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Use of Modern Analytical Methods in Designing Induction Heating Devices

Abstract: The article presents induction heating and its application range, discusses factors significantly affecting the course of an induction heating process and characteristic phenomena such as electromagnetic induction, skin effect and proximity, enumerates the advantages and downsides of modern numerical and experimental methods as well as characterises (giving emphasis to FEM) and compares numerical methods used during designing induction heating systems and devices. The article also contains an overview related to FEM-based commercial software applications used for analysing issues connected with the simultaneous presence of electromagnetic and thermal phenomena.

Keywords: induction heating process, induction heating devices, numerical analytical methods, FEM, Finite Element Method;

Introduction

Induction heating consists in heating conducting materials located in a variable magnetic field. Heat is mainly generated by eddy currents flowing through a thin subsurface layer of materials (charge); currents are caused by electromagnetic induction. In the case of ferromagnetic materials, part of the emitted heat is the result of hysteresis losses.

The design principle of an induction heating device is presented in Figure 1.

The conversion of electric energy into thermal one takes place in a charge and is caused by the flow of eddy currents having high density and frequency. The source of eddy currents is an electromagnetic field generated by a properly formed set of conductors referred to as an inductor. An inductor and an

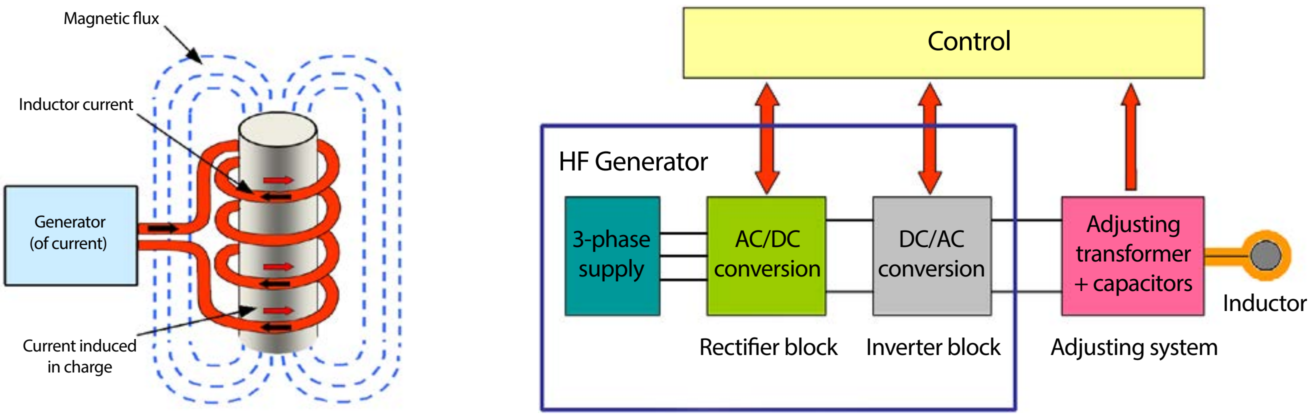

Fig. 1. Induction heating method and heating device block diagram

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element being heated form an induction heating system, i.e. inductor-charge. An inductor is powered by an adjusting system (transformer) from a generator (frequency converter). The presence of a transformer enables obtaining the optimum adjustment of power source in relation to load, the reduction of losses in transmission lines and an increase in operational safety.

The most important induction heating advantages include the following [3]:
- flexibility connected with the possibility of heating only selected zones of an element (charge) which often makes it possible to obtain significant reduction of unitary energy consumption in comparison with traditional heating technologies,
- high efficiency resulting from the ease of obtaining high surface (or volumetric) density of active power emitted in the charge thus obtaining a high temperature increase rate,
- ease of automation, as a result of which usually high costs of induction technology implementation are to a considerable extent compensated by low operating costs.

Induction Heating in Industry – Applications

The advantages of the method combined with the state-of-the-art power electronics solutions have contributed to the intense development of devices (frequency converters) intended for induction heating, widely used in many sectors of industry ranging from jewellery through metallurgy to shipbuilding. The most typical induction heating applications include the following:
- surface hardening,
- through heating,
- melting,
- heating before forging,
- annealing,
- tempering,
- soldering and brazing,
- induction heating,
- pre-weld heating,
- post-weld heating and heating after pressing to remove detrimental stresses,
- hot forming,
- production of semiconducting crystals.

As regards welding engineering, the most important applications include brazing (brazing metals: copper, brass, silver alloys) and soldering (solders: tin and its alloys). Induction heating is mainly used to apply sintered carbides on cutting tools and while joining pipes, flanges, can and box edges and cable terminals in electric machines. Other important welding-related induction heating applications include pre-weld and post-weld heat treatment such as tempering and stress relief annealing of pressure welded and fusion welded joints. Figure 2 presents examples of induction heating applications.

Obtaining optimum induction heating conditions depends on the following:
- proper adjustment of generator electric parameters,
- adjustment of system parameters,
- inductor design,
- tooling selection and design.

Fig. 2. Examples of induction heating applications: surface hardening, brazing, coat remelting, stress relief annealing, heating before forging, rotor soaking after varnishing, induction welding of sheets, production of amorphous and nanocrystalline strips.

Particularly important induction heating parameters are the following:
- efficiency of the inductor–charge system,
- heating zone,
- material temperature distribution,
- efficiency.

In order to obtain these parameters it is of key importance to properly design and create an appropriate inductor. Figure 3 presents examples of inductors for various heating technologies.

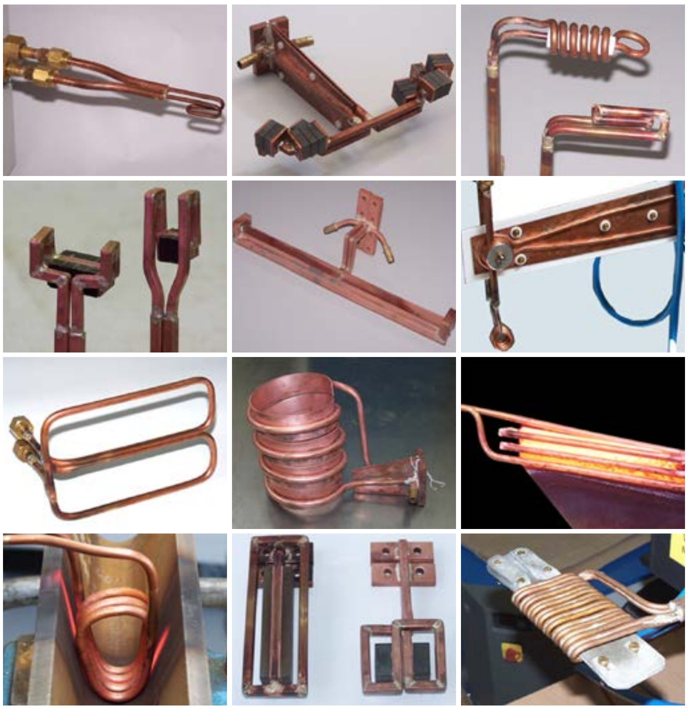

Fig. 3. Examples of inductors for various heating technologies

Characteristics of induction heating

Induction heating includes a number of interconnected phenomena – electromagnetism, emission and conduction of heat, structural transformations, electrodynamic effect etc. Induction heating is predominantly conditioned by the presence and interaction of electromagnetic and thermal fields.

In order to determine the thermal effect in an induction-heated medium it is necessary to know the following:
- heat source distribution,
- system geometry,
- thermal boundary conditions,
- physical parameters of all materials composing the system under consideration.

The description and analysis of the problem presented require using the system of Fourier–Kirchoff equations. The solution to this system is not an easy task taking into account the fact that in the temperature range considered, the electric and thermal parameters of system elements undergo changes (usually non-linear ones) [27].

The process of induction heating is dominated by three phenomena [26], [27]:
- electromagnetic induction,
- skin effect,
- proximity.

Electromagnetic induction consists in the generation (induction) of electromotive force in a conductor. The value of the force depends on the rate of magnetic field flux changes. The phenomenon of induction is described by Faraday's law of electromagnetic induction:

$$e = -\frac{d\Phi_B}{dt} \qquad (1)$$

where
- e – induced electromotive force in volts,
- Φ_B – magnetic induction flux flowing through the conductor surface (B * S).

The Skin effect phenomenon is manifested by the non-uniform current distribution in the conductor; current is displaced towards the conductor surface. The greater specific conductance, permeability and frequency, the greater the non-uniformity of the system.

$$\delta = \sqrt{\frac{2}{2\pi \cdot f \cdot \sigma}} \qquad (2)$$

where:
- δ – penetration depth,
- σ – specific electric conductance ($\sigma = 1/\rho$),
- ρ – specific resistance,
- f – frequency,
- μ – magnetic permeability.

The **Proximity phenomenon** is strongly connected with the skin effect and consists in the interaction of currents flowing in neighbouring cables, causing current density changes in the subsurface layer.

The flow of alternating (usually sinusoidal) current through the inductor generates a time-changing magnetic field around it. This is the first energy conversion in the induction heating system described by **Maxwell's 1st equation**. A variable magnetic field induces electromotive force in the charge. The force, in accordance with Ohm's law, causes eddy currents to flow in the environment. This is the successive conversion of electromagnetic field energy mathematically expressed by **Maxwell's 2nd equation**.

The field energy in the charge undergoes conversion into thermal energy in accordance with **Joule's law**. The non-uniform skin distribution of currents concentrates emitted active power and thermal power in the subsurface layers of the charge.

The theoretical analysis of the induction heating process requires very in-depth knowledge of problems related to the theory of electromagnetic field and the use of a complicated method of mathematical analysis. Due to a great variety of heating systems and the fact that at the temperatures under consideration thermal and electrical parameters undergo changes, which requires the conjugation of electromagnetic and temperature fields, analytical solutions are either highly complicated or impossible to carry out.

As mentioned before, Maxwell's equations describing the properties of electric and magnetic fields, as well as dependences between them, are the basis for the analysis. Such equations are general and not limited to a specific system. Obtaining detailed results requires first, defining the system geometry, boundary conditions, material and energy source parameters and second, the transformation of Maxwell's equations into an appropriate form. As a rule, while calculating real systems many simplifying assumptions are made.

Design of Induction Heating Devices

Apart from basic generator output parameters such as current intensity and frequency, inductor shape and parameters are decisive for the distribution of electromagnetic field intensity and, as a result, for charge temperature distribution, which is of key importance to the final technological effect. It is on the inductor, or more precisely, on the inductor-charge induction heating system, that the aforementioned calculation- and design-related problems are concentrated. The use of magnetic concentrators improving the inductor charge coupling additionally complicates the mathematical system description. At the same time, due to the cost of concentrators and the greater complexity of the object composed of the inductor and the concentrator, experimental methods become more time-consuming and costly.

Depending on the geometry and properties of the system, the purpose of research, available technical and computational resources, as well as the designer's knowledge and experience, the calculation and analysis of induction heating systems require the use of various methods:

- analytical methods,
- equivalent circuit diagram methods,
- physical modelling methods,
- numerical methods,
- combined methods.

Until recently the course of an induction heating process depended mainly on the experience of the designers and technologists, who initially assessed process parameters such as frequency, power or shape of the inductor (dimensions, number of coils, a gap between the material being heated and an inductor) and next verified these data using the trail-and-error method. The results of experimentation were assessed by determining temperature, measuring power and time of heating, measuring hardness, etc.

While using such a method, in order to obtain the optimum inductor parameters and to obtain required charge heating it is sometimes necessary to modify the shape, inter-coil distance or other parameters of the inductor. "Technologically" difficult cases require the use of time-consuming and expensive methods, as well as extensive experience.

The use of various, long-available analytical tools such as analytical methods, equivalent circuit diagram methods, physical modelling methods or combined methods depends on the geometry and properties of the system, technical and computational resources as well as designer's knowledge and experience.

Due to the significant complexity of dependences describing thermal and electromagnetic phenomena, the use of analytical methods or equivalent circuit diagram methods, the adoption of often far-reaching, simplifying assumptions is often a commonly applied rule. In spite of this it is possible to obtain fairly satisfying results only for relatively simple cases. In turn, experimental methods are time-consuming and costly, and require significant experience. Data and parameters obtained by means of this method, even if satisfactory, still fail to determine to what extent the process is optimised. It is only possible to assess the technological result.

Where traditional methods of designing and optimising the inductor design prove unsatisfactory, the use of present numerical modelling-based analytical methods may turn out successful.

Modern Computational Methods in Induction Design

The past decade or so have seen the growing importance of numerical computer-aided process modelling in many areas of science, technique, and technological preparation and development.

Due to significant difficulty of analytical solutions related to induction heating, the use of numerical methods in this area was also attempted. Works on inductor-related computer simulations date back to 1960s. Due to limited access to computers, their insufficient memory, inadequate computational power and poor programming methods, computer simulations were unsuitable for industrial applications until 1980.

Presently, computer simulation has become a practical tool used also in designing induction heating systems and devices. Computer simulation makes it possible to separate and analyse the effect of one or more variables considerably faster and easier than it is possible using experimentation, enables better understanding of phenomena taking place in a given process, which, among others, facilitates designing inductors. Numerical modelling enables users

Table 1. Comparison of experimental and numerical methods of designing induction heating systems

Computer-aided simulations	Experimental methods
Advantages: • Possible applications for various shapes and working conditions • Possibility of demonstrating process dynamics • Possibility of using models and data many times • Unlimited accuracy of calculations • Use of special devices is not necessary • Lower costs and time consumption • Possible future improvements	**Advantages**: • Reliability of results • Presentation of results with unexpected effects and problems • Material databases are not necessary • Providing physical examples for verification
Limitations and disadvantages • Necessity of using special software and databases • Not all processes can be simulated (as of today) • Lack of physical examples	**Limitations and disadvantages** • Expensive equipment • Lack of possibility of adequate understanding of processes • Hindered transfer of knowledge • Case-dependent accuracy • Limited access to devices (costs)

to optimally design systems, improve devices, significantly reduce design time and costs, and better understand process dynamics. Significant differences between experimental and computer-aided design methods are presented in Table 1.

Numerical modelling, or computer-aided simulation, consists in reproducing and predicting the course of phenomena, physical processes, operation of systems and devices based on previously known initial and boundary parameters using special simulation programmes. The description of a simulated object or phenomenon is presented in the form of a diagram, figure or mathematical equations with the results usually obtained in the graphic form.

Computational methods can be divided into the following groups (Fig. 4):
- FDM – Finite Difference Method,
- BEM – Boundary Element Method,
- FVM – Finite Volume Method,
- FEM – Finite Element Method.

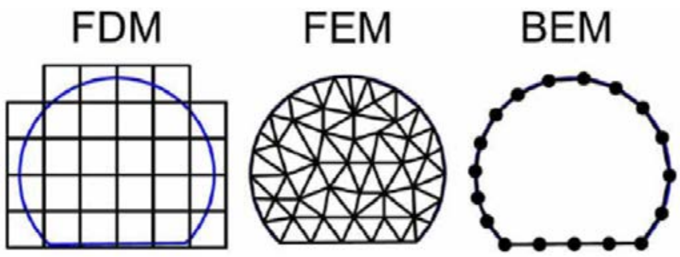

Fig. 4. Discretisation manner depending on simulation method

To put it simply, these methods consist in dividing a continuous area under consideration into a finite number of subareas (creating a mesh) and, next, in searching and finding an approximate solution in the subareas. A solution at any point in space is obtained by interpolating results obtained previously. The basic difference between the methods enumerated above is the manner of searching for a solution, the manner of analysis and the manner of defining boundary conditions.

Presently, the market offer of software applications utilising the aforesaid numerical methods is significant. The greatest number of commercial programmes is based on the Finite Element Method (FEM) consisting in approximate solving of partial differential equations. This method has found applications in solving problems in many areas of science and technique, e.g. in the mechanics of deformable bodies, fluid mechanics, thermal conductance analysis, vibroacoustics, analysis of various fields etc.

Today, FEM, as one of modern analytical methods, is also used in software developed for analysing electromagnetic fields, this includes induction heating. In 1980s and early 1990s it was possible to observe software developments based on BEM (Boundary Element Method) [2], solutions based on FDM (Finite Difference Method) and, as regards induction heating, solutions based on MIM (Mutual Impedance Method), as well as combinations of various methods [2], [23].

The Finite Difference Method (FDM) was the first numerical technique used in the mathematical modelling of various processes. The method, referred to as the "squares method", was proposed by A. Thom in the 1920s for solving a non-linear hydro-dynamic equation. Ever since, the method has been used in solving various field problems, including heat exchange issues and electromagnetic problems.

It is one of the simplest numerical methods for solving problems expressed by differential equations. This method consists in replacing derivatives present in equations by appropriate difference quotients. Some difficulties in using this method are connected with boundary conditions and irregular boundary shapes. This method is relatively easy to use, as an area being modelled has either a cylindrical or rectangular shape [2]. In this method a regular mesh (rectangular, circular, oblique or triangular) of nodal points is placed onto an area in which an equation is to be solved. Usually, an area to be modelled, i.e. inductor, charge, concentrator, is digitalised by an orthogonal mesh. The algorithm of discretisation accompanying the use of the orthogonal mesh is simple. The values of a function searched for in the mesh nodes are the set of unknowns. It is necessary to determine difference quotients corresponding to the order of equation. These quotients enable the conversion of a differential equation into the system of algebraic equations. Obtaining such quotients requires the expansion of a function searched for into the Taylor series around nodal points. According to Taylor, for two variables the value of a variable in a mesh node can be expressed in relation to neighbouring values taking into consideration a constant step (constant distance between nodes).

The technique of finite differences is based on approximations enabling the replacement of a differential equation with equations of finite differences (in digitalised space) which can be derived directly from the difference quotient or from the Taylor expansion. Such approximations have an algebraic form, bind the value of a dependent variable in the point of the region of the solution with values in several neighbouring points. Algebraic equations obtained can also be solved using iterative techniques (Jacobi method, Gauss-Seidel method etc.) and direct methods. These issues are described in numerous publications on computer modelling and will not be the subject of detailed consideration in this study. In all the methods the accuracy and stability of calculations is significantly affected by the selection of a mesh and time step. These parameters are particularly critical in FDM. The smaller the step of a distance and time, the greater the accuracy, yet also the greater the level of complication.

The Boundary Element Method (BEM) [2] [30], is one of the methods of the numerical solution of partial differential equations. It consists in reducing the system of differential equations with pre-set boundary conditions (i.e. so-called boundary problem) to the system of integral equations determined on the boundary of an area under consideration. Unlike other popular methods, i.e. the Finite Element Method (FEM) and the Finite Difference Method (FDM) the use of BEM does not require the discretisation of the interior of an area, but only of its boundary. The Boundary Element Method developed in two directions, which is connected with the manner of formulating boundary integral equations. In a so-called direct approach, a task is formulated directly, i.e. by means of physical quantities, whereas in an indirect approach a boundary task is formulated for functions without physical importance, where values searched for are determined on the basis of such functions. As solving integral equations is usually very difficult, the method of solution is usually approximated; this method consists in the discretisation of a boundary and introducing a finite number of boundary elements. On each element, boundary functions are approximated by means of nodal values and interpolation functions referred to as shape functions. The result is to obtain a finite number of algebraic equations. An advantage of this method is the generation of a mesh only on the boundary of an area, easy modelling of geometry and boundary conditions, obtaining an accurate solution inside the area, and the lack of necessity of the local densification of a mesh on a boundary.

The disadvantages of this method include the necessity of existence and knowledge of a fundamental solution, difficulties while taking into consideration the non-uniformity of the area and anisotropy. BEM was used in induction heating in late 1980s and early 1990s.

The Finite Element Method (FEM) is today one of the most extensively used methods in solving various engineering problems. Its versatility, consisting in the ease of the schematisation of various geometrically complicated areas, non-uniform and anisotropic among others, makes it a good tool for modelling physical problems.

The development of FEM has coincided with the development of computer technology. The first works utilising the Finite Element Method were published in 1940s. Initially, calculations carried out using FEM concerned objects of very simple geometry (usually modelled as one-dimensional) and constant material properties, as well as phenomena described by means of linear differential equations.

From 1970s FEM was gradually applied in solving non-linear problems, yet still for objects of relatively simple geometry, modelled as 1D or 2D. A rapid development of computer technology in 1980s, connected with greater computing power and possibility of processing and storing great sets of information, enabled FEM to be used in calculating non-linear problems for objects with geometry of any complexity, particularly 3D. It was then that the method became useful for induction heating modelling.

Finite Element Method (FEM) – characteristics

The basic FEM idea consists in divining an area into a finite number of subareas (elements). Each element has nodes, with which field quantities searched for are connected. These nodes are usually located on the sides and in the corners of elements in such a manner that a given node and its field quantities are common for two or more neighbouring elements. The area being solved is digitalised and presented as a mesh of elements, usually triangular ones (2D). Each point of an element has an assigned vector potential on the basis of its values in nodes composing the element. The basic advantage of FEM is the possibility of obtaining results for complicated shapes, for which analytical calculations are impossible to carry out. As a result, a given problem can be simulated in the computer memory without the necessity of building a prototype, which significantly facilitates a design process.

The division of an area into increasingly small elements usually leads to more accurate calculation results, yet at the cost of greater demand for computing power. It is also necessary to allow for overlapping calculation errors resulting from multiple approximations (roundings) of values being processed. If an area is composed of several hundred thousand elements having non-linear properties, calculations must be modified accordingly in successive iterations so that the final solution could be proper. For this reason, in exceptional cases it may appear that accumulated calculation errors are not negligible. In order to minimise such errors between different versions of the same problem (e.g. changes of material parameters while maintaining the same dimensions) it is necessary to use the identical discretisation of a problem so that rounding-related errors, if any, are the same, and possible calculation differences result really from the changes of material properties.

FEM simulations cannot be carried out in real time as for very complicated systems the solution of a given problem may take a very long time (depending on the level of complication and computing power this time can range from several seconds to several days and beyond).

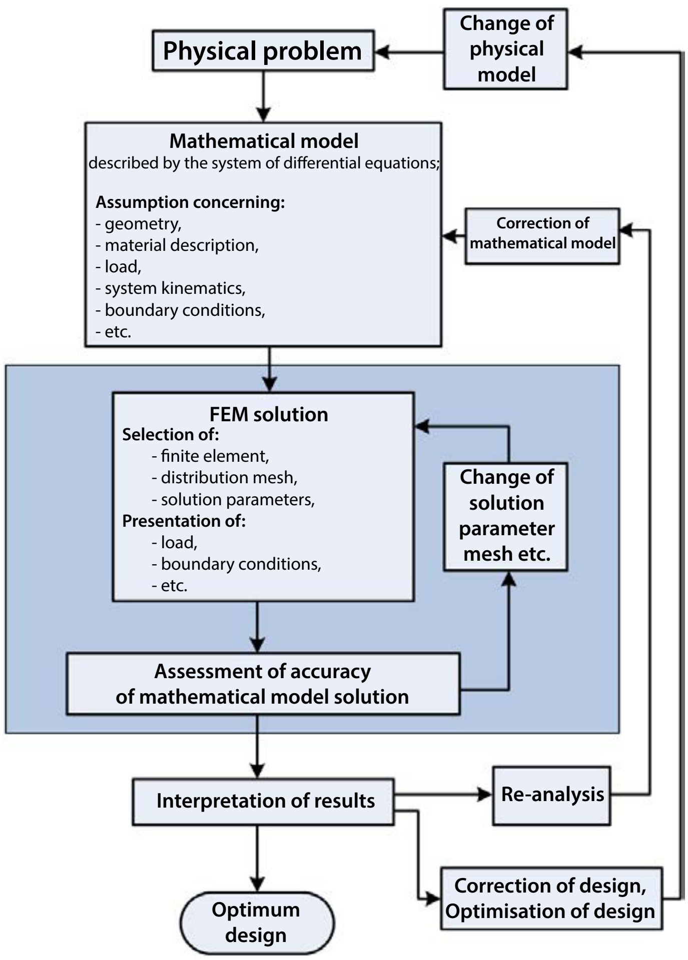

Fig. 5. Method used for FEM-based computer modelling

In addition, values calculated by means of FEM can be encumbered with errors, the value of which depends on assumptions adopted during the formulation of a problem to be solved and on the accuracy of available material data. For this reason, as much as possible, calculated data should be verified against data measured by means of a real device or a system.

FEM-based Software in Induction Heating Processes

Recent years have proved that FEM is a universal method for solving equations in engineering knowledge and physics. Numerical methods are used for solving a number of problems difficult to solve with analytical methods both at the device design stage and while analysing a technology.

Presently, the market offers relatively many finite element-based software packages applicable for designing induction heating systems [10], [11], [12], [13], [14], [18], [21], [22], [24], [25]. Most 2D (two-dimensional) and 3D (three-dimensional) analytical programmes come from companies specialising in the development of general purpose software intended for calculations of physical fields in machine and electrotechnological industries. Usually, only part of the software package is used for the simulation of a heating process – a thermal unit, an eddy current unit or their combination. These programmes do not usually feature proper databases necessary for modelling and simulating induction heating. Such software usually requires adaptation to the simulation of a real heating process.

There are also some specialised programmes for solving electromagnetic problems and thermal problems associated with them. Such programmes are provided with optional modules for modelling auxiliary phenomena (such as motion, structural transformations etc.).

Commercial software applications used for modelling induction heating processes include, among others, the following programmes:

- FLUX2D/3D (by Cedrat Research from France), [4], [5], [11], [25];
- ANSYS (by ANSYS Inc. from the USA) [21], [22];
- Maxwell 2D/3D [18], [22] (by ANSOFT from the USA) for calculating electromagnetic fields (Maxwell 2D/3D is usually used for analysing the operation of electric equipment);
- Opera 2D/3D (VectorFields Ltd. from Oxford) used mainly for field analysis in complicated physical objects;
- QuickField (by Tera Analysis from Denmark).

There is a big number of programmes, such as the Prometheus system [18] (Ilmenau University, Germany), usually developed to address the own needs of various technical universities or companies dealing with induction heating.

On the basis of the analysis of available publications and other sources of information it is possible to state that the most efficient and popular programmes for solving tasks related to the simultaneous presence of electromagnetic and thermal phenomena are ANSYS and FLUX.

ANSYS has very complex and complicated packages, most of which are focused on solving mechanical and thermal problems. The programme is also provided with elaborate tools for the analysis of electric and electromagnetic fields.

Among others, FLUX software can be used for analysing magnetic, electric and thermal fields, static states and mutual magnetic-thermal and dielectric-thermal problems. This programme is also used in the analysis of phenomena in rotating machines (motors, generators), devices for converting and transporting electric energy (inverters, transformers, overhead lines), in industrial processes such as induction heating, resistance heating, dielectric heating and heat treatment.

In Poland, QuickField software is used mainly by students, who have the possibility of obtaining a free version of this programme for 2D analysis of stresses, magnetism, electrostatics, conduction of current and heat.

In spite of numerous advantages, computer simulations in induction heating processes are not as commonly used as in the machine-building industry, electrotechnological industry or in building engineering. This results from the following reasons:

– induction heating processes are very complex. Their simulation requires gathering mutual non-linear and multidimensional problems (electromagnetic fields, thermal fields, cooling process, structural changes, deformations, supply systems etc.) (Fig. 6);
– induction processes, particularly heat treatment processes have very diversified features which may require various structures of software or various simulation methods;
– solving problems in induction processes requires the use of very good and fast computers featuring considerable size memory;
– induction heating market is relatively small if compared with other industry sectors, thus developing specialist commercial software for simulating induction heating processes is difficult and not very profitable.

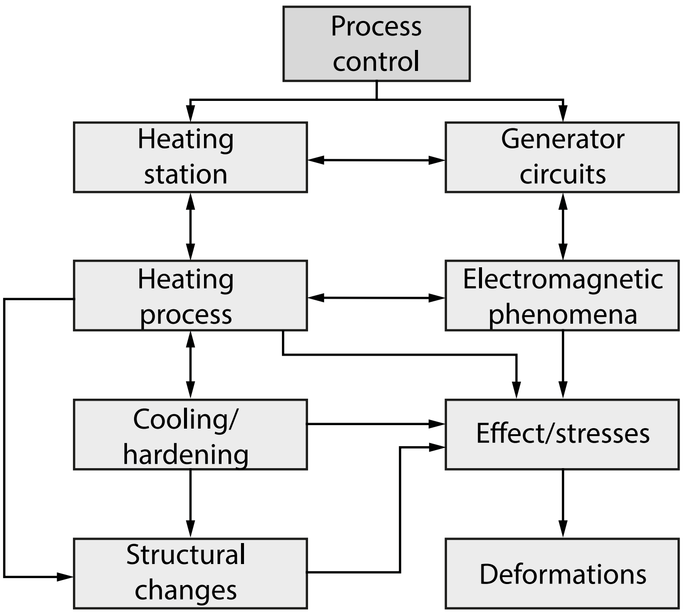

Fig. 6. Structure of induction heating process model

Summary

Comparing simulation methods used in induction heating it should be stated that the Finite Element Method (FEM) and Finite Difference Method (FDM) have a lot in common. FDM consists in replacing partial derivatives (differential equations connected with a given physical problem) with difference quotients in order to be able to carry out approximation at a given point. FEM starts from creating basic equations based on, e.g. variational methods. Both methods digitalise continuous functions (e.g. magnetic potential vector or temperature) and result in the creation of algebraic equations. Differences lie in the manners of mesh generation. The INDUCTOHEAT [2] experience of using both methods for problems connected with induction heating indicate that computer operation time with FDM is shorter, particularly for modelling typical and regular shapes. However, it should be noted that while comparing methods it is necessary to take into consideration the type of a physical problem to be solved. Important is also the structure of specific software based on these methods. FDM is not suitable for modelling problems related to the heat treatment of elements having complicated shapes or while using various materials and shapes (e.g. camshafts, crankshafts, gear wheels and other critical elements). Both methods require generating the mesh of an area being modelled. Such a mesh includes inductors, charges and capacitors. In order to meet the conditions of continuity of differential equations it is also necessary to generate a mesh and carry out calculations inside an area not conducting electricity, such as air space. This necessity can be regarded as a disadvantage characteristic of both methods mentioned above. Another difficulty related to electromagnetic field calculation in both methods is the manner of treating the outside area extending to infinity.

In turn, the Boundary Element Method (BEM) does not require analysing the electromagnetic field in the air, which can be considered as an advantage. As this method requires discretisation only on the boundary of induction system elements, the generation of a mesh is relatively easy and simple.

In conclusion it can be stated that each of the simulation methods has its strengths and weaknesses. In some solutions it is advisable to combine various methods. The selection of a given method depends on a specific application and process characteristics. The INDUCTOHEAT company, known for having a record of extensive experience in applying various methods tends to use FEM and/or MEB.

The issues related to modelling the distribution of temperature fields are less complicated than those concerning electromagnetic fields. If the boundaries of elements being heated are properly defined, temperature fields can be calculated both my means of FEM and FDM, where FEM is more popular due to the possibility of modelling also in the case of complicated shapes.

It should be noted that using modern software does not guarantee proper calculation results. Maintaining required accuracy demands not only experience of numerical techniques but also experience and engineering knowledge of induction heating.

Computer modelling enables predicting how various factors affect heat exchange conditions as well as makes it possible to determine what changes should be introduced to a system being designed in order to ensure obtaining required results.

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