 kHz. In this manner one can trigger additional transition processes in the areas of net current passage through zero. After registering the courses, the parameters of the Mayr model [7] can be calculated from the following dependence:

$$\theta = -\frac{g}{\frac{dg}{dt}}, \quad \text{if } i = 0 \tag{13}$$

$$P_M = ui, \quad \text{if } \frac{dg}{dt} = 0 \tag{14}$$

where P_M – constant value of the dissipated power of the model. As the determination of the time constant is carried out at point $i = 0$, when conductance g has an indefinite value, the value of the time constant in the expression (13) is calculated from the following interpolation:

$$g = \frac{g(t_0 - \Delta t) + g(t_0 + \Delta t)}{2} \tag{15}$$

where t_0 – time instant in which $i = 0$ A; Δt – time interval, at which the recording of current and voltage values takes place.

According to another method, net current does not have to pass through zero. In such a case the value of power is determined using the formula (14), and the time constant is calculated from the dependence below:

$$\theta = -\frac{g}{\frac{dg}{dt}}\left(\frac{i^2}{gP_M} - 1\right) \tag{16}$$

It is also possible to determine the damping factor function on the basis of the reaction of the arc column on various length disturbances. They should also be appropriately synchronised and shifted in the phase in relation to the course of current. In laboratory conditions the changes of length can be relatively easy to induce by means of properly selected rotating electrodes (commutators) [7]. The excitation of high-frequency disturbances by electrode vibrations is more difficult, especially if electrodes are massive. It also facilitates the sputtering of electrode material and increases electrode erosion. A relatively high frequency of such changes can be obtained by the crosswise action of variable magnetic field on the arc [27]. Due to some binding of the column to electrode spots (especially of the cathode) only the central part of a long arc is the preferable area of this action.

In an electric arc with stabilised current it is possible to trigger voltage changes by modifying the conditions of heat exchange with the surroundings [21]. The longitudinal pulse flow of gas around the column causes momentary changes of dissipated power and of the arc column diameter. In turn, the transverse or slant pulse flow of gas around the column causes its temporary elongation and contraction. The changing motion of the arc in relation to the gaseous environment can also be triggered by means of a modulated magnetic field properly synchronised with the course of discharge current. The use of parallel ring electrodes with moving arc spots enables maintaining almost the whole length of the column.

Artificially introduced arc disturbances should be characterised by a limited range of amplitude due to the strong non-linearity of static and dynamic characteristics as well as because of discharge instability. For this reason, the depth of modulation usually amounts to a few percent. Too high an amplitude of current disturbances changes the character of arc discharge from the "dc-" to "ac-type" [18]. Also, too high a frequency of current disturbances changes the character of discharge from the ac-type arch discharge with thermal plasma to the "RF-type discharge" with non-equilibrium plasma. For this reason there should be an inverse proportionality between the amplitude of periodic disturbance and its frequency.

It is also technically possible to carry out synchronised disturbance of the arc column with two or more types of external factors at the same time. In this manner one can obtain the deepened modulation of courses, which however may facilitate the occurrence of discharge instability. Due to this fact such solutions are not applied for testing electrotechnological devices. In addition, the greater complexity of the design and operation of the testing station entail the greater complexity of necessary analyses and measurements. Such inputs cannot be compensated by the improved accuracy of obtained results. In such situation, the methods introducing electric disturbances are the simplest, most accurate and, consequently, most popular [6, 15, 16, 18, 28].

The second part of the article focuses on the assessment of the usability of methods used for measurements of dynamic characteristics by simulating processes in circuits with modified and hybrid models of the electric arc.

Conclusions:

1. The results of the so-far experimentation and theoretical analysis of such physical quantities as the damping factor function and arc geometrical dimensions often do not confirm adopted assumptions, nor do they offer the possibility of obtaining simple and direct relationships between θ and the diameter d_a or the cross sectional area S of the column.

2. Most of the experimental methods for determining the dynamic characteristics of the electric arc enable only the determination of the time constant in the areas of current decay. The constant constitutes the maximum value of damping function and, as such, is predominantly useful for modelling the joining arc.

3. The pursuit of more and more precise reproduction of processes in the circuits of welding and electrothermal equipment with an electric arc causes the popularisation of complex hybrid models increasing the usability of non-linear damping functions.

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