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# Influence of quasi-static gas pressure changes on electric arc modelling in electrotechnological devices

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**Abstract:** The article presents exemplary applications of devices with electric arc in gases of varying pressure used in industry practice and variants of the approximation of static arc characteristics. The article also contains an analysis of the influence of pressure changes on an arc damping factor function and presents mathematical arc models with a quasi-statically variable pressure parameter. The tests also involved simulations of processes in a simple circuit with an arc. The article also contains the results of calculations in the form of the evolution of dynamic voltage-current arc characteristics influenced by pressure changes.

**Keywords:** electrical arc, damping factor, time constant, gas pressure, mathematical model

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## Introduction

Although the effects of the vast majority of theoretical deliberations and experimental investigation of the electric arc primarily refer to arc burning in gases of atmospheric pressure, this value is far from being exceptional as regards the mathematical modelling of electrothermal processes. The arc maintains its structure in a wide range of pressure changes starting from  $10^3$  Pa. The use of atmospheric pressure results from the easy implementation of technological processes, adequate efficiency of energy conversions, low manufacturing costs and higher handling safety. However, the external conditions of technological processes or obtained manufacturing results not always allow using such a pressure value.

Electric arc burning in gases of pressure lower than atmospheric is common in normal operating conditions of many devices carrying out various technological processes, e.g. in

plasma furnaces with an arc of a pressure below  $2 \cdot 10^4$  Pa, used for the carbothermal reduction of refractory metals (Nb, Ta, Zr, Ti) [1]. The use of pulse excitation for energising the arc discharge enables the nitration of the surface layer of metals in gas mixtures ( $N_2+H_2$ ) of a very low pressure, even close to the atmospheric one [2]. Lower pressure and arc or plasma heating facilitate the removal of detrimental gas admixtures from a metal bath during refining in extra-furnace (ladle) steel treatment [3]. Plasma welding with an electric arc is recognised as a very attractive method of joining metal elements in outer space. In comparison with laser welding or electron beam welding this method is characterised by significantly lower costs, easy handling and safe operation [4]. It can be implemented in special hypobaric chambers.

Hyperbaric welding is usually imposed by the conditions of repair and renovation of underwater objects and systems. It can be dry,

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i.e. in a special chamber immersed in water, or wet, i.e. directly in the water environment. Dry welding is carried out inside a dry chamber surrounding elements being welded. Such a chamber is usually filled with a mixture of gases having a strictly controlled composition and being under high pressure dependent on an immersion depth. Dry welding, which can be carried out by properly skilled personnel or by robots, is characterised by similar conditions and results as atmospheric welding and may include MMA, TIG and MIG/MAG. Wet welding is carried out without additional equipment and chambers, which provides the welder wearing a special "dry suit" with the necessary freedom of movement. However, such welding does not ensure high quality of work as the rapid cooling of elements and welds saturated with hydrogen and oxygen results in the formation of cracks. In addition, wet welding is not very efficient or precise due to the impaired visibility of the process connected with the significant volume of generated gases. Such welding is carried out by means of special electrodes or wires provided with proper protective coatings. The elevated pressure of gas atmosphere is also used in plasma metallurgy for the deep and uniform nitration of a steel bath [5]. The pressure is also present in gas areas confined by molten slag and charge in arc furnaces (e.g. for the production of ferroalloys).

In gas at atmospheric pressure, the column of a high-current arc consists of a conical (near-cathode) part and a cylindrical part – distant from the cathode. A decrease in pressure does not affect the existence of the conical part, yet the cylindrical part becomes wider. Also the areas of cathode and anode spots grow bigger. This decreases the density of current in the column and electrode spots. At the same time the arc column radiation power diminishes significantly. In turn, the power transferred from the arc to the anode declines only slightly. As the decrease in pressure leads to decreased electric field intensity in the arc

and decreased voltage in the column, it is necessary to change the supply system parameters in a way which would make it possible to provide the electric arc with greater power in order to effectively carry out a technological process. Decreasing gas pressure significantly affects cathode processes. Low pressure facilitates material erosion through the greater evaporation intensity of ionising admixtures from the electrode surface (which increases electronic work function) and facilitates the evaporation of a liquid metal pool. A significant decrease in pressure to the boundary level ( $6 \cdot 10^3$  Pa) leads to a significant increase in temperature and the change of material structure [1]. Below this threshold value it is necessary to prevent rapid erosion intensity by using a hollow (tubular) cathode and supplying plasma-creating gas in the amount of 10-12% of the total gas mass to the cathode duct. The formation of a crater or the use of a tubular cathode enables the plasma torch to operate with greater power and with a significantly longer electrode life than in the case of a rod cathode. Along with decreasing pressure, the arc surface boundary becomes increasingly diffuse, and after exceeding the critical value ( $p < 10^3$  Pa) the discharge becomes diffusive.

In turn, an arc in gas of pressure higher than atmospheric is characterised by a great voltage gradient, which impedes arc ignition and requires changing the external characteristics of a supply source. Elevated pressure leads to the significant compression of the column, which also depends on the type of gas. This phenomenon is more visible with helium than with argon [6]. At the same time the areas of cathode and anode spots decrease. The increased density of current leads to a significant increase in the temperature of plasma and of electrode surface, which, in turn leads to a significant increase in arc column radiation power, commonly used in high-pressure discharge lamps. High pressure affects the geometrical dimensions of a weld pool (the depth of fusion and the

height of the weld projection increase, whilst the seam width decreases). The adsorption of gases increases and so does the simultaneous evaporation of weld admixtures.

As the efficiency of melting, welding, surface treatments etc. depends on the proper association of the values of pressure and that of electric discharge current, mathematical models and computer-aided macromodels are helpful in analysing, designing and controlling such devices. In most electrotechnological devices the character of pressure changes is usually quasi-static. Relatively big volumes of furnace chambers, melting pots and heaters eliminate leap effect of vacuum pumps and gas flow damping systems on the arc.

This work extends the results of research on modelling an electric arc in a gas of reduced pressure [7]. It is assumed that the lower limit ensuring the existence of a thermal equilibrium in plasma is  $10^4$  Pa. This requires maintaining the proper value of electric field intensity and leads to the development of specific assumptions and models used for approximating dynamic states in the circuits of selected electrotechnological devices. In this work the simulation of processes in an electric discharge involved a very wide range of pressure, i.e. from  $10^3$  Pa to several MPa.

### Arc static characteristics in gas of quasi-statically variable pressure

Due to selected technological applications, the experimental determination of arc static characteristics usually includes specific ranges of gaseous atmosphere pressure and of exciting current. For this reason there are separate investigations of arc discharges in gases of reduced, normal  $10^5$  Pa and elevated pressure. Starting from the pressure of  $10^3$  Pa the changes of arc energy characteristics are smooth and the value of  $10^5$  is not exceptional. That is why data diagrams obtained in the neighbourhood of this value can be extrapolated outside an adopted range.

Arc voltage is the function of three variables

$$U=U(l,p,I) \tag{1}$$

where  $l$  – arc column length;  $p$  – plasma pressure;  $I$  – electric field intensity.

However, the precise determination of all these quantities tends to be very difficult, especially in the case of industrial machinery of great power. Therefore, researchers must usually rely on very approximate and easier measurable values such as voltage between electrode terminals, distances between electrodes  $L$ , pressure inside a discharge chamber  $p$ , the root-mean-square AC brought from the input side to the load side of a transformer  $I_2 = I_1 \cdot \vartheta$ , where  $\vartheta$  - transformer voltage ratio. The quasi-static pressure changes significantly affect the shape of static current-voltage characteristics [8] (Fig. 1 a,b). This effect is very similar to that of arc length changes (Fig. 1 c). Also the chemical composition of plasma-creating gas plays a significant role in this case. Reducing pressure greatly affects long arcs (Fig. 1 d).

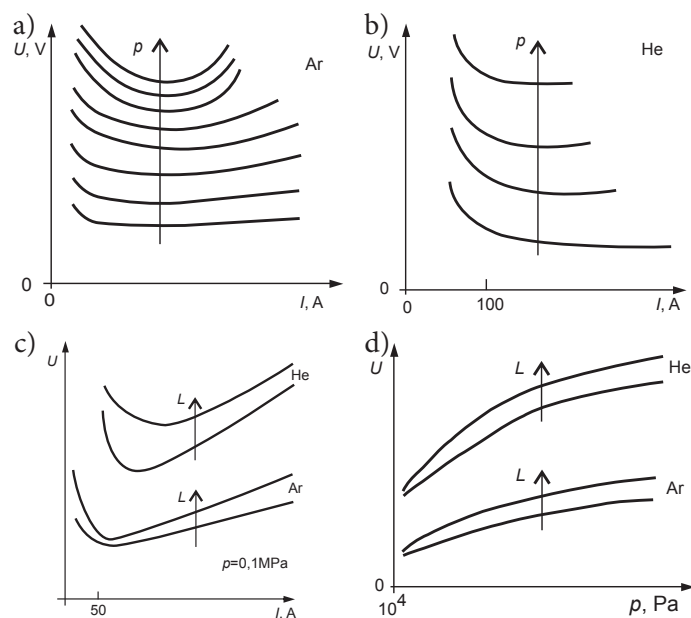


Fig. 1. Effect of pressure and arc column length changes on the families of discharge static characteristics in gases

Experimental tests of an arc in gas of elevated pressure led Hechtfisher to propose an approximation

$$U(L,p,I) = a + \frac{bL^\alpha p^\beta}{I^\gamma} \tag{2}$$

where  $a$  – sum of near-electrode voltage drops;  $b$  – constant coefficient;  $\alpha, \beta, \gamma$  - approximation exponents. On this basis the power-current characteristic (preferred in some arc mathematical models) can be expressed by means of a general formula

$$P(L, p, I) = aI + \frac{bL^\alpha p^\beta}{I^{\gamma-1}} \quad (3)$$

During the creation of arc models, especially of extended ones, particularly important are the families of electric field intensity characteristics. In such case they adopt the following form

$$E(L, p, I) = \frac{\partial U}{\partial L} = \frac{\alpha b L^{\alpha-1} p^\beta}{I^\gamma} \quad (4)$$

The formula (2) and its resultant formulas (3) and (4) are very similar as they cannot approximate a static characteristic in a wide range of current changes, particularly if such a characteristic contains a quasi-linear section.

Other experimental tests of a free high-current arc burning in argon [9] give the following dependences:  $U(I) \propto I$ ,  $U(p) \propto p^{2/3}$ ,  $U(L) \propto L^{2/3}$ . On this basis it is possible to conclude that  $U(Lp) \propto (Lp)^{2/3}$ . In this work the proposed approximation of the current-voltage static characteristics has the following form

$$U(L, p, I) = a(Lp)^{2/3} + bI(Lp)^{2/3} + cIp^{2/3} + dI + e \quad (5)$$

where  $a, b, c, d, e$  – approximation coefficients. The power emitted on the arc is expressed by the formula

$$P(L, p, I) = U \cdot I = [a(Lp)^{2/3} + e] I + [b(Lp)^{2/3} + cIp^{2/3} + d] I^2 \quad (6)$$

In turn, the intensity of an electric field can be calculated using the formula

$$E(L, p, I) = \frac{\partial U}{\partial L} = \frac{2}{3}(a + bI)L^{-1/3} p^{2/3} \quad (7)$$

The parameters  $a, b, c, d$ , and  $e$  depend on the type of gas and cathode (rod or hollow). Along with decreasing pressure, the efficiency of rod (pointed) cathode decreases and the use a hollow

cathode becomes advisable. The characteristic (5) includes only the linear range of current changes. Tests related to an arc in a low current range indicate [10] that the electric field intensity diagram can be approximated using the function

$$E = f \cdot \left(\frac{I}{1A}\right)^{-m} \cdot \left(\frac{p}{1Pa}\right)^n, \frac{V}{cm} \quad (8)$$

where  $f$  – constant dependent on the type of gas,  $V/cm$ ;  $p$  – pressure, Pa;  $I$  – current.

On the basis of data contained in [11] it was possible to calculate the values of these coefficients (see Table 1). As can be seen in the formula, an increase in plasma pressure is accompanied by a significant increase in field intensity. It is also possible to observe an increase in the arc re-ignition value. Due to strongly unstable discharges, the parameters of the dependence (8) are characterised by a significant scatter of values. Knowing the distance between the electrodes  $L$  it is possible to calculate the voltage on the arc column  $U_{stat} = E \cdot L$ .

A relatively accurate representation of processes in the whole range of high AC arc excitation can be obtained by associating the static characteristics (5) and (8), which leads to the expression

$$U(L, p, I) = a(Lp)^{2/3} + bI(Lp)^{2/3} + cIp^{2/3} + dI + e + fL \frac{p^n}{I^m} \quad (9)$$

After assuming a constant arc length  $L = \text{const.}$ , it is possible to use the approximation of the static characteristic with a simpler dependence

$$U(p, I) = a' \cdot \left(\frac{p}{1Pa}\right)^{2/3} + b' \left(\frac{I}{1A}\right) \cdot \left(\frac{p}{1Pa}\right)^{2/3} + d \left(\frac{I}{1A}\right) + e + f' \cdot \left(\frac{I}{1A}\right)^{-m} \left(\frac{p}{1Pa}\right)^n \quad (10)$$

where  $a' = aL^{2/3}$ ,  $b' = bL^{2/3} + c$ ,  $d, f' = fL$  have the dimension V.

The approximation of arc current-voltage static characteristics in gas with quasi-statically variable pressure can also be obtained using a known formula presented by Nottingham

Table 1. Values of approximation coefficients in the formula (8)

Approximation parameters	Environment						
	Air	He	Ar	N <sub>2</sub>	CO <sub>2</sub>	H <sub>2</sub> O	H <sub>2</sub>
f	2.26	16.00	3.00	2.85	100·10 <sup>-5n</sup>	230·10 <sup>-5n</sup>	10.05
m	0.6	0.73	0.54	0.6	0.6	0.59	0.7
n	0.31	0.2	0.2	0.3	-	-	0.32

$$U_a = a_0 + a_1L + \frac{b_0 + b_1L}{I^n} \quad (11)$$

where  $a_0$  – sum of near-electrode voltage drops;  $a_1$  – voltage gradient;  $b_0, b_1,$  - approximation coefficients;  $n$  – approximation exponent. The modification of this formula consists in such a variation of coefficients that they constitute the functions of two variables

$$U_a(I, L, p) = \alpha(L, p) + \frac{\beta(L, p)}{I^n} \quad (12)$$

Their expansion into a series around  $L=0$  m and  $p=0$  Pa gives

$$\alpha(L, p) = a_0 + a_1L + a_2p + a_3Lp + \dots \quad (13)$$

$$\beta(L, p) = b_0 + b_1L + b_2p + b_3Lp + \dots \quad (14)$$

By getting limiting to linear terms it is possible to obtain [12]

$$U(I, L, p) = a_0 + a_1L + a_2p + a_3Lp + \frac{b_0 + b_1L + b_2p + b_3Lp}{I^n} \quad (15)$$

### Electric arc damping factor function in gas of quasi-statically variable pressure

The amount of internal energy accumulated in an arc depends inter alia on the pressure of plasma-creating gas. Therefore, the finite times of reactions to the excited changes of thermal states also depend on pressure [13]. This can be expressed by means of a dependence by associating a damping factor with the cross-section area of a plasma column

$$\theta(p, T) \propto \frac{\rho(p, T) \cdot c_p(p, T)}{\lambda(p, T)} r_a^2(p, T) \quad (16)$$

where  $\rho$  - gas density;  $c_p$  – specific heat of gas of

pressure  $p$ ;  $\lambda$  - gas heat conduction coefficient;  $r_a$  – arc column radius.

As can be seen, these quantities also depend on the temperature  $T$ , which in turn, usually depends on discharge current

$\theta(p, I)$ . In atmospheric pressure conditions low current values (below 20 A) trigger the attenuation of the column contraction effect and the expansion of the plasma area (Fig. 2a). Particularly significant arc diameter changes take place in the low pressure range, which also affects the damping factor. An increase in pressure above normal intensifies this phenomenon even stronger (Fig. 2b). In the range of high current and high pressure the changes of the damping factor are only slight.

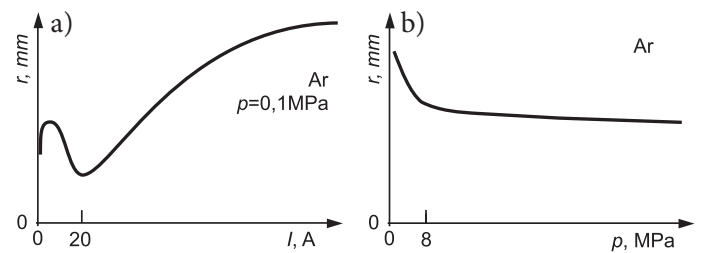


Fig. 2. Changes of the arc column radius affected by the changes of a) electric field intensity [14]; b) gas atmosphere pressure [8]

The dependence  $\theta(I)$  is a decreasing function with a local minimum [14, 15]. In order to approximate the single diagram presented Figure 3a the following formula can be used [16]

$$\theta(i) = \theta_0 + \theta_1 \cdot \exp(-\alpha_i |i|) \quad (17)$$

where approximation parameters  $\theta_0, \theta_1, \alpha_i$  depend on pressure  $p$  and the type of gas.

Advantageous approximation properties, particularly in a low current range, can also be found in the dependence  $\theta(i) = \theta_0 + \theta_1 \cdot \exp(-\alpha_i i^2)$ . The experimental tests of a free arc burning in various gases (nitrogen, air, CO<sub>2</sub>, SF<sub>6</sub>) [17] revealed that the dependence  $\theta(p)$  is a monotonically increasing function (Fig. 3b). A strong effect of the pressure value  $p$  on the coefficient  $\theta$  can be observed in the case of arcs in nitrogen and air.

In turn, arcs in CO<sub>2</sub>, and particularly in SF<sub>6</sub>, are not poorly sensitive to pressure changes. A decrease in pressure significantly below atmospheric is accompanied by a sharp decrease in the time constant, which can be ascribed to the double-layer structure of the arc column [15, 18]. Initially viscous plasma in the surface layer of a high pressure arc is diffused along with decreasing pressure, as is the case with the diffusive state of a thermally-insulated high-current arc. A decrease in pressure is also accompanied by a deterioration in the conditions of heat dissipation, which leads to the improvement of arc burning stability due to the linearization of arc characteristics. However, a further decrease in pressure has an inverse effect and, in accordance with Paschen curves, facilitates the suppression of an arc. Decreasing damping factor values are also affected by the stream of gas mass washing around the arc column and on the temperature of the gas environment [19, 20].

As a decrease in pressure particularly strongly affects the column of a low-current arc, in the first instance it is possible to use a variation of the coefficient  $\theta_i$  and suggest the following approximation

$$\theta(i, p) = \theta_0 + \theta_1 \cdot \exp\left(-\frac{\beta_i}{p}\right) \cdot \exp(-\alpha_i |i|) \quad (18)$$

where  $\beta_i$  – constant approximation coefficient.

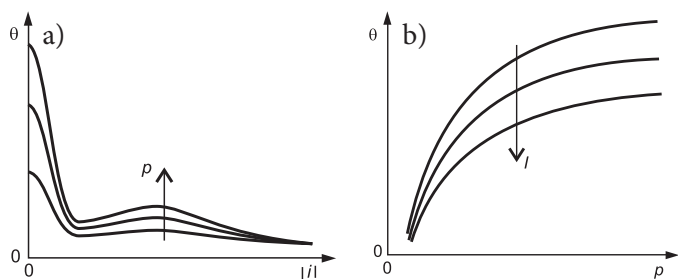


Fig. 3. Families of electric arc damping function diagrams as dependences on a) electric current  $\theta(I, p)$ ; b) plasma-creating gas pressure  $\theta(p, I)$

### Models of an electric arc in gas of quasi-statically variable pressure

For the purpose of these deliberations it is necessary to treat pressure as a quasi-statically variable parameter ( $p \approx \text{var}$ ) and adopt a constant

arc length  $l = \text{const}$ . If discharge current is low, process simulations require the use of the modified Mayr model in the conductance form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta(i(t), p)} \left[ \frac{P_{kol}(t)}{P_{dys}(t, p)} - 1 \right] \quad (19)$$

where  $g$  – column conductance;  $i$  – AC;  $P_{kol}$  – electric power supplied to the plasma column;  $P_{dys}$  – power of energy dissipation from the column;  $\theta_M(i)$  – damping function corresponding to the time of heat process relaxation.

The electric power supplied to the thermal plasma amounts to

$$P_{kol}(t) = u_{kol} i = \frac{i^2}{g} \quad (20)$$

where  $u_{kol}$  – voltage drop in the arc column.

In the classical Mayr model it is assumed that  $P_{dys}(i(t)) = P_M = \text{const}$  and  $\theta(i(t)) = \theta_M = \text{const}$ . In the case under discussion it is assumed that  $P_{dys}(t, p) = P_M(p)$  and  $\theta(t, p) = \theta_M(p)$ , which leads to

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_M(p)} \left[ \frac{u_{kol} i}{P_M(p)} - 1 \right] \quad (21)$$

where  $\theta_M(p) = Q_0(p) / P_M(p)$  – Mayr model time constant;  $Q_0(p)$  – value of gas enthalpy reference coefficient.

On the basis of the approximation (6) it is possible to propose the simplified dependence for the Mayr model power

$$P_M(p, I) \approx P_M(p) = a_p \left( \frac{p}{1 \text{ Pa}} \right)^{2/3} \quad (22)$$

where  $a_p$  – coefficient, W.

In the case of a high-current arc better results of process approximations can be obtained using the Cassie model. Its modification, dependent on the parameter  $p$ , has the following form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_C(p)} \left[ \frac{u_{kol}^2}{U_C^2(p)} - 1 \right] \quad (23)$$

where the value of the Cassie voltage is expressed

by the formula: 
$$U_C(p) = l \sqrt{\frac{P_{dys}(p)}{\sigma(p)}}; \theta_C(p) = \frac{q(p)}{\sigma(p)}$$

- the Cassie model time constant;  $q$  - plasma enthalpy volumetric density. The effect of pressure changes on arc pressure is very similar to column length changes, demonstrated by the formula (5).

Hence, per analogy with the Cassie-Berger model [21] it is possible to propose

$$U_C^2(p, I) \approx U_C^2(p) = b_p \left( \frac{P}{1 \text{ Pa}} \right)^{2/3} \quad (24)$$

The modification of the formula (24) makes it possible to take into consideration the energy dissipation through plasma radiation [22]

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_C(p)} \left( \frac{u_{kol}^2}{U_C^2(p)} - 1 - \frac{P_r(i, p)}{g U_C^2(p)} \right) \quad (25)$$

On the basis of experimental data related to a poorly compressed arc (TIG) [8] it is possible to use the approximation of radiation power  $P_r$  with the function

$$P_r(i, p) = K |i| p^{1/2} = K g |u_{kol}| p^{1/2} \quad (26)$$

where  $K$  - approximation coefficient dependent on the type of gas,  $\text{V} \cdot \text{Pa}^{-1/2}$ . Then, the differential equation (25) can be written in the conductance form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_C(p)} \left[ \frac{u_{kol}^2 - K |u_{kol}| p^{1/2}}{U_C^2(p)} - 1 \right] \quad (27)$$

The stronger compression of the arc column in plasma devices with properly selected gas chemical composition (admixture of hydrogen, metal vapours) corresponds to increased radiation power which can be expressed by means of the dependence close to the expression  $P_r \propto i^2$  [23]. This affects the deformation of static characteristics in the high-current range (Fig. 1a). After taking into consideration  $P_r \propto p^{1/2}$  it is possible to use the approximation

$$P_r(i^2, \sqrt{p}) = e_0 + e_1 i^2 + e_2 p^{1/2} + e_3 i^2 p^{1/2} + \dots \cong e_0 + e_1 (u_{kol} g)^2 + e_2 p^{1/2} + e_3 (u_{kol} g)^2 p^{1/2} \quad (28)$$

In such a case the model describing the arc column dynamics is more complex

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_C(p)} \cdot \quad (29)$$

$$\left\{ \frac{g u_{kol}^2 - [e_0 + e_1 (u_{kol} g)^2 + e_2 p^{1/2} + e_3 (u_{kol} g)^2 p^{1/2}]}{g U_C^2(p)} - 1 \right\}$$

As heat dissipation processes in the arc slowly react to external disturbances, it is possible to roughly assume that the power of losses is basically determined by the static characteristics [19],  $P_{dys}(t, p) \approx P_{stat}(i(t), p)$ , i.e.

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_M(i, p)} \left[ \frac{u_{kol} i}{P_{stat}(i, p)} - 1 \right] \quad (30)$$

where  $P_{stat}$  - takes into consideration power losses only in the column plasma, without near-electrode areas. However, more commonly used are the current-voltage static characteristics (3), (9) or (15), and then

$$P_{dys}(t, p) = U_{stat}(i, p) \cdot i = \frac{i^2}{G_{stat}(i, p)} \quad (31)$$

After substituting (31) to (30) it is possible to obtain the modified Mayr equations in the conductance form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_M(i, p)} \left[ \frac{G_{stat}(i, p)}{g} - 1 \right] \quad (32)$$

At this moment, when conductance does not change in time, the arc static characteristic in this model has the following form

$$U_{stat}(i, p) = \frac{P_{stat}(i, p)}{i} \quad (33)$$

$$G_{stat}(i, p) = \frac{i^2}{P_{stat}(i, p)} = \frac{i^2}{U_{stat}(i, p) \cdot i} \quad (34)$$

Thus, on this basis it is possible to write the model (31) in the conductance form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_M(i, p)} \left[ \frac{i}{g \cdot U_{stat}(i, p)} - 1 \right] \quad (35)$$

Taking into consideration a wide range of current changes is possible thanks to the hybrid arc model TWV [16]. It is formed by the system of two connected non-linear conductances

corresponding to the Mayr and Cassie models. The activity of each of them is affected by the tapering function dependent on the total current value

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta(i, p)} \cdot \left\{ [1 - \varepsilon(i)] \frac{u_{kol} i}{g U_c^2(p)} + \varepsilon(i) \frac{u_{kol} i}{P_M(p)} - 1 \right\} \quad (36)$$

with the designation of the tapering function

$$\varepsilon(i) = \exp \left( - \frac{|i|^w}{I_0^w} \right) \quad (37)$$

where  $I_0$  – boundary value of switching current between the Mayr and Cassie models, A;  $w > 0$ .

Pressure changes relatively poorly affect this value, therefore it was adopted that  $I_0(p) = \text{const}$ . The form of the tapering function can vary and depends on the type of an electrotechnological device as well as on the device operating conditions [24]. As it is possible to approximate the dynamic characteristics of an arc within a wide range of excitation current, the damping factor function  $\theta$  should not adopt a constant value. It is selected along with other model coefficients. The real damping factor function is strongly non-linear in relation to current (Fig. 3a). The Cassie voltage  $U_c(p)$  can be determined using the formula (24), whereas the Mayra power  $P_M(p)$  can be determined using the formula (22) and used in the model (36).

After adopting the approximation of the power radiation function (26), it is possible to obtain the hybrid arc model of a constant column length in the form of the following equation

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta(i, p)} \left\{ \frac{1 - \varepsilon(i)}{g U_c^2(p)} \cdot \left[ u_{kol} i - K |i| p^{1/2} \right] + \varepsilon(i) \frac{i^2}{g P_M(p)} - 1 \right\} \quad (38)$$

A more complex modification of the twv model can be obtained using the approximation (28).

If the arc length changes are quick, the arc dynamic characteristics can be described by means of the modified Kulakov model [21] being the expansion of the model (35)

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta(p)} \left( \frac{i}{g l E_{st}(i, p)} - 1 \right) - \frac{1}{l} \frac{dl}{dt} \quad (39)$$

If the arc length changes are slow  $dl/dt \approx 0$  the formula gets simplified to the following form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta(p)} \left( \frac{i}{g l E_{st}(i, p)} - 1 \right) \quad (40)$$

### Simulations of processes in circuits with an arc in gas of quasi-statically variable pressure

The practical use of electric arc models for simulating the operating conditions of electrotechnological devices can be significantly facilitated by implementing them in popular MATLAB-Simulink software. Due to the application of electric energy supply sources of properly selected external characteristics (acting on the principle of current excitations), it is comfortable to use transformed arc models in which  $u_{kol} = i \cdot g^{-1}$ .

Figure 4 presents the evolution of the current-voltage dynamic characteristic described by means of the modified hybrid twv model (36) during the changes of gas pressure within the limits  $10^3 \div 10^6$  Pa. The system under analysis took into consideration the sum of near-electrode voltage drops  $U_{AB} = 20$  v. The parameters of the simple series circuit energising the arc were the following: sinusoidal excitation current  $I_{max} = 250$  A,  $f = 50$  Hz,  $R = 0,1 \Omega$ ,  $L_m = 1$  mH. The adopted model parameters had the following values:  $a_p = 0.4$  W;  $b_p = 2 \cdot 10^{-3}$  V<sup>2</sup>;  $I_0 = 5$  A;  $G_0 = 20$  S,  $w = 2.2$ . The damping factor function was approximated using the dependence (18) with the parameters:  $\theta_0 = 10^{-4}$  s,  $\theta_1 = 10^{-3}$  s,  $\alpha_t = 0.05$  A<sup>-1</sup>;  $\beta_t = 50$  000 Pa. The time of simulation was 0.5 s.

Figure 5 presents the evolution of the current-voltage dynamic characteristic described by means of the modified hybrid twv model taking into consideration thermal radiation (38)



during the changes of gas pressure within the limits  $10^3 \div 10^6$  Pa. The system under analysis took into consideration the same sum of near-electrode voltage drops  $U_{AB} = 20$  V. The parameters of the simple series circuit energising the arc were the following: sinusoidal excitation current  $I_{max} = 250$  A,  $f = 50$  Hz,  $R = 0,1 \Omega$ ,  $L_m = 1$  mH. The adopted model parameters had the following values:  $a_p = 0.4$  W;  $b_p = 2 \cdot 10^{-3}$  V<sup>2</sup>;  $K = 0.05$  VPa<sup>-1/2</sup>;  $I_0 = 5$  A;  $G_0 = 20$  S,  $w = 2.2$ . The damping factor function was approximated using the dependence (18) with the parameters:  $\theta_0 = 10^{-4}$  s,  $\theta_1 = 10^{-3}$  s,  $\alpha_t = 0.05$  A<sup>-1</sup>;  $\beta_t = 50\ 000$  Pa. The time of simulation was 0.5 s.

Comparing the obtained static and dynamic characteristics of a free or quasi-free arc in selected gas with the results of previously conducted tests [21-24] it is possible to state that pressure changes, i.e. increasing or decreasing, in relation to normal ( $\sim 10^5$  Pa) have a similar effect as changes, i.e. shortening or lengthening, of the arc column. A similar situation can be observed in the case of the power characteristics (15). In turn, there is no such compatibility in the case of gradient characteristics (electric field intensity) (16).

Taking into consideration the effect of pressure changes on the damping factor function value leads to the expected stronger non-linearity of dynamic characteristics in the range of low current and that of high pressure.

Therefore, an increase in the pressure of gas environment in an electrotechnological device causes static and dynamic effects. The static effects include shifting current-voltage static characteristics towards higher voltage values and, in selected gas environments, the possibility of significant deformations of static characteristics within the range of very high pressure and that of very high discharge current. The dynamic effects include the increased non-linearity of characteristics within the range of low current, and in some gases, also within the range of high current. Due to the fact that similar electric effects in an electrotechnological device can be evoked by lengthening the arc column or washing it around with gas flowing, a clear control signal of an electrotechnological device should come from a pressure sensor. However, this measurement, similarly as in the case of the environment temperature, appears highly problematic due to the possible occurrence of the heterogeneous distributions of these quantities, and worse still, with very significant gradients. In most electrotechnological devices with non-free (stabilised) arc discharge, the column is in the heterogeneous gas pressure field and the distribution of plasma pressure in the cross-section and along the column is not homogenous either. For these reasons it is necessary to locate sensors properly and use averaged values.

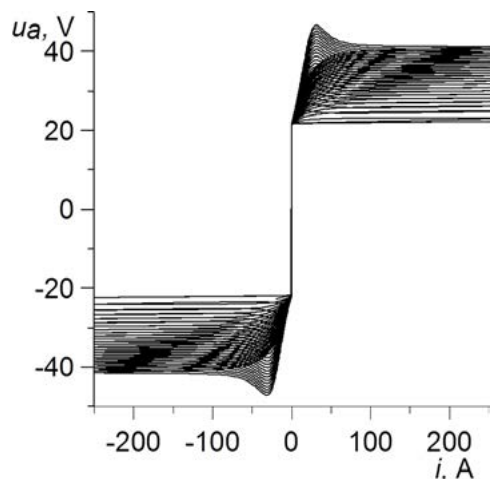


Fig. 4. Evolution of the arc current-voltage dynamic characteristic described by means of the modified TWV equation (36) affected by gas pressure changes

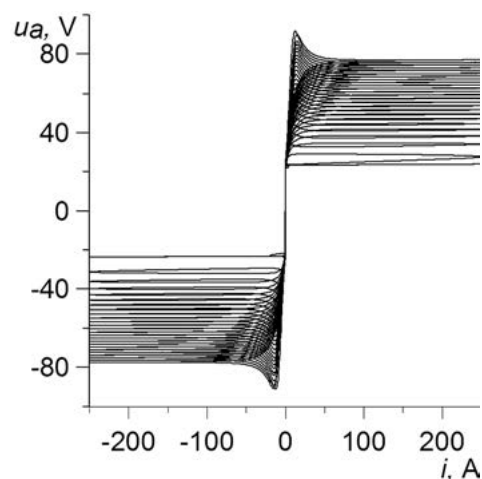


Fig. 5. Evolution of the arc current-voltage dynamic characteristic described by means of the modified TWV equation with radiation (38) affected by gas pressure changes

## Conclusions

1. Presently developed welding and electrothermal technologies use arc discharges in gas environments of various or quasi-variable pressure, which requires the proper modelling of such technologies.

2. As quasi-static pressure changes strongly affect the basic electrothermal characteristics of the plasma column and of electrodes, the previously used simplified mathematical models of the electric arc cannot be used for the precise description of processes taking place in electrotechnological devices operating with gases of various pressure.

3. The approximations of the families of experimental static (current-voltage) and dynamic (damping factor function) characteristics enable such modifications of the known mathematical models of the arc so that these models can be successfully used in describing processes taking place in electrotechnological devices operating with gases of various pressure.

4. Both decreasing and increasing pressure in relation to the normal value triggers such transformations of static and dynamic characteristics which are similar as in the case of arc column length changes or the changes of the washing gas stream.

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