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The Modelling of Electric Arc with Stochastic Disturbances

Part 1. The Mapping of Stochastic Disturbances in Mathematical Models of Electric Arc

Abstract: The heterogeneous physical structure of electric arc inspired a proposal to consider separately the generation of random disturbances of electric waveforms in the plasma column and in near-electrode areas. The article presents selected mathematical correlations which could be used to assess parameters of noisy signals in arc devices. Depending on the type of generated noise (white, pink, red), it is necessary to apply appropriate filters (which can be modelled using the ordinary differential equations presented in the article). The article discusses in detail the methods enabling the mapping of stochastic disturbances affecting the electric arc column. 1. By assuming specific systems of the physical effect of disturbances and thereby interfering in the input equation of energy balance it was possible to obtain the noisy Mayr-Voronin and Cassie-Voronin models of arc characterised by variable geometrical dimensions. 2. By assuming specific systems of the mathematical effect of parameter disturbances and thereby interfering in previously developed deterministic mathematical models it was possible to obtain the modified noisy Mayr, Cassie and Schwarz models as well as a model with the radius of a cylindrical column as a state variable. 3. The assumption of specific systems of the mathematical effect of disturbances on the deterministic load of the circuit with electric arc made it possible to consider macro-models with current or voltage noise generating sources additionally connected to arc.

Keywords: electric arc, stochastic disturbances of elecric waveforms, mapping

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Introduction

Electrotechnical equipment may be "troubled" by various disturbances affecting production processes. Such disturbances, which could be of deterministic or stochastic nature, are characterised by various intensity and are triggered by various physical, i.e. thermal, mechanical, electromagnetic, optical, etc. factors. Deterministic disturbances often result in the change of the point of operation of a given element within a system or could even change the operation of the entire system. Usually, the aforesaid disturbances can be described using standard methods. In turn, the mapping of devices with

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stochastically disturbed elements requires the use of additional sources, generating random waveforms.

The burning of electric arc in electrotechnical devices is usually accompanied by relatively intense stochastic disturbances resulting from the transformation of electric energy into heat dissipated and accumulated in gases, electrodes, walls of conduits, chambers, barriers, etc. There is also optical radiation, acoustic waves, magnetic fields, gasodynamic pumping, etc. Only in certain devices (e.g. spectrometric plasmatrons, discharge lamps) it is possible to appropriately minimise the level of disturbances. Natural stochastic disturbances are usually overlapped by external disturbances and deterministic disturbances of various intensity. The intensity of the aforesaid disturbances depends on the design of electrotechnical devices and their operating conditions during technological processes.

Electrotechnical arc devices, even without the effect of deterministic disturbances (having the form of additional waveforms and external effects) are troubled by random disturbances, intensified along with deterministic changes.

In terms of the physical structure of arc, disturbances can be generated in the near-cathode area, in the arc column and in the near-anode area. Because of the fact that arc can be treated as the series connection of non-linear elements, it is necessary to take into account various possibilities when creating macromodels of electric arc using controlled current and voltage sources [1].

Random disturbances in the near-cathode area include:
a) displacements of the cathode spot and changes of its area,
b) changes of the cathode material structure,
c) disturbances of the thermal state of the cathode spot.

Similar disturbances may also affect the anode spot. Because of the passive role of the anode spot in the system, coming down to the neutralisation of the moving spot, the effect of disturbances in the aforesaid area on arc characteristics can be significantly lower than that in the near-cathode area.

Disturbances of the plasma column can be triggered by:
a) changes of the column length,
b) changes of the cross-sectional area of the column,
c) changes of the physical properties of the plasma-forming gas.

The changes of the column length may be triggered by column movements (linear growths, lateral deflections or flexions) caused by, e.g. displacements of electrodes, magnetic field activity, gas blasts, movements of diaphragms, laser beam effect, etc.

Changes of the column cross-sectional area may result from changes of current, gas mass stream or gas pressure, movements of diaphragms or channels (e.g. in plasmatrons).

Changes of physical properties of plasma-forming gases may result from various admixtures to such gases near the cathode spot. The sources of such admixtures could include the non-homogenous structure of the cathode material or the non-homogenous chemical composition of the plasma-forming gas.

The high rate and wide range of changes of column parameters are observed during the use of alternating current of relatively high frequency and amplitude. In the aforesaid situations, even periodic changes of arc parameters are accompanied by intense stochastic disturbances of electric parameters.

Stochastic disturbances may also affect parameters of active and passive elements of electric circuits powering arcs. The randomness of parameters of only one element of the electric circuit translates into the randomness of all electromagnetic processes in the circuit. The forgoing results from, among other things, stochastic Thevenin and Norton theorems, constructed with appropriate assumptions of system linearity [2]. Usually one substitute random disturbance of an element or a subsystem in the modelled circuit is derived from such definitions.

The methods enabling the mapping of stochastic disturbances affecting the column of electric arc can be divided into:

a) methods assuming the physical effect of disturbances and thereby interfering in the input equation of energy balance,
b) methods assuming the mathematical effect of disturbances and thereby interfering in previously developed deterministic mathematical models,
c) methods assuming the mathematical effect of disturbances on the deterministic load of the circuit by electric arc.

In many analyses of electric systems, the occurrence of random disturbances is ignored. Such an approach may result from the low intensity of such disturbances or their insignificant effects, limited by the activity of subsystems of appropriate damping structures. However, the knowledge of stochastic processes in arcs of electrotechnical devices may be useful when diagnosing and designing control systems of such devices.

Selected correlations necessary for the assessment of noisy signals in arc devices

Tests of non-linear systems with sinusoidal excitation often involve the identification of the total harmonic distortion (*THD* of distorted voltage waveforms. If noise is absent, the total harmonic distortion is expressed by the following dependence:

$$THD = \frac{\sqrt{\sum_{k=2}^{n} U_k^2}}{U_1} \tag{1}$$

where U_1 – root-mean-square voltage of the primary component and U_k – root-mean-square voltage of a k-th harmonic. The occurrence of random disturbances increases *THD* measurement results. In such cases, the total harmonic distortion is calculated using the following formula:

$$THD + N = \frac{\sqrt{U_n^2 + \sum_{k=2}^{n} U_k^2}}{U_1} \tag{2}$$

where U_n – root-mean-square noise in the measurement band.

The signal-to-noise ratio (SNR) is a non-dimensional parameter [3]:

$$SNR = \frac{P_{signal}}{P_{noise}} = \left(\frac{A_{signal}}{A_{noise}} \right)^2 \tag{3}$$

where A – appropriate amplitudes of the deterministic signal and noise.

In some cases it is necessary to determine the minimum and maximum value of signals within the interval of observations. Statistical indicators of various signals change in time. The time interval of signal observation is referred to as the window. The window may be displaced in a stepped or sliding manner in relation to the input signal. Based on the foregoing it is possible to determine local parameters describing the signal (average values, local minima and maxima). The identified component is referred to as the trend and can be used to calculate the peaks and drops of signal edges [4].

Methods used in the frequency analysis of random disturbances use the balance of harmonic waveforms in systems with oscillations. Such methods are useful in the mapping of poorly or relatively non-linear circuits.

The method of noise analysis in the time domain [5] enables the simulation of strongly non-linear circuits. Also in the above-named case it is advisable to assume the slight effect of noise.

Parameters describing random signals are expressed by the formulas presented below.

It is assumed that $x_k(t)$ is the k-th realisation of a process creating a set of random functions

$\{x(t)\}$. A function of probability density is referred to as a limit indicating that the probability of the momentary value of random signal $x_1(t)$, restricted within the range of $(x_1(t_1)$ to $x_1(t_1) + \Delta x)$, amounts to:

$$p(x_1,t_1) = \lim_{\Delta x \to 0} \frac{P(x_1(t_1) < x(t) < x_1(t_1) + \Delta x)}{\Delta x} \qquad (4)$$

The average value (expected value) is expressed by the following formula [6]:

$$M_{av}(t_1) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x_k(t_1) \qquad (5)$$

where t_1 – selected time instant, $x_k(t_1)$ – value of the random function of signal x_k in time instant t_1, k – number of random function (realisation) and N – number of realisations.

The root-mean-square value expresses the following dependence:

$$M_{x2}(k) = \lim_{T \to \infty} \frac{1}{T} \sum_{k=1}^{N} x_k^2(t) \qquad (6)$$

The variance (dispersion) is expressed by the following formula:

$$D_x(t_1) = \sigma_x^2(t_1) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} (x_k(t_1) - M_{av}(t_1))^2 \qquad (7)$$

where $\sigma_x(t_1) = \sqrt{D_x(t_1)}$ – mean square deviation.

The function of autocorrelation [6] constitutes a limit:

$$R_x(t_1, t_1 + \tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x_k(t_1) \cdot x_k(t_1 + \tau) \qquad (8)$$

where τ – value of time shift.

If the average value of random signal $M_x(t_1)$ and the function of autocorrelation $R_x(t_1, t_1 + \tau)$ change along with the change of time instant t_1, such a signal is **non-stationary**. The examination of the system requires a large number of realisations (which is technically difficult). The random non-stationary signal is characterised by the multidimensional distribution law $p_N(x_1,t_1; x_2,t_2;...; x_N,t_N)$.

A signal is **stationary** if the N-dimensional probability density does not depend on the selection of time instant, but only on the size of time intervals of signal: $M_x(t) = M_x = const$, $D_x(t) = D_x = const$, $\sigma_x(t) = \sigma_x = const$. Using these features, the above-named correlations can be expressed in new forms:

- mean value (expected value):

$$M_x = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)dt \qquad (9)$$

- root-mean-square value:

$$M_x = \lim_{T \to \infty} \frac{1}{T} \int_0^T x^2(t)dt \qquad (10)$$

- variance (dispersion):

$$D_x = \sigma_x^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T (x(t) - M_x)^2 dt \qquad (11)$$

where $\sigma_x = \sqrt{D_x}$ – mean square deviation.

The function of the autocorrelation of signal x(t) [6] is the following:

$$R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) \cdot x(t + \tau)dt \qquad (12)$$

The above-named function reaches a maximum at point $\tau = 0$ and is even $R_x(-\tau) = R_x(\tau)$.

The condition of signal ergodicity can be expressed as follows:

$$\lim_{\tau \to \infty} R_x(\tau) = 0 \qquad (13)$$

The degree of ergodicity b_{per} is determined using the following formula:

$$R_x(\tau_{max}) < b_{per} \qquad (14)$$

The correlation coefficient can be defined as follows:

$$k_x(\tau)=\frac{R_x(\tau)}{R_x(0)} \tag{15}$$

One of more important energy characteristics of signal is power spectral density. Power spectral density can be calculated from autocorrelation function $R_x(\tau)$ using the direct Fourier transform:

$$S_x(f)=\int_{-\infty}^{+\infty} R_x(\tau)\cdot\cos(\omega\tau)d\tau \tag{16}$$

where pulsation $\omega = 2\pi f$. Including the parity of the autocorrelation function it is possible to use the following formula:

$$S_x(f)=2\int_{0}^{+\infty} R_x(\tau)\cdot\cos(\omega\tau)d\tau \tag{17}$$

The above-named function makes it possible to identify the power of individual spectrum components. Noise power spectral density at the system output is calculated as the superposition of input power density of various sources of noise.

In cases necessitating the more detailed investigation of the properties of deterministic signal in the presence of noise, methods enabling an increase in the signal-to-noise ratio include:

a) use of filtration,

b) averaging.

Generation of noises in arc devices

Noises are divided into statistically stationary and non-stationary. In statistically stationary processes, the value of signal does not depend on time, thus making the ergodic hypothesis correct. In accordance with the aforesaid hypothesis, measured values of macroscopic parameters are equal to values of microscopic parameters averaged after time. However, they are equal to average values, calculated in relation to operating conditions of the system.

In arc models it is often necessary to consider the occurrence of noises having white noise or coloured noise.

White noise (also referred to as broadband noise or noise of constant power spectral density) has a correlation time of 0 and a totally flat spectrum, i.e. the same power in a band of preset frequency. Physical systems are nearly never disturbed by white noise, although such noise constitutes the useful theoretical presentation of physical phenomena. In terms of white noise it is assumed that:

$$\xi(t)=\sqrt{2D}n(t) \tag{18}$$

where n(t) – normalised white Gaussian noise ($\langle n(t)\rangle \equiv 0$, $\langle n(t)n(t+\tau)\rangle=\delta(\tau)$ – Dirac function),

D – constant constituting a measure of noise intensity $\xi(t)$. Angle brackets <…> designate the averaging of the entire set of data obtained in experiment.

Presently, white noise generation methods involve the use of digital shift registers with feedback and programmable logic systems. Simulation programs use blocks providing white noise effect. For this purpose, it is necessary to use the random sequence of data with correlation time significantly shorter than the shortest time-constant of the system. The aforesaid sequence is generated by the white noise block of the limited band.

Coloured noise has the non-flat frequency and the finite value of correlations. It is the discrete analogue of the Ornstein-Uhlenbeck process with the exponential damping of the correlation function. It can be regarded as the result of the filtration of white noise of normal distribution [7].

Pink noise has a down-slope spectrum. In such a case, power spectral density is inversely proportional to frequency in a band under consideration. Then, power is constant for each octave of the band. The spectrum of red noise (Brown noise) falls inversely proportionally to squared frequency.

Coloured noise can be obtained by the appropriate filtration of white noise. It is assumed that:

$$\frac{d\xi}{dt} = f(\xi,t) + noise \tag{19}$$

Reference publications present examples of simple low-pass filters.

An example can be the filter expressed by the following equation [7, 8]:

$$\frac{d\xi}{dt} = -\frac{1}{\tau_c}\xi + \frac{\sqrt{2D}}{\tau_c}n(t) \tag{20}$$

where D – parameter defining noise intensity and τ_c – coloured noise correlation time.

In asymptotic limit $\tau_c \to 0$, process $\xi(t)$ is the stationary Gaussian process, with stationary probability density [7].

Another case of the low-pass filter is described by Langevin equation [9]:

$$\frac{d\xi}{dt} = \eta \tag{21}$$

$$\frac{d\eta}{dt} = -\gamma\eta - \omega_1^2\xi + \sqrt{2D}\cdot n(t) \tag{22}$$

It is, in fact, a line oscillator excited by additive white Gaussian noise $n(t)$. The oscillator has resonant frequency ω_1 and is characterised by damping γ. As a result of filtration, the output signal represents coloured Gaussian noise $\xi(t)$. If damping is weak, $\xi(t)$ is narrowband ("harmonic") noise, used as the signal of parametric modulation. The spectral density of the noise is similar to the Lorentzian function with the maximum corresponding to resonant frequency ω_1 and spectral width $\Delta\omega = \gamma$ (at the half-power level). The dispersion of the process [9] amounts to:

$$\sigma_\xi^2 = \frac{D}{\gamma\omega_1^2}$$

The introduction of disturbance into a selected parameter is performed using a simple dependence:

$$P_{a\xi} = P_a(1+K\xi(t)) \tag{23}$$

where P_a – parameter of the undisturbed system, $\xi(t)$ – disturbance signal and K – non-dimensional coefficient controlling the amplitude of disturbance signal ($0 \le \xi < 1$) [9].

If sources of noise are not correlated, their amplitudes do not sum up arithmetically. In the case of two sources of noise there is relationship $V_{res} \neq V_A + V_B$. The amplitude of resultant noise can be defined as the square root of the sum of squares of amplitudes of individual noises. The aforesaid value is also referred to as the root sum square (RSS). For instance, the amplitude of the sum of three sources of noise amounts to:

$$V_{rss} = \sqrt{V_A^2 + V_B^2 + V_C^2} \tag{24}$$

If amplitudes of one noise are higher than amplitudes of remaining noises, the remaining noises can be ignored. If the amplitudes of noises n_n are equal to V_A, then:

$$V_{rss} = \sqrt{n_n}V_A \tag{25}$$

Random disturbances in the near-electrode areas of electric arc

Random disturbances in near-electrode areas accompany arc discharges in states assumed as stationary or non-stationary. During the modelling of electric arc it is usually assumed that the material structure of electrodes is homogenous. In fact, there are several factors making the electrode material structure heterogeneous. The aforesaid factors include technological processes connected with electrode production, the selection of the design of electrodes and plasma devices, changes of electrode material properties resulting from the time of their operation and the use of electrodes in various technological operations.

Displacements of electrode spots can take place in a forced or natural manner.

Forced displacements are caused by:
a) electrode movements,
b) movements of the near-electrode area of the arc column,
c) electrode and arc column movements.

Factors triggering movements of the near electrode area of the arc column can be of mechanical, electromagnetic or optical nature. Figures 1 and 2 present examples of electrode spot displacements caused by electrode movements.

Figure 3 presents exemplary movements of electrode spots triggered by movements of arc columns, which, in turn, can be induced by magnetic fields and gas flows.

Changes of arc voltage and current can also trigger displacements of arc spots. Higher voltage corresponds to the greater length of the column and, under certain conditions, also displacements of arc spots. In turn, higher current is responsible for increased areas of electrode spots and more intense interaction between the column plasma and the external magnetic field, (which leads to displacements of the column plasma and, consequently, to displacements of arc spots).

The inertia of thermal processes in the electrode depends on the following factors:

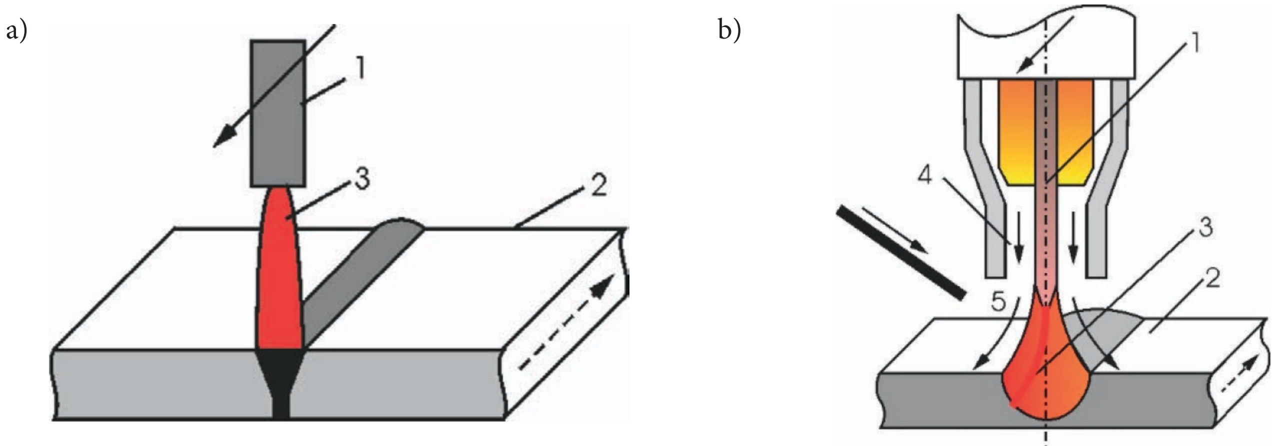

Fig. 1. Translational movements of electrodes triggering movements of electrode spots of free arc: a) in the MMA welding system and b) in the TIG welding system (1, 2 – electrodes (cathode and anode), 3 – electric arc, 4 – inflow of gas (also plasma-forming gas) and 5 – outflow of gas)

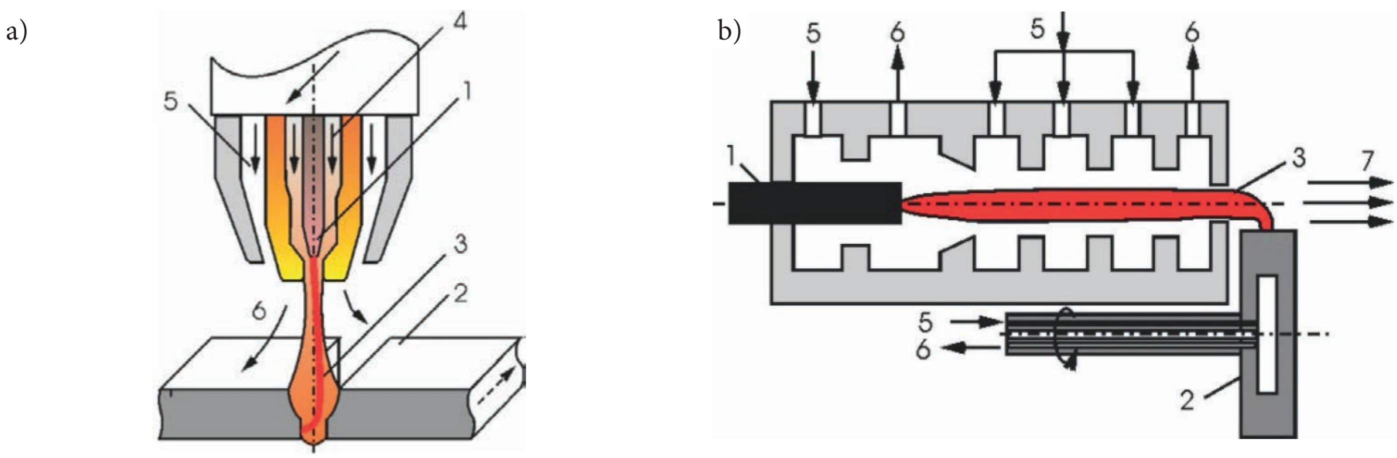

Fig. 2. Movements of electrodes triggering movements of electrode spots of compressed arc: a) in the plasma cutting system (1 – cathode, 2 – anode, 3 – electric arc, 4 – inflow of plasma-forming gas, 5 – shielding gas and 6 – outflow of gas) and b) in the system with plasma made of water and with the rotating anode (1 – cathode, 2 – anode, 3 – electric arc, 5 – inflow of water, 6 – outflow of water and 7 – ionised gases from steam)

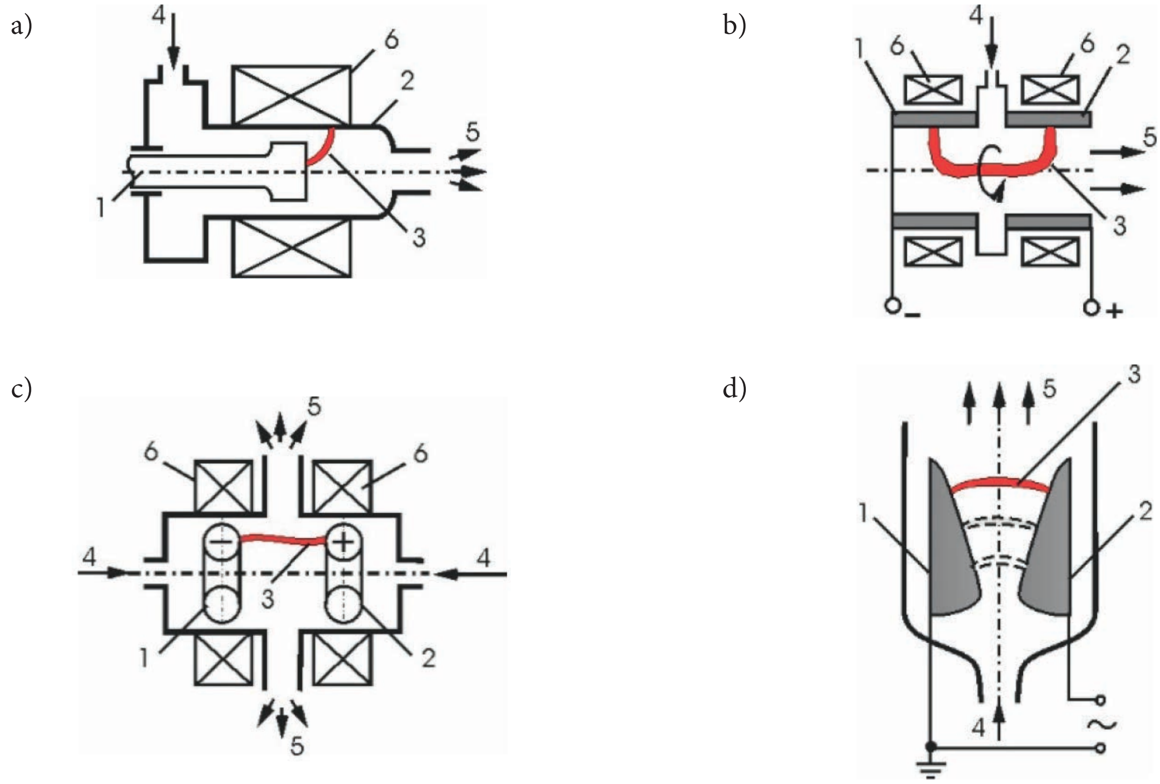

Fig. 3. Movements of the arc column triggering movements of electrode spots in plasmatrons: a) with one cylindrical electrode, b) with two cylindrical electrodes, c) with ring electrodes and e) with divergent electrodes (*gliding arc*) (1 – cathode, 2 – anode, 3 – electric arc, 4 – inflow of cold gas, 5 – outflow of got gas and 6 – electric inductor)

a) electrode material,
b) electrode geometric dimensions (mass),
c) screening, cooling or additional heating of the electrode,
d) velocity of electrode and near-electrode arc area displacement.

The inertia of thermal processes in fixed (immovable) electrodes is usually significantly higher than the inertia of processes in arc plasma. However, very fast movements of electrodes or of the plasma column may trigger random disturbances characterised by arc voltage spectra within wide frequency ranges. There are also possible effects of voltage disturbances resulting from high-frequency laser impulses affecting the arc spot and its surroundings.

Related examples show that changes in values of near-electrode voltage drops may have both deterministic and stochastic features.

The anodic arc area has a large range and smaller voltage gradient in comparison with the cathodic area. Voltage drop in the anodic area (U_A) results from the extraction of electrons from the plasma column and the acceleration of these electrons until they enter the surface of the anode. Because of a small number of negatively charged ions (characterised by lower velocity than that of electrons), the anodic area is primarily dominated by electron current. The striking of an electron against the anode surface provides not only the influx of kinetic energy but also energy equal to work function. The foregoing combined with radiation heat makes the temperature of the anode always higher than that of the cathode.

Voltage U_A depends on conditions affecting the creation of positive ions and their diffusion to the column. For this reason, U_A depends on the geometry of the discharge area and of the anode as well as on current and the composition and pressure of gas. Because of this, U_A may be both positive and negative. If the dimensions of the anode are small, U_A is usually positive. The value of anode voltage drop in welding arc is usually restricted within the range of 5.5 V to 6.5 V.

An increase in current density is accompanied by an increase in the temperature of the near-cathode area. The separation of electrons from the surface of the cathode spot is triggered primarily by thermionic emission and autoemission. Lost energy is comparable with energy received from the column by means of the stream of positively charged ions [10].

The value of near-cathode voltage drop depends (U_C) on the type of gas (gas ionisation potential), the material and shape of the cathode as well as the condition of the cathode surface. Voltage does not depend on the distance between electrodes or changes of discharge current values within a wide range. Depending on electric arc burning conditions, authors of technical studies provide various ranges of U_C values. The foregoing is affected by the concentration of metal vapours. According to publication [10], the value of U_C is restricted within the range of 10 V to 16 V. In turn, according to publication [11] the value of U_C in welding arc is restricted within the range of 5 V to 21 V. A decrease in gas pressure leads to a significant increase in the above-named voltage. Authors often provide the total value of voltage drop U_{AC}. During CO_2-shielded metal arc welding, U_{AC} is restricted within the range of 17 V to 19 V. In turn, during Ar-shielded metal arc welding, U_{AC} is restricted within the range of 16 V to 18 V.

In engineering calculations, including or ignoring near-electrode voltage drops depends on the length of the column, i.e. on resultant voltage applied to arc. Because of the fact that in manual welding machines arc is usually short (and low-voltage), it is therefore necessary to take into account voltage U_{AC}. In turn, because of the fact that in other high-voltage electrotechnical and electrical power devices arc is long (and high-voltage), near-electrode voltage drops are usually ignored. Such an approach is also adopted to noise accompanying near-electrode voltage drops.

Because of quasi-constant values of near-electrode voltage drops, the macromodelling of arc usually involves the mapping of arc using uncontrolled voltage sources (regardless of AC or DC power supply). In the latter case, depending on physical properties of electrodes and changes of current flow directions, the sum of voltage drops may be symmetric or asymmetric. In the symmetric case, arc voltage amounts to:

$$u_a(i) = u_{\text{col}}(i) + u_{A\xi}\,\text{sgn}(i) \tag{26}$$

where u_{col} – column voltage,
$u_{A\xi} = (1 + \xi(t))U_{AC}$ – sum of voltages (along with noise) in near-electrode areas and $\xi(t)$ – non-dimensional stochastic process. In the asymmetric case, the value of voltage w amounts to:

$$u_a(i) = u_{col}(i) + \\ +0.5[u_{A\xi 1}(\text{sgn}(i) - 1) + u_{A\xi 1}(\text{sgn}(i) + 1)] \tag{27}$$

where $u_{A\xi 1} = (1 + \xi_1(t))U_{AC1}$, $\xi_1(t)$,
$u_{A\xi 2} = (1 + \xi_2(t))U_{AC2}$, $\xi_1(t)$
$\xi_2(t)$ – non-dimensional stochastic processes, $U_{AC} > 0$ V, $U_{AC1} > 0$ V, $U_{AC2} > 0$ V.

Mayr-Voronin model of the low-current arc column

The Voronin [12, 13] model involves the following reduction assumptions: i) arc column is cylindrical, ii) plasma is homogenous in terms of the cross-section and arc axis, iii) heat is dissipated only from the side surface of arc and iv) arc column length can change in time.

The basis enabling the creation of a mathematical model of arc involves the introduction of the simplified equation of plasma column thermal balance:

$$\frac{dQ}{dt} = P_{el} - P_{dis} = u_{col} i - P_{dis} \tag{28}$$

where Q – plasma enthalpy, J, P_{el} – electric power supplied to the column (W), P_{dis} – thermal power dissipated from the arc column (W) and u_{col} – arc voltage (V).

The conductance of arc is the function of arc enthalpy $g = F(Q)$. The adoption of one of the assumptions of the Mayr model [14] leads to the obtainment of:

$$\sigma = \sigma_{0M} \cdot \exp\left(\frac{q_V}{q_{0M}}\right) \tag{29}$$

where σ_{oM} and q_{oM} – coefficients of the approximation of the plasma conductivity function. Because of the above-named assumption, the model will appropriately approximate low-current arc characteristics. In such a case, the equation of the Mayr-Voronin (MV) mathematical model of arc with changeable length $l(t)$ and cross-section $S(t)$ has the following form:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{Ms}(S)}\left(\frac{u_{col}i}{P_{Ms}(l,S)} - 1\right) - \frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \\ + \frac{1}{S}\frac{dS}{dt}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right) \tag{30}$$

where

- damping function (s):

$$\theta_{Ms}(S) = \frac{q_{0M}}{p_S}\sqrt{\frac{S}{4\pi}} \tag{31}$$

- function of dissipated power (W):

$$P_{Ms}(l,S) = p_S l\sqrt{4\pi S} \tag{32}$$

The designations are as follows:

$$Q = q_V ls \tag{33}$$

$$g = \frac{\sigma S}{l} \tag{34}$$

$$P_{dis} = P_{Ms} = p_S l\sqrt{4\pi S} \tag{35}$$

where q_V – volumetric density of enthalpy (J/m³), g – conductance (S), σ – arc column specific conductivity (S/m), l – arc column length (m), p_S – density of power dissipated by the side surface of the column (W/m²), P_{dis} – power dissipated by the side surface of the column (W), S – cross-sectional area of arc (m²).

If condition (35) is replaced with the assumption, whereby dissipated power is proportional to the volume of the column:

$$P_{dis} = p_V ls \tag{36}$$

the following correlations for auxiliary functions are obtained:

- damping constant (s):

$$\theta_{Mv} = \frac{q_{0M}}{p_V} = const \tag{37}$$

- function of dissipated power (W):

$$P_{Mv}(l,S) = p_V lS \tag{38}$$

where p_V – volumetric density of dissipated power (W/m³). In equation (30), only designations of auxiliary functions are subject to change.

The numerical integration of differential equation (30) enables the calculation of values of plasma column conductance. The division of the local value of current by conductance leads to the obtainment of $u = i/g$, i.e. voltage of controlled source (constituting the load of the circuit).

The modelling of arc of selected electrotechnical devices sometimes involves the integral forms of mathematical model equations. In the general case, the integral equivalent to equation (30) is the following:

$$g = g_{oM} \times \\ \times \exp\left\{\int_0^t \left[\frac{1}{\theta_{Ms}(S)}\left(\frac{u_{col}i}{P_{Ms}(l,S)} - 1\right) - \frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \\ + \frac{1}{S}\frac{dS}{dt}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right)\right] d\tau\right\} \tag{39}$$

The calculation of integral (39) makes it possible to obtain the value of plasma column conductance. The multiplication of the local value of voltage by the value of conductance makes it possible to obtain the value of controlled source current $i = gu$ (constituting the load in the circuit).

Cassie-Voronin model of the high-current arc column

Similar to the Voronin model [12], the obtainment of the Cassie-Voronin model of the high-current arc column requires the adoption of reduction assumptions. Also in this case, the model is based on the simplified equation of the energy balance of the arc column (28) and conditions designated as (32)–(35). The dependence of arc conductance on plasma enthalpy $g = F(Q)$ is consistent with the assumption of the Cassie model [14]:

$$\sigma = \sigma_{0C} \cdot \frac{q_V}{q_{0C}} \tag{40}$$

where σ_{oC} and q_{oC} – coefficients of the approximation of the plasma conductivity function. Because of the above-named assumption, the model will appropriately approximate high-current arc characteristics. The equation of the Cassie-Voronin mathematical model of arc with changeable dimensions of $S(t)$ and $l(t)$ has the following form:

$$\frac{1}{g}\frac{dg}{dt} = \frac{\sigma_{0C}}{q_{0C} g l^2}\left(u_{col} i - P_{dis}(l,S)\right) - \frac{2}{l}\frac{dl}{dt} \tag{41}$$

After taking into account equation (35) and related transformations, the following equation is obtained:

$$\frac{1}{g}\frac{dg}{dt} = \frac{\sigma_{0C} p_S \sqrt{4\pi S}}{q_{0C} g l}\left(\frac{u_{col} i}{p_S l \sqrt{4\pi S}} - 1\right) - \frac{2}{l}\frac{dl}{dt} \tag{42}$$

After the introduction of new designations, the final form of the new Cassie-Voronin model is the following:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{Cs}(S)}\left(\frac{u_{col}^2}{U_{Cs}^2(l,S)} - 1\right) - \frac{2}{l}\frac{dl}{dt} \tag{43}$$

where the damping function (s):

$$\theta_{Cs}(S) = \frac{q_V}{p_S}\sqrt{\frac{S}{4\pi}} \tag{44}$$

function of squared voltage on the arc column (V²):

$$U_{Cs}^2(l,S) = E_{Cs}^2(S) \cdot l^2 \tag{45}$$

where E_{Cs} – electric field intensity in the arc column (V/m).

The replacement of condition (35) with condition (36) leads to the obtainment of the following damping function independent of S (instead of obtaining (44)):

$$\theta_{Cv} = \frac{q_V}{p_V} = const \tag{46}$$

In turn, the function of squared voltage on the arc column adopts the following form:

$$U_{Cv}^2(l) = E_{Cv}^2 l^2 \tag{47}$$

where E_{Cv} – electric field intensity in the arc column (V/m).

In the general case, the integral form of the model, equivalent to (43), is as follows:

$$g = g_{0M} \exp\left\{\int_0^t \left[\frac{1}{\theta_C(S)}\left(\frac{u_{col}^2}{U_C^2(l,S)} - 1\right) - \frac{2}{l}\frac{dl}{dt}\right] d\tau\right\} \tag{48}$$

where formula (35) corresponds to $\theta_C = \theta_{Cs}$ and $U_C = U_{Cs}$, whereas formula (36) corresponds to $\theta_C = \theta_{Cv}$ and $U_C = U_{Cv}$.

Introduction of random disturbances into geometrical dimensions of the arc column

The gradient of voltage in the plasma column surrounded by gas of atmospheric pressure is expressed by the following dependence:

$$E = \frac{K_C}{I^{1/3}} \quad \text{for} \quad I > I_{\min} > 0 \qquad (49),$$

where K_C depends on the effective ionisation potential of the gas mixture and weight functions of statistical cross-sections of collisions of electrons and ions with atoms [11]. The minimum value of current in welding machines with free arc usually amounts to approximately 20 A. If values of current are lower, arc is unstable and the gradient of voltage grows very fast.

In addition to being influenced by current, the gradient of welding arc voltage is affected by:

a) type of plasma-forming gas,
b) concentration of metal vapours,
c) electrode cross-section diameter.

In metal arc welding, the gradient of arc voltage depends (to a little extent) on the type of shielding gas applied in the process:

a) in the atmosphere of
CO_2 – E_{CO_2} = 2.4–2.8 V/mm,
b) in the atmosphere of
Ar – E_{Ar} = 2.2–2.4 V/mm.

The reason for the above-presented situation is the relatively high concentration of metal vapours.

In accordance with the channel model of arc, the radius of the column can be calculated using the following formula:

$$r_c(I) = K_r I^{2/3} \quad \text{for} \quad I > I_{\min} > 0 \qquad (50)$$

where K_r depends on the effective ionisation potential of the gas mixture and weight functions of statistical cross-sections of collisions of electrons and ions with atoms [11].

The application of formulas (30) and (39) in the modelling of the column poses certain difficulties connected with formula (50) in cases where arc is powered by alternating current. The reduction of current to zero is accompanied by a decrease in the column diameter. In such a case, damping functions (31) also decrease (instead of increasing) [15]. For this reason, it is suggested to approximate the radius of the column using the following simple correlation:

$$r_{col} = r_0\exp(-k_i i^2) + r_c(i) \qquad (51)$$

where $k_i > 0$. It is possible to use more precise approximation through a more complex non-linear function [14]:

$$r_{ü} = r_0\varepsilon_{ṙ}(i) + r_c(i)\cdot(1-\varepsilon_{ṙ}(i)) \qquad (52)$$

where ε_{ri} – tapering function which can adopt the following form:

$$\varepsilon_{ṙ}(i) = ü \left(h(k_r)\frac{i^2}{I_{r0}^2} \right) \qquad (53)$$

where r_0 – preset constant value of the radius ($r_0 > r_c(i)$), k_r – coefficient of the fraction of individual components r_0 and r_c in the function of the radius at the point with abscissa I_{r0}, ($0 < k_r < 1$), I_{r0} – value of current corresponding to the minimum value of the damping function (determined experimentally), e.g. 20 A [15].

The application of formulas (43) and (48) could be preferred when modelling the high-current arc column. In such a case, changes of arc conductance are affected by the cross-sectional area of the plasma column as the dependence of plasma column conductance on high temperature is restricted primarily within the range of saturation. In turn, low-current arc is characterised by significant changes of plasma temperature (responsible for changes of its conductivity), therefore it can be assumed that the value of the cross-sectional area of the plasma column is constant and low.

It is possible to obtain the stochastic process accompanying changes of the plasma column length $l(t)$ by introducing additional non-dimensional stochastic disturbance $\xi_l(t)$:

$$l = l_{st} = l(t) + \xi_l(t)l(t) = (1 + \xi_l(t))l(t) \qquad (54)$$

Similarly, it is possible to obtain the stochastic process accompanying changes of the cross-sectional area of the plasma column $S(t)$ by introducing additional non-dimensional stochastic disturbance $\xi_S(t)$:

$$S = S_{st} = S(t) + \xi_S(t)S(t) = (1 + \xi_S(t))S(t) \quad (55)$$

It is also possible to introduce stochastic disturbances of the column length and cross-section. Depending on the design and operating conditions of an electrotechnical device, the effectivity of both disturbances may be comparable or may vary significantly. In the latter case, weak disturbances can be ignored.

Modification of deterministic mathematical models of the arc column through the introduction of disturbances

In order to obtain the more realistic mapping of processes taking place in an electrotechnical device, at least one of the arc model parameters should be described by the stochastic process. The forgoing leads to the formation of a random differential equation. If it is assumed that current excitation is sinusoidal, also in the above-named case it is possible to obtain the process of stochastic changes of voltage.

The (assumed) local thermal balance in the arc column of a constant length is expressed by power balance equation (28). Electric power (supplied) is expressed by the following formula:

$$P_{el} = ui = gu^2 = ri^2 \quad (56)$$

Then, equation (28) can be expressed as follows:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta(g)}\left(\frac{gu^2}{P_{dis}(g)} - 1\right) \quad (57)$$

where g – column conductance, θ – process damping function and P_{dis} – dissipated power. After the adoption of appropriate reduction assumptions [16] from equation (57) it is possible to obtain mathematical models in the conductance or resistance form and with current or voltage excitation:

- Mayr model:

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M} = \frac{g^2u^2}{P_M} \quad (58)$$

or

$$\theta_M \frac{dr}{dt} - r = -\frac{r^2i^2}{P_M} = -\frac{u^2}{P_M} \quad (59)$$

- Cassie model:

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2} = \frac{g^2u^2}{U_C^2} \quad (60)$$

or

$$\theta_C \frac{dr^2}{dt} - r^2 = -\frac{r^4i^2}{U_C^2} = -\frac{r^2u^2}{U_C^2} \quad (61)$$

where $r = 1/g$ – resistance, P_M – power of the Mayr model, θ_M – time constant of the Mayr model, U_C – voltage of the Cassie model, θ_C – time constant of the Cassie model. Because models are dedicated either to low-current arc or to high-current arc with the identified length of the column, in order to satisfy the needs of the wide range of changes of excitation parameters and deterministic disturbances it is sometimes necessary to use the Schwarz model expressed in the following form:

$$\theta(g)\frac{dg}{dt} + g = \frac{i^2}{P_{dis}(g)} = \frac{g^2u^2}{P_{dis}(g)} \quad (62)$$

where approximating functions $\theta(g) = \theta_0 g^\alpha$ and $P_{dis}(g) = Pog^\beta$, α and β – constant parameters.

Some publications concerning the modelling of arc [17, 18] contain examples of the introduction of random disturbances. In such cases, instead of using constant parameters it is possible to use designations of stochastic parameters:

$$P_{st}(t) = P_{dis} + \xi_M(t)P_{dis} = (1 + \xi_M(t))P_{dis} \quad (63)$$

or

$$U_{st}(t) = U_C + \xi_C(t)U_C = (1 + \xi_C(t))U_C \quad (64)$$

where $\xi_M(t)$ and $\xi_C(t)$ – non-dimensional stochastic disturbances.

The Mayr model extended by two stochastic disturbances is referred to as the Sporckmann model [17]:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_0(1+\xi_\tau(t))}\left(\frac{ui}{P_0(1+\xi_p(t))} - 1\right) \quad (65)$$

where ξ_p and ξ_τ – non-dimensional stochastic disturbances.

The power balance equation in the arc column with current $i(t)$ or voltage $u(t)$ excitation enables the obtainment of a channel mathematical model in the form of a differential ordinary non-linear equation [18, 19]:

$$k_1 r_a^n(t) + k_2 r_a(t)\frac{dr_a(t)}{dt} = \frac{k_3 i^2(t)}{r_a^{m+2}(t)} = \frac{r_a^{m+2}u^2(t)}{k_3} \quad (66)$$

where arc radius $r_a(t) > 0$. Model coefficients k_1, k_2 and k_3 adopt positive values.

Parameters m and n belong to set {0, 1, 2} and reflect various operating conditions of arc devices. The value of voltage on the arc column can be calculated using the following dependence:

$$u = \frac{i}{g} = ri = \frac{k_3}{r_a^{m+2}}i \quad (67).$$

Stochastic changes may also take place in relation to coefficients and exponents of the above-presented equation. Publication [18] presents the modelling of coefficient k_3 as the stochastic process:

$$k_{st}(t) = k_3 + \xi_3(t)k_3 = (1 + \xi_3(t))k_3 \quad (68)$$

where $\xi_3(t)$ – non-dimensional stochastic disturbance.

There is a more general method of transforming the deterministic mathematical model of electric arc, making it possible to take into account random disturbances. The dynamic deterministic system defined by the finite number of non-linear differential equations has the following form:

$$\dot{x} = f(x,t) \quad (69)$$

where $f(x,t)$ – sufficiently smooth n-dimensional vector function. In relation to the behaviour of dynamic system (69) affected by random disturbances it is possible to consider the corresponding Ito system of stochastic equations (Ito Kiyosi) [17]:

$$\dot{x} = f(x,t) + \sqrt{2D}\xi(t) \quad (70)$$

where $\xi(t)$ – noise and D – noise intensity. Such a random differential equation is usually examined using the Monte Carlo method.

Modification of the load with deterministic mathematical models of column through the introduction of disturbances into resultant voltage drops

The mapping of stochastic disturbances is concerned not only with models of plasma systems. In terms of resistors it is necessary to take into account thermal and current noises. In relation to semiconductors, it is necessary to take into account shot noise, generation-recombination noise, $1/f$ type noise and telegraphic noise. In accordance with the Nyquist theorem [20, 21], the thermal noise of resistor $R = 1/G$ at absolute temperature T can be represented by the current source of white Gaussian noise connected in parallel to conductance G or by the voltage source of white Gaussian noise connected in series to resistance R (Fig. 4a,b).

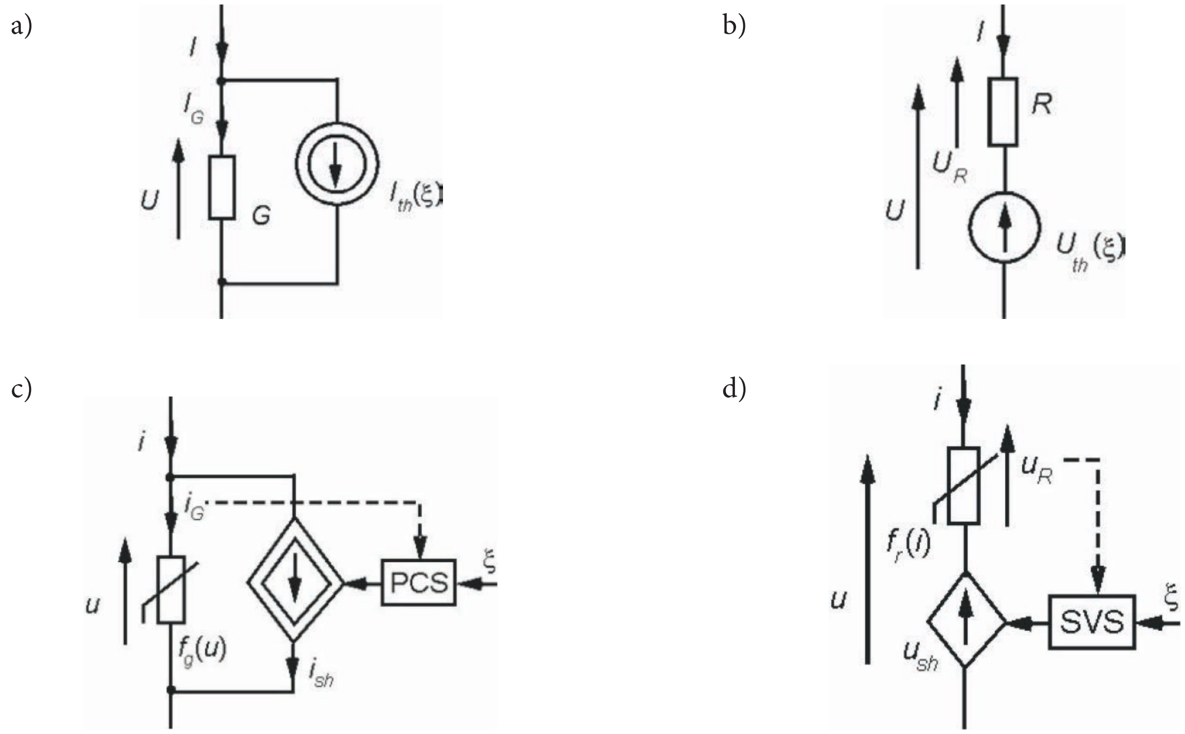

Fig. 4. Circuital mapping of noise generated in: a) and b) linear resistor and c) and d) non-linear resistor (PCS block performs mathematical operation expressed by formula (75), whereas SVS block performs operation expressed by formula (76))

Spectral densities of such sources of noise are the following:

$$S_i(f) = 2kTG \quad (71)$$

or

$$S_u(f) = 2kTR \quad (72).$$

Then, it is possible to identify the parameters of additional sources of noise:

$$U_{th} = \sigma_{Uth} \cdot \xi(t) = \sqrt{S_u(f)} \cdot \xi(t) \quad (73)$$

or

$$U_{th} = \sigma_{Uth} \cdot \xi(t) = \sqrt{S_u(f)} \cdot \xi(t) \quad (74)$$

The resistance element can be non-linear and have characteristic expressed in the form of function $I = f_g(U)$ or $U = f_r(I)$. These correlations can be expressed algebraically or analytically. In the first case, they will be static characteristics, whereas in the latter – differential or integral equations.

If an element is non-linear, the intensity of thermal noise depends on its characteristic (Fig. 4c,d) [22]:

$$i_{sh}(t, u(t)) = \sqrt{q_u |f_g(u)|} \cdot \xi(t) \quad (75)$$

or

$$u_{sh}(t, i(t)) = \sqrt{q_i |f_r(i)|} \cdot \xi(t) \quad (76)$$

where q_u and q_i – constant coefficients.

In terms of the parallel connection of elements, the resultant load of the circuit is expressed by the following formula [23]:

$$i(t) = i_G + i_{sh}(t, u(t)) \quad (77)$$

In turn, in the case of the series connection of elements, the resultant load of the circuit is expressed by the following formula:

$$u(t) = u_R + u_{sh}(t, i(t)) \quad (78).$$

Non-linear elements of characteristics $I = f_g(U)$ or $U = f_r(I)$ can be mapped using controlled sources. It should be noted that, because of the non-linearity of the elements, it is not possible to simply use the principle of circuit duality. It was assumed that thermal processes in electric arcs are the primary source of noise. In cases of the mathematical models described by equations (58)–(61), the values of functions $f_g(u)$ and $f_r(i)$ can be calculated by the integration of the equations. In the simplified variant of calculations it is possible to use static characteristics of the arc column.

The column of electric arc is the generator of stochastic disturbances in the circuits as a result of various (internal and external) random factors. Higher voltage (greater column length) increases sensitivity to external gasodynamic and electromagnetic disturbances. A similar effect is triggered by an increase in current, which is connected by an increase in plasma volume and increased forces of electrodynamic effects. Another consequence is the increased intensity of noise in technological processes (welding and melting of metals).

In a manner similar to (75) and (76) it is possible to propose the mapping of the random disturbance of arc models using the following formula:

- Mayr model (with static characteristic $U = P_M/I$):

$$i_{sh}(t,i(u(t))) = \sqrt{K_{Mi} g |u(t)|} \cdot \xi(t) = \sqrt{K_{Mi} \frac{|u(t)|}{r}} \cdot \xi(t) \approx \sqrt{K_{Mi} \frac{P_M}{|u(t)|}} \cdot \xi(t) \quad (79)$$

or

$$u_{sh}(t,u(i(t))) = \sqrt{K_{Mu} \frac{|i(t)|}{g}} \cdot \xi(t) = \sqrt{K_{Mu} r |i(t)|} \cdot \xi(t) \approx \sqrt{K_{Mu} \frac{P_M}{|i(t)|}} \cdot \xi(t) \quad (80)$$

- Cassie model (with static characteristic

$$i_{sh}(t,i(u(t))) = \sqrt{K_{Ci} g |u(t)|} \cdot \xi(t) = \sqrt{K_{Ci} \frac{|u(t)|}{r}} \cdot \xi(t) \approx \sqrt{K_{Ci} g U_C} \cdot \xi(t) = \sqrt{K_{Ci} \frac{U_C}{r}} \cdot \xi(t) \quad (81)$$

or

$$u_{sh}(t,u(i(t))) = \sqrt{K_{Cu} \frac{|i(t)|}{g}} \cdot \xi(t) = \sqrt{K_{Cu} r |i(t)|} \cdot \xi(t) \approx \sqrt{K_{Cu} U_C} \cdot \xi(t) \quad (82)$$

where K_{Mi} and K_{Mu} as well as K_{Ci} and K_{Cu} – constant values, $I \neq f(U_C)$.

The model of the arc column with the radius as state variable (66) has a static characteristic described by the following formula [19]:

$$U = \frac{k}{I^q} \qquad \text{or} \qquad I = \sqrt[q]{\frac{k}{U}} \quad (83)$$

where $q = \frac{m+2-n}{m+2+n}$, $k = k_3 \left(\frac{k_3}{k_1} \right)^{\frac{-m-2}{m+n+2}}$

In a manner similar to (79) and (80) it is possible to propose the mapping of the random disturbance of arc models using the controlled source of current, the value of which is expressed by the following formula:

$$i_{sh}(t,i(u(t))) = \sqrt{K_{ri} \frac{|u(t)| r_a^{m+2}}{k_3}} \cdot \xi(t) \approx \sqrt{K_{ri} \cdot \sqrt[q]{\frac{k}{|u(t)|}}} \cdot \xi(t) \quad (84)$$

or the controlled source of voltage, the value of which is expressed by the following formula:

$$u_{sh}(t,u(i(t))) = \sqrt{K_{ru} \frac{k_3}{r_a^{m+2}} |i(t)|} \cdot \xi(t) \approx \sqrt{K_{ru} \frac{k}{|i(t)|^q}} \cdot \xi(t) \quad (85),$$

where K_{ri} and K_{ru} – constant parameters.

Publication [24] presents the model of arc (66) with static characteristic (83). The macromodel of arc was built using a controlled voltage source with a voltage source (connected in series) representing random disturbance $\xi(t)$ $u(t)$:

$$u_{st} = u(t) + \xi(t)\, u(t) = (1 + \xi(t))u(t) \tag{86}$$

where $\xi(t)$ – non-dimensional stochastic disturbance. In the aforesaid case, disturbance effects do not depend directly on the electric characteristic of arc.

Conclusions

1. Because of the heterogeneous structure of electric arc and differences in mathematical descriptions of the structure components, it is possible to separately take into account stochastic disturbances in the plasma column and in near-electrode areas.
2. The current state of research related to the modelling of electric arc with deterministic disturbances enables the use of three various methods enabling the mapping of stochastic disturbances of the plasma column.
3. The non-linearity of electric arc affects not only deterministic but also stochastic components of signals (which is sometimes ignored).
4. The introduction of stochastic disturbances through the modification of mathematical models may lead to the non-fulfilment of a previously assumed energy balance of arc.

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