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Method for Determining the Parameters of the Pentegov Model Describing an Electric Arc with Hyperbolic-Linear Static Characteristic

Abstract: The article presents reasons for nonlinear static voltage-current characteristics of an electric arc; the characteristics consist of an initial voltage drop followed by a voltage rise in the range of strong currents. The article suggests that overcoming difficulties in mathematical modelling of electric processes in circuits with arcs requires the use of Pentegov assumptions and the building of an arc model utilising a static hyperbolic-linear characteristic. The article also presents a method for the experimental determination of arc model parameters with sinusoidal excitation as well as describes a macromodel built using the Matlab-Simulink programme. The correctness of analytical expressions specifying mathematical model parameters was verified through simulation. The study also involved testing the resistance of the proposed method to arc length random disturbances.

Keywords: electric arc, Pentegov model, static characteristics, time constant

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Introduction

One of the main reasons for the non-linearity of electric arc static and dynamic characteristics and for the deformation of AC and voltage waveforms in circuits with an arc is the high voltage of discharge ignition, the value of which depends on many physical factors present in the inter-electrode space as well as on the character of current excitation (e.g. frequency). By appropriately affecting the inter-electrode gap space, it is possible to change the voltage of discharge ignition. For instance, in devices for used for welding under flux it is possible to effectively isolate an arc thermally and maintain a high temperature of ionised gases, thus maintaining the uninterrupted burning of AC-energised discharge. However, in many other cases the obtainment of low-value ignition voltage would require undertaking actions which could seriously disturb technological processes. For this reason, the limitation of detrimental effects of missing compatibility between devices and mains (power supply) takes place outside an arc, and, often, even outside an electrotechnological device. These negative effects are the strongest in devices with low-current arcs or arcs burning in the strongly cooled environment. In most electrotechnological devices, high-current arc burning is accompanied by a voltage stabilisation effect, also responsible for the non-linearity of characteristics and electric quantity waveform deformations. The presence of the effect

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or its range also depends on many physical and chemical factors coexisting in the inter-electrode space [1, 2].

The use of selected gases (e.g. argon or hydrogen), high pressure of gases, the use of pointed electrodes, a constrictor with an edge in a plasma torch etc. can lead to the effect of a rise in static or dynamic characteristics in a high-current range [3, 4]. The positive differential resistance of an arc facilitates the obtainment of discharge stability and, as a result, the moderation of requirements as regards parameters and characteristics of selected power sources. The association of the Mayr and Cassie models of a dynamic arc of hyperbolic and linear-horizontal static characteristics into a hybrid model (TWV) causes significant difficulties with selecting a non-linear tapering function and a damping factor as well as with determining its parameters. It seems better to make use of the Pentegov assumptions [5, 6] as they enable building arc models of any static characteristics, all the more that in many electrotechnological devices these arc characteristics can be relatively easily determined through experimentation. At the same time, in selected cases of static characteristics it is possible to develop methods of the experimental determination of the Pentegov model parameters [7]. An example can be the method of "three measurements" described in [8] or the spectral method presented in [9]. This article is focused on an arc of a static characteristic being the sum of a hyperbolic function and of a linear increasing function. These deliberations assume that the description of dynamic characteristics takes into consideration only phenomena taking place in arc column plasma. One of the known methods [1] was used to determine near-electrode voltage drops. Their influence on arc modelling depends on many factors and can be small in relation to long arcs.

Basic Assumptions of the Pentegov Model of an Electric Arc of a Selected Static Characteristic

In a real inert arc, a stepped current change $i(t)$ triggers a stepped voltage impulse and a quasi-exponential change of resistance. Similar to the structure of most known models (e.g. Mayr, Cassie, and Zarudi), the basis of the Pentegov input assumptions [5] is an energy balance equation. A hypothetical arc model is introduced instead of a real arc model. At the same time, unlike the original, this model is electrically inertialess. Resistance is determined not using real current, but certain lagging current $i_\theta(t)$, changing with a specific time constant θ and being a kind of real current modelling $i(t)$. For high frequency $f >> 1/\theta$ of an AC source, an arc thermal state is determined using root-mean-square current. At a steady state, state current $i_\theta(t)$ should coincide with real current $i(t)$. All energetically identical states are characterised by one variable – arc state current $i_\theta(t)$. Similarly, using the same variable it is also possible to determine arc parameters and dynamic characteristics (Table 1).

The relation between the state current square and the real arc current square is described by a first-order linear equation

$$\theta \frac{di_\theta^2}{dt} + i_\theta^2 = i^2 \tag{1}$$

Table 1. Definitions of physical quantities of a real arc column and of the Pentegov model (U_a – arc voltage; ΔU_{AK} - sum of near-electrode voltage drops

Physical quantity	Real arc	Pentegov model
Current	$i(t)$	$i(t);\ i_\theta(t) \geq 0$
Dynamic voltage $U_{dyn}(t) = U_a(t)\text{-}\Delta U_{AK} =$	$U_{dyn}(i)$	$U_{dyn}(i_\theta) = \frac{U_{st}(i_\theta)}{i_\theta} i$
Dissipated power $P_{dys}(t)$	$P_{dys}(i)=U_{dyn}(i)\cdot i= =R_{dyn}(i)\cdot i^2$	$P_{dys}(i_\theta) = U_{st}(i_\theta)\cdot i_\theta = = \frac{U_{st}(i_\theta)}{i_\theta} i^2 = R_{st}(i_\theta)\cdot i_\theta^2$
Dynamic resistance $R_{dyn}(t)$	$R_{dyn}(i) = \frac{U_{dyn}(i)}{i}$	$R_{dyn}(i_\theta) = \frac{U_{st}(i_\theta)}{i_\theta} = R_{dyn}(i)$

The Pentegov model represents a non-linear circuit two terminal network being
1. energetically balances;
2. first-order thermally inert, linear and stationary;
3. electrically inertialess.

The advantages of this model include the possibility of using any approximation of a voltage-current static characteristic and, at the same time, a constant damping factor function value (so-called time constant). In comparison with other models, [10, 11] this model, in a physically justified manner, can use more accurate approximations of arc static characteristics, observed during welding with, both inert and active, gases. Examples of arc characteristics in such gases are presented in Figure 1.

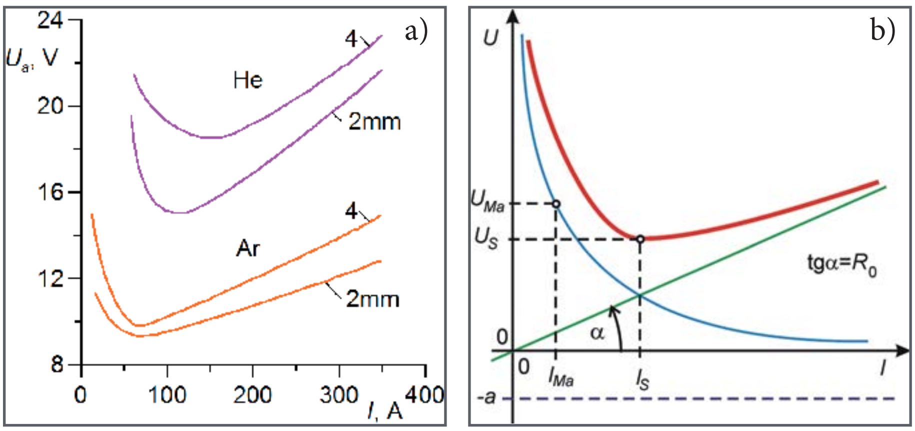

Fig. 1. TIG welding arc static characteristics: a) physical characteristics; b) example of hyperbolic-linear approximation

As can be seen in Figure 1, in high-current ranges, families of characteristics are increasing. Therefore, they can be approximated using a simple dependence

$$U_{st}(I, L_a) = a + bL_a I + \frac{c + dL_a}{I} = \\ = U_C(L_a, I) + \frac{P_M(L_a)}{I} \tag{2}$$

The parameter a represents the sum of near-electrode voltage drops. The value of this parameter can be relatively easily determined experimentally [1]. In cases of sort arcs, it is possible to reduce arc voltage by the value a, thus obtaining a necessary voltage drop on a plasma column. In turn, in cases of appropriately long arcs, the low value of the parameter can be ignored. As a result, it is possible to write

$$U_{st}(I, L_a) = R_0(L_a)I + \frac{P_M(L_a)}{I} \tag{3}$$

In the further part of this publication the parameter *a* will be ignored in arc voltage expressions. An arc under consideration is a constant length arc (L_0=const.), the static characteristic of which can be expressed using the dependence

$$U_{st}(I) = R_0 I + \frac{P_M}{I} \tag{4}$$

where $R_0 = bL_0$, $P_M = c + dL_0$ – constant coefficients of approximation. For the value b=0 Ω/m, the Pentegov model of an arc of a hyperbolic static characteristic, as in the Mayr model, is obtained.

As regards the Mayr model [1, 2], the value of a Mayr power parameter P_{Ma} can be determined using coordinates (I_{Ma}, U_{Ma}) of any point (Fig. 1b) located on a static characteristic ($P_{Ma} = U_{Ma} \cdot I_{Ma}$). In order to determine the parameters of the presently considered Pentegov model, it is possible to use a point (I_S, U_S) located on the static characteristic minimum extreme. The point coordinates are defined by formulas

$$I_S = \sqrt{\frac{P_M}{R_0}} \tag{5}$$

$$U_S = 2\sqrt{P_M R_0} \tag{6}$$

On the basis of these formulas it is possible to calculate static characteristic parameters:

$$R_0 = \frac{1}{2}\frac{U_S}{I_S} \tag{7}$$

$$P_M = \frac{1}{2}U_S I_S \tag{8}$$

The experimentally determined values R_0 and P_M do not need to correspond to the optimum approximation of a static characteristic within a wide range of excitation current changes. In this respect, the method described below offers more possibilities.

Method for Determining the Parameters of the Pentegov Model Describing an Arc of Hyperbolic-Linear Static Characteristic

If it is assumed that in a circuit with an arc of a static characteristic (2) there is variable sinusoidal current excitation of pulsation ω

$$i = I_m \cos\left(\omega t + \frac{\varphi}{2}\right) \tag{9}$$

the arc state current can be described using the following dependence

$$i_\theta^2 = I_{sk}^2\left(1 + \cos\varphi \cos 2\omega t\right) \tag{10}$$

where $I_{sk} = I_m/\sqrt{2}$ – root-mean-square value. On the basis of the definitions provided by the Pentegov [5] it is possible to calculate root-mean-square voltage values and the average value of arc resistance (Table 2). The value of a phase angle φ is related to the time constant of an arc

$$tg\varphi = 2\omega\theta \tag{11}$$

Table 2. Definitions of measurement values

Physical quantity	Real arc	Pentegov model
Root-mean-square current $I_{sk}^2 =$	$\frac{1}{T}\int_0^T i^2 dt$	$\frac{1}{T}\int_0^T i_\theta^2 dt$
Root-mean-square voltage $U_{sk}^2 =$	$\frac{1}{T}\int_0^T u^2 dt$	$\frac{1}{T}\int_0^T U_{st}^2(i_\theta) dt$
Momentary power average value $P=$	$\frac{1}{T}\int_0^T uidt$	$\frac{1}{T}\int_0^T U_{st}(i_\theta)\cdot i_\theta dt$
Average resistance value $R=$	$\frac{1}{T}\int_0^T \frac{u}{i} dt$	$\frac{1}{T}\int_0^T \frac{U_{st}(i_\theta)}{i_\theta} dt$

In accordance with the Pentegov assumptions the following dependences are obtained

$$U_{sk}^2 = \frac{P_M^2}{I_{sk}^2 \sin\varphi} + 2P_M R_0 + R_0^2 I_{sk}^2 \tag{12}$$

$$P = P_M + R_0 I_{sk}^2 \tag{13}$$

$$R = \frac{P_M}{I_{sk}^2 \sin\varphi} + R_0 \tag{14}$$

After solving this system of equations the following formulas for calculating model parameters are obtained [7]

$$P_M = \frac{\left(U_{sk} I_{sk}\right)^2 - P^2}{R I_{sk}^2 - P} \tag{15}$$

$$R_0 = \frac{RP - U_{sk}^2}{R I_{sk}^2 - P} \tag{16}$$

The time constant is expressed by the following formula

$$\theta = \frac{1}{2\omega\sqrt{\left[\frac{(R - R_0) I_{sk}^2}{P_M}\right]^2 - 1}} \tag{17}$$

which, after substituting (15) and (16) can be expressed in the following form

$$\theta = \frac{1}{2\omega\sqrt{\left[\frac{\left(U_{sk} I_{sk}\right)^2 + R I_{sk}^2\left(R I_{sk}^2 - 2P\right)}{\left(U_{sk} I_{sk}\right)^2 - P^2}\right]^2 - 1}} \tag{18}$$

which, in turn, enables the direct use of measured quantities of current, voltage and power.

Testing the Efficiency of the Method for Determining the Pentegov Model Describing an Arc of Hyperbolic-Linear Static Characteristic

The data from Figure 1 were used to determine the values of parameters *a, b, c* and *d*. Depending on a gas type, these dependences will adopt the values of:
a_{Ar} = 8.823 V, b_{Ar} = 3.786·10^{-3} VA^{-1} mm^{-1},
c_{Ar} = 4.994 W, d_{Ar} = 15.467 Wmm^{-1},
a_{He} = 11,471 V, b_{He} = 8.929·10^{-3} VA^{-1} mm^{-1},
c_{He} = 263.172 W and d_{He} = 32.393 Wmm^{-1}.
If constant arc lengths are selected, the values of parameters presented in Table 3 are obtained.

Table 3. Values of parameters R0 and PM of approximations of static characteristics of a column of an arc having various length (L) values and burning in selected gases

Parameter	$L = 2$ mm	$L = 4$ mm
R_{0Ar}, Ω	7,573·10^{-3}	15,145·10^{-3}
P_{MAr}, W	35,927	66,861
R_{0He}, Ω	17,859·10^{-3}	35,718·10^{-3}
P_{MHe}, W	327,958	392,743

Testing the efficiency of the proposed method for determining the parameters of an electric arc model of a static, in some segments hyperbolic-linear, characteristic involved creating a macromodel of a simple circuit with an electric arc using Matlab-Simulink software. The power source used had the parameters I_m= 150 A and f= 50 Hz. The value of a time constant θ= 1·10^{-3} s was assumed. On the basis of voltage and current waveforms in time it was possible to calculate root-mean-square voltage and current values (U_{sk}, I_{sk}) as well as average values (P, R). Formulas (15)-(18) were used to calculate values of arc parameters R_0, P_M, and θ; afterwards the results obtained were compared with the present values (see Table 3). Errors resulting from numerical calculations are presented in Table 4. To some extent, the errors represent disturbances in measurement systems.

In real electrotechnological devices, disturbances are present in power supply systems, electric arcs and supply systems. The first group of disturbances, of a determined or random character, is responsible for supply current waveform deformations, causing the divergence of current from a sinusoidal form. However, by undertaking appropriate actions (e.g. use of filters, fast control systems) it is possible to reduce or even eliminate such disturbances. It is significantly more difficult to achieve such results when arc is affected, without influencing the course of a technological process. The most frequent disturbances include arc column changes triggered by various factors such as gas flows, magnetic fields, electrode vibration, electrode material drop transfer, weld pool surface fluctuations etc. Arc length random disturbances can be described using the following dependence

$$L_a = L_0 + \Delta L_z = L_0 \cdot \left(1 + \frac{\xi_{L\%}}{100}\right) \tag{19}$$

where L_0 – determined arc length value; ΔL_z, ζ_L% – absolute and percentage value of arc length random disturbance. Such disturbances lead to disturbances of static characteristic parameters

$$R_0(L_a) = bL_0 \cdot \left(1 + \frac{\xi_{L\%}}{100}\right) = R_{0L}\left(1 + \frac{\xi_{L\%}}{100}\right) \tag{20}$$

$$P_M(L_a) = c + dL_0 \cdot \left(1 + \frac{\xi_{L\%}}{100}\right) = P_{0c} + P_{0M}\left(1 + \frac{\xi_{L\%}}{100}\right) \tag{21}$$

where $R_{0L} = bL_0$; $P_0c = c$; $P_{0M} = dL_0$. The simulations of electric processes taking place in a circuit were repeated with arc length random disturbances. The assumed arc length amounted to L_0= 4 mm. The pre-set level of random disturbances amounted to $\zeta_{L\%}$ = 1% and 5%. Disturbances $\zeta_L(t)$ were generated using a random generator with a timing frequency of 300 Hz, connected in a cascade manner with the first-order inert

Table 4. Values of errors in determining the Pentegov model parameters related to various lengths (L) of a column of an arc burning in selected gases

Gas	Ar		He	
L, mm	2	4	2	4
δ_R,%	0,877·10^{-3}	0,814·10^{-3}	3,358·10^{-3}	2,006·10^{-3}
δ_P,%	2,53·10^{-3}	2,522·10^{-3}	2,504·10^{-3}	2,5·10^{-3}
δ_θ,%	0,11·10^{-3}	0,106·10^{-3}	0,053·10^{-3}	0,039·10^{-3}

Table 5. Values of errors in determining Pentegov model parameters related to the disturbed length (L) of an arc burning in selected gases

Gas	Ar		He	
L, mm	4		4	
$\zeta_{L\%}$,%	1	5	1	5
δ_R,%	0.116	0.508	0.121	0.571
δ_P,%	0.248	1.43	0.088	0.475
δ_θ,%	0.042	0.324	0.015	0.095

module of a time constant T= 0.00035 s. As before, the calculations of parameters R_0, P_M and θ were made using the formulas (15)-(18). The results were compared with the pre-set values in Table 3. The errors resulting from numerical calculation and arc length disturbances are presented in Table 5. As can be seen, errors of the Pentegov model parameter determination have very low values. It can be stated that in the cases under consideration the errors depend on a gas type to a very little degree. Despite a significant increase in disturbance intensity, errors remain on an acceptably low level.

Conclusions

1. Pentegov model allows to take into consideration both strong non-linearity of static characteristic of an electric arc and disturbances of arc column length.
2. Measurement method described above allows to determine parameters of selected Pentegov model, which, at the same time, determine coefficients of approximation of static characteristics using hyperbolic-linear function.
3. Despite complex static characteristic, it is enough to use single time constant to create Pentegov model. The time constant can be easily determined by measurement mode.
4. Measurement method described above demands using sinusoidal current excitation, which can be natural or admissible excitation in selected electrotechnical devices with electric arcs.
5. To determine parameters of selected Pentegov model using proposed method, it is possible to use simple and easily available measuring instruments and direct analytical dependences, which makes it relatively inexpensive and quick in realization.
6. The computer simulations carried out show great accuracy of proposed method of determining Pentegov model electric arc parameters, even in conditions of intensive column length disturbances

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