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Damping Factor Function in AC Electric Arc Models. Part. 4. Criteria for Selection of Arc Models and Determination of Low-current Arc Time Constant

Abstract: Causes of problems in strict classification of arcs into two categories: low and high current have been described. Assumption has been taken that processes in column can be approximated with one of two models: Mayr or Cassie. A new criterion of arc classification has been proposed, based on minimal deviation of harmonic relation of real voltage on arc column comparing to theoretically estimated data. Using MATLAB-Simulink program, errors in determining time constant value have been analyzed in Mayr model of arc described with different mathematical models with and without random noise.

Keywords: electric arc, Mayr model, Cassie model, damping factor, time constant, spectral analysis

Introduction

Significant differences in the parameters and shapes of static and dynamic characteristics of low and high-current arcs necessitate undertaking various activities in order to stabilise or destabilise discharges in electrotechnological devices and electric equipment. Such activities can include the appropriate design and functioning of supply sources with controlled and non-linear external characteristics and systems of electrodes, nozzles, ducts or discharge chambers (including extinguishing chambers).

Due to significant difficulties with the adequate description of processes in electric circuits with control systems and simplified arc models, it was necessary to adopt a conventional division of models into those of low and high-current. The needs for the generalisation of analyses and simulation tests of the systems using models associating these “sub-models” into one common hybrid model [1] require the determination of at least approximate applicability threshold of constituent models. So far this threshold has been determined intuitively and therefore with a high likelihood of error.

One of more important characteristics of arc clamp models is a damping factor function. Simplified models adopt the constant value of a damping factor function in the form of a time constant. Due to the necessity of maintaining a burning AC arc in many types of electrotechnological devices, the time constant value near the area of current transition through zero is of significant importance. Experimental tests have revealed [2, 3] that in such cases the damping factor function obtains values close to the maximum. However, most methods of the experimental determination of the function are complex, requiring additional calculations and difficult to carry out in the on-line mode. At

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the same time the damping factor functions of high-current arcs have very low values and do not play such a significant role in maintaining arc burning stability as is the case with low-current arcs.

Low and high-current arcs in electrotechnological devices

In general cases, static voltage on an electric arc is a function of electric current I and physical parameters, including the distance between electrodes l, pressure p, a gas volume stream washing around a column $\dot{V}$ etc. [4]. By adopting some of these parameters as constant values (e.g. $p=\text{const}$, $\dot{V}=\text{const}$) it is possible to write a formula for voltage in the form of the sum of two summands

$$U_a=f_1(l)+f_2(l, I) \tag{1}$$

One of voltage constituents strongly depends on current, whereas the other one only on an arc length, which significantly affects the shapes of static and dynamic characteristics. Depending on the range of applied excitations and existing disturbances it is possible to determine the appropriate approximations in the form of the family of characteristics. One of the best known is the dependence provided by Nottingham

$$U_a = a_s + b_s l + \frac{c_s + d_s l}{I^n} \tag{2}$$

where a_s – the sum of near-cathode and near-anode voltage drop; b_s – arc voltage gradient; c_s, d_s, n – constant approximation coefficients. A particular case of this formula, where $n=1$, was provided by H. Ayrtona

$$U_a = a_s + b_s l + \frac{c_s + d_s l}{I} \tag{3}$$

where U_a – voltage; I – electric current (3÷14 A); l – the distance between electrodes (0÷7 mm). In the case of high currents it is possible to adopt

$$U_a=a_s+b_s l \tag{4}$$

The characteristics of a long electric arc are significantly affected by the distance between electrodes, which shifts the diagrams of these characteristics towards higher voltage. If l = const, the formula (2) adopts a simpler form

$$U_a = A + \frac{B}{I^n} \tag{5}$$

where $A=a_s+b_s l$; $B=c_s+d_s l$. In such cases it is often adopted that $d_s l >> a_s \cong 0$.

In turn, the properties of a short welding arc are greatly influenced by near-electrode voltage drops depending on the material of electrodes, their shape and thermal state as well as on the type of plasma-creating gas. Their presence affects the balance of arc dissipated energy, particularly on electrodes. In such cases it is often adopted that $a_s \cong d_s l > 0$. At the same time, near-electrode voltage drops pose a considerable difficulty in diagnosing electric and thermal processes in an arc. There are a few methods which can be applied to experimentally determine such processes [5]. Within the deliberations of this study it is assumed that the values of near-electrode voltage drops are known and the analysis presented concerns a column with thermal plasma.

In the theory and practice of using various electrotechnological devices, electric equipment and discharge lamps with electric arcs existing in the main electric circuit, the terms of low and high-current devices are often used; what is meant thereby, as a rule, is the constant or root-mean-square current usually affecting the design, parameters, state variables and the operation of devices. Due to strong non-linearities of characteristics, very low damping factor values, sensitivity to various disturbances and difficult-to-maintain discharge burning stability, the arc is particularly sensitive not only to the changes of current values but also to external influence including, e.g. ambient temperature, pressure, chemical composition and streams of gases washing around the column, the changes of column length and diameter, the vertical and horizontal positions of electrodes in relation to each other etc. For this reason, while classifying an arc and devices with an electric arc, environmental parameters should also be taken into account. The easiest is to observe the effect of these parameters on the families of static current-voltage characteristics. Figure 1 presents effects caused by some external factors. It can be seen that such factors influence not only the position, but most importantly, the non-linearity of arc characteristics. This, in turn, requires continual matching of (very often non-linear) supply source characteristics with arc characteristics. Therefore, even the minimum value of DC at which the arc is burning is determined not only by the characteristics and parameters of the supply circuit but also by non-electric factors.

Due to the strong non-linearity of arc static and dynamic characteristics and frequent use of two different classical mathematical models by Mayr and Cassie, the division into low and high-current arcs is used in the technique for the purpose of approximating electric processes. On the basis of the degree of adequacy with measurement data low-current arcs are described using the Mayr model, whereas the high-current arcs are described by means of the Cassie model.

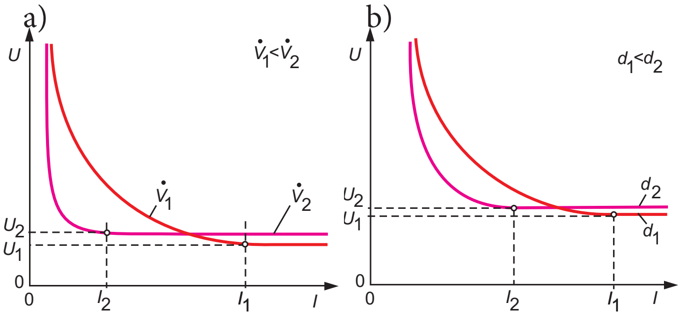

Fig. 1. Current-voltage characteristics of an electric arc a) not stabilised and stabilised by a gas flow in low power devices ($\dot{V}_2>\dot{V}_1\cong 0$ l/min - gas volume streams); b) not stabilised and stabilised by massive electrodes in high power devices($d_2 >> d_1$ – diameters of flat frontal surfaces of electrodes)

The needs of diagnosing and automatic control of electrotechnological devices justify the efforts aimed at the experimental determination (often in situ) of arc parameters and characteristics, among which of particular importance is a damping factor function value as it strongly influences the stability of discharge in gases. The effects of the experimental investigation of an electric arc can be the damping factor function values of

1) real arc,

2) mathematical model of an arc.

In the first case they represent the inertia of heat dissipation processes in the environment, whereas in the second, in strict connection with other parameters of the model, they determine the dynamic characteristics of an arc. There can be very significant discrepancies between these functions. The first function depends solely on physical factors and the accuracy of measurement methods, whereas the second also depends on an adopted mathematical model [6].

The effect of the strong non-linearity of the damping factor function on the dynamic properties of arcs can be significantly reduced in cases of small bipolar excitation amplitude, unipolar pulse excitation or DC excitation. In such situations possible vibration of current and damping factor take place only in the low or high-current range.

Experimental tests of an electric arc in the wide range of current changes [1, 2] reveal a strong arc damping function non-linearity. In turn, constant damping factor values are assumed in simplified mathematical models of an arc. This means that mathematical models very roughly approximate the course of physical processes in limited ranges not only of current but also of other physical disturbances.

The value of boundary current, I_0, separating the ranges of low and high-current models, depends on the scale of device parameters (electrode dimensions, inter-electrode distance, electrode thermal state, surrounding gaseous environment, electric power etc.). During simulating welding arcs this value tends to be insignificant, usually 5 A, and in testing circuits with high-pressure low-power discharge lamps (sodium and mercury) the value of I_0 can be even 10 times lower [7]. In turn, during the analysis of a 1MW plasma torch supply system the value of I_0 obtained even 2 kA [8].

The experimental determination of a DC arc damping factor function value requires the artificial introduction of disturbances into the arc circuit. Due to the strong non-linearity of arc characteristics they should have relatively low amplitudes. This however, in the conditions of intense natural disturbances, reduces the accuracy of measurement methods. A similar situation can be observed in the case of an AC arc, where it is possible to observe the overlapping of high-frequency harmonics [9] and natural noise.

If the amplitude of variable current excitation is relatively low $I_m/I_0 \leq 1$, there is no doubt as to the purposefulness of Mayr model application. In the case of strong current ($I_m/I_0 > 1$) having the same frequency as in the previous case, the electric and thermal states of the environment (primarily of the arc column and electrodes) change periodically and the time of low-current discharge decreases. However, the presence of discharge can significantly affect the quality of electric energy supplied to electrotechnological devices. For this reason, using only the Cassie model can prove insufficiently accurate. A good solution in such cases can be the use of a hybrid arc model associating two models, i.e. those of low and high-current arc [1, 10, 11].

Basic assumptions of arc classification criteria

In the practice of calculating electrotechnological devices, one of the two electric arc classical models is often used. These models are

– Mayr model

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_M}\left(\frac{u_{kol}i}{P_M} - 1\right) \tag{6}$$

– Cassie model

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_C}\left(\frac{u_{kol}^2}{U_C^2} - 1\right) \tag{7}$$

where P_M – constant value of the Mayr model power; θ_M – Mayr model time constant; U_C – constant value of Cassie model voltage; θ_M – Cassie model time constant. They result in two different static characteristics:

– hyperbolas from the Mayr model

$$U_{Mkol} = \pm\frac{P_M}{I} \tag{8}$$

– rays from the Cassie model

$$U_{Ckol} = \pm U_C \tag{9}$$

The adoption of many simplifying assumptions [6] leads to discrepancies of both dynamic and static characteristics of mathematical models (6) and (7) in comparison with the real arc characteristics.

In order to determine the selection of an electric arc model various criteria can be suggested:

1. Criterion of model selection on the basis of static characteristics

The abscissa of the intersection point of the static characteristics of both models ($U_{Mkol}(I_0) = U_{Ckol} = U_C$) is defined by the limiting current).

$$I_g = \frac{P_M}{U_C} \tag{10}$$

Due to the discrepancy of the real static characteristics of an arc both in the low-current range in relation to the Mayr model and in the high-current range in relation to the Cassie model, the dependence (10) is not fulfilled in practice, which is demonstrated in Figure 2. In addition, such an approach entirely ignores the influence of the damping factor function $\theta(i)$ on the selection of an arc model. If the value of the product θ and of current excitation pulsation ω is considerable ($\theta\omega \geq 4$), the waveform of voltage on the column becomes almost sinusoidal, and this happens regardless of the selected mathematical model of an arc. Significant differences appear along with a decreasing damping factor value. Each (even high--current) AC arc in the neighbourhood of the point of current transition through zero behaves in a similar manner as a low-current arc.

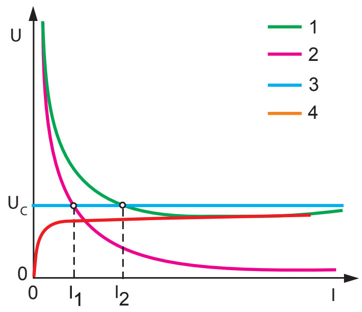

Fig. 2. Static current-voltage characteristics of (1 – a real intensively cooled arc; 2 – an arc described by the Mayr model; 3 – an arc described by the Cassie mode; 4 – a real thermally insulated arc)

The formula (10) also reveals that devices of various arc power can have the same limiting current as it also depends on the value of voltage (arc length). For this reason, the technological effects of the operation of high-power arc devices are sometimes assessed on the basis of the product of power and voltage (analogically to the W.E. Schwabe indicator of lining erosion in arc furnaces). In the study [1] the value of limiting current adopted in the hybrid model is almost five times greater than that resulting from the formula (10), in the study [8] it is six times and even seventeen times greater. At the same time, on the basis of reference publications it is possible to observe the approximate dependence $I_0 \propto P_M U_C$.

In comparison with the adopted approximations of static characteristics (8) and (9), more accurate (particularly in the terminal range of low current) is the approximation expressed by the formula (5). In such a case the criterion of transition between the models can adopt the following form

$$\frac{\left| A + \frac{B}{I_g^n} - U_C \right|}{U_C} \leq \delta \tag{11}$$

and the limiting current can be determined using the formula

$$I_g = \sqrt[n]{\frac{B}{U_C(1+\delta) - A}} \tag{12}$$

where δ - very low pre-set value. On the basis of these deliberations it is possible to improve this criterion by taking into consideration the influence of time constants in the form of the function $I'g = f(I_g, \theta_M, \theta_C)$.

The current I_0 is used for the creation of tapering functions in arc hybrid models. These functions adopt different forms [12] and define smooth transition along with an increase in a current module, i.e. from the low-current model to the high-current model.

2. Criterion of model selection on the basis of spectrum of voltage on real arc column

The use of sinusoidal current excitation $I_m \sin\omega t$ in the Mayr and Cassie models corresponds to various voltage waveforms in time on a column and thus to various characteristics of their spectrum. The voltage waveform being the analytical periodical solution in the Mayr model is the function

$$u_{kol}(t) = \frac{2P_M \sin\omega t}{I_m \left[1 - \frac{1}{1+4(\omega\theta_M)^2}(\cos 2\omega t + 2\omega\theta \sin 2\omega t)\right]} \tag{13}$$

The voltage waveform being the analytical periodical solution in the Cassie model is the function

$$u_{kol}(t) = \frac{\sqrt{2}U_C \sin\omega t}{\sqrt{1 - \frac{1}{1+(\omega\theta_C)^2}(\cos 2\omega t + \omega\theta \sin 2\omega t)}} \tag{14}$$

These solutions can be presented by means of the Fourier infinite trigonometric series of odd harmonics (1, 3, 5, 7...) with decreasing amplitudes. The heights of the spectral lines of the Mayr model periodical solution form a geometric progression meeting the condition

$$\frac{U'_{2k+1}}{U'_{2k-1}} = \chi_M(k) = const \tag{15}$$

where $2k + 1$, $2k - 1$ – numbers of neighbouring odd harmonics; U" – amplitude of appropriate voltage harmonic.

In the extreme case of a very low damping factor value θ_C, the Cassie model voltage waveform becomes similar to a bipolar rectangular wave with the amplitude U_C. The harmonic spectra of such a wave meet the condition

$$\frac{U^{"}_{2k+1}}{U^{"}_{2k-1}} = \frac{2k-1}{2k+1} \quad (16)$$

This dependence can be expressed in the form

$$\frac{U^{"}_{2k+1} \cdot (2k+1)}{U^{"}_{2k-1} \cdot (2k-1)} = \chi_C(k) = 1 \quad (17)$$

where the multiplier $(2k+1)/(2k-1)$ is a strongly non-linear function having the shape of a shifted hyperbola.

The spectrum of real arc voltage has many harmonics with noise overlapped on them. In addition, the real arc characteristics differ from the idealised ones, i.e. the ones expressed with the Mayr and Cassie models. For this reason, the dependences (15) and (17) can be fulfilled only roughly. However, this can cause a significant difficulty in the classification of an arc. Therefore, the average value estimate is calculated from the dependence (15) by approximating it (taking into consideration n + 1 of the first harmonics) using the expression

$$\overline{\chi}_M = \frac{\sum_{k=1}^{n} \chi_M(k)}{n} \quad (18)$$

It is recommended that the measures of deviation of real arc harmonic indicators χ from its mathematical models in the form presented below should be introduced:

– measure of deviation in relation to the Mayr model

$$\delta_M = \max{}_{k=1}^{n} \left| \chi_M(k) - \overline{\chi}_M \right| \quad (19)$$

– measure of deviation in relation to the Cassie model

$$\delta_C = \max{}_{k=1}^{n} \left| \chi_C(k) - 1 \right| \quad (20)$$

The same value of current amplitude usually corresponds to various values of deviations $\delta_M(I_m)$ and $\delta_C(I_m)$. The initial criterion of dividing arcs into low and high-current can have the following form: if $\delta_M(I_m) < \delta_C(I_m)$, the approximation with the Mayr model is more accurate and the arc meeting this condition is a low-current one; if if $\delta_M(I_m) > \delta_C(I_m)$, the approximation with the Cassie model is more accurate and the arc meeting this condition is a high-current one.

The arc division criterion proposed in this form is not perfect. While the Mayr model damping factor is not subject to any limitations, the Cassie model damping factor should be very close to zero, which is possible only roughly. The strong non-linearities of the static characteristics and of a damping factor function [2] are responsible for the fact that the condition (17) can be roughly fulfilled by real arc models in the range of very high current and with effective thermal insulation. For this reason the boundary condition of dividing arcs into low and high-current should have a weaker form

$$\delta_M(I_m) = \delta_C(I_m) \Rightarrow I^{"}_g = I_m / \sqrt{2} \Rightarrow I_g = K \cdot I_0 \quad (21)$$

where K > 1.

Verification of arc classification criteria

In order to verify the efficiency of the method proposed, a TWV hybrid model was used [1]

$$\frac{1}{g} \cdot \frac{dg}{dt} = \frac{1}{\theta} \left\{ \left[1 - \varepsilon(i)\right] \frac{u_{kol} \cdot i}{g \cdot U_C^2} + \varepsilon(i) \frac{u_{kol} \cdot i}{P_M} - 1 \right\} \quad (22)$$

assuming, following publications by various authors, that this model enables the most accurate approximations of real arc dynamic characteristics. In this model a variable damping factor function is assumed

$$\theta(i) = \theta_0 + \theta_1 >> \theta_0 \cdot \exp(-\alpha |i|) \quad (23)$$

where $\alpha > 0$, $\theta_1 >> \theta_0$ – are constant factors of this function and the tapering function $\varepsilon(i)$ can have various forms [10]. In the TWV model it was adopted that

$$\varepsilon(i)=\exp(-i^2/I_0^2) \qquad (24).$$

Another form of the tapering function can be the following

$$\varepsilon(i)=\exp\left[-\left(\frac{|i|}{I_0}\right)^5\right] \qquad (25)$$

Due to a greater rate of rise in the area of the inflexion point (I_0, 1/e) [8] this function enables obtaining much clearer boundary of transition between mathematical models.

After assuming that $dg/dt = 0$ the static characteristic of the model (22) is described by the dependence

$$U_{stat}(I)=\frac{U_C^2}{2P_M(1-\varepsilon(I))}\times$$
$$\times\left[-\varepsilon(I)I\pm\sqrt{\varepsilon^2(I)I^2+4\frac{P_M^2}{U_C^2}(1-\varepsilon(I))}\right] \qquad (26)$$

The diagrams of the function (26) using various values of model parameters are presented in Figure 3. It can be seen that the shapes of the curves diverge from known experimental characteristics. It is particularly visible in the transition current areas between the Mayr and Cassie models. The consequence of this divergence is the deterioration of precision in the simulated verification of criteria.

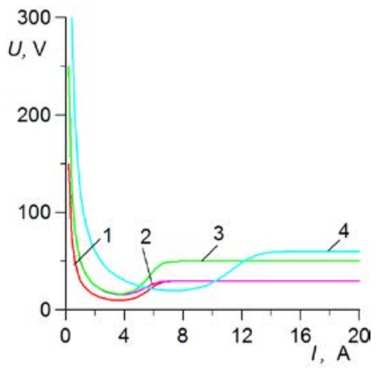

Fig. 3. Hybrid model static characteristics
(1 – U_C = 30 V, P_M = 30 W, I_0 = 5 A;
2 – U_C = 30 V, P_M = 50 W, I_0 = 5 A;
3 – U_C = 50 V, P_M = 50 W, I_0 = 5 A;
4 – U_C = 60 V, P_M = 120 W, I_0 = 10 A)

Parameters adopted as the pre-set arc model parameters were $\theta_0 = 4\cdot10^{-4}$ s, $\theta_1 = 1\cdot10^{-3}$ s, $\alpha = 10$ A^{-1}, $U_C = 30$ V, $P_M = 30$ W. The case of model switch-over current $I_0 = 5$ A was considered as first. The pre-set values of sinusoidal excitation amplitude were between 5 A and 300 A; the frequency f = 50 Hz. The voltage spectral analysis took into consideration odd harmonics up to 9 inclusive. The obtained diagrams of harmonic indicators and deviation measures are presented in Figure 4a. It can be seen that the boundary criterion (21) is fulfilled in the abscissa of approximately I_g = 60 A. This means that the value of limiting current determined by means of this method is twelve times greater than the value pre-set with current I_0.

Afterwards, the case for I_0 = 10 A was considered. The model parameters were P_M = 150 W, U_C = 120 V. The remaining parameters were the same as previous ones. The current excitation amplitude was increased to as many as 600 A. The diagrams of harmonic indicators and deviation measures are presented in Figure 4b showing that the boundary criterion is fulfilled in the abscissa of approximately 140 A, i.e. 14 times greater than pre-set current I_0.

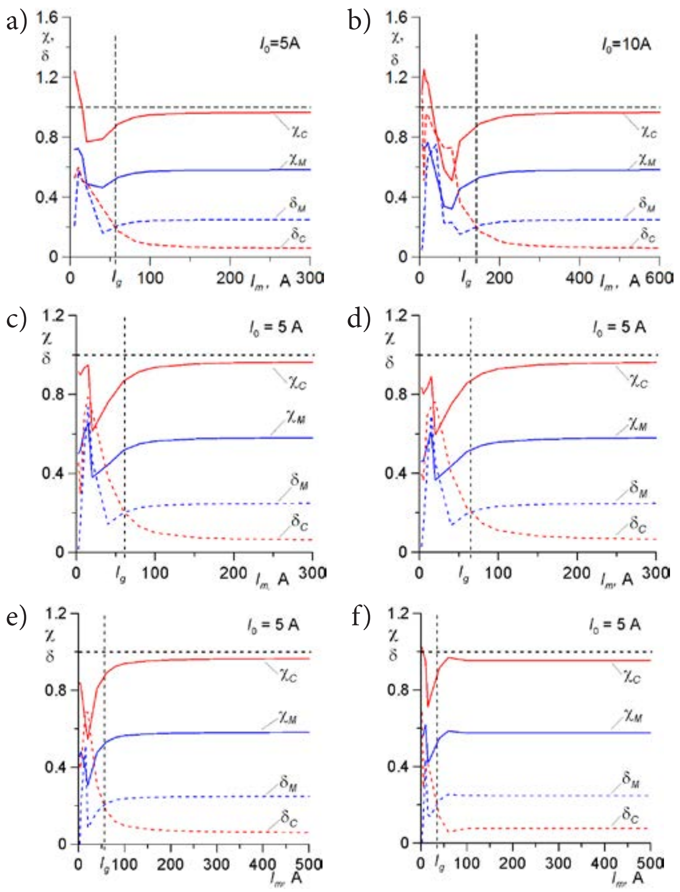

Fig. 4. Diagrams of harmonic indicators $\chi_M(I_m)$ and $\chi_C(I_m)$ and measures of deviation $\delta_M(I_m)$ and $\delta_C(I_m)$ from Mayr and Cassie models

The TWV model tests were repeated with the parameters as presented in Figure 4a, except the one related to the damping factor function amounting to $\alpha = 0{,}3\ A^{-1}$. It can be assumed that the factor was closer to the values observed in the experiments [2]. However, taking this change into consideration did not significantly affect the limiting current value $I_g \cong 70$ A, which is shown in Figure 4c.

In addition, the simulation tests indicate that in the range of excitation amplitude changes I_m to approximately $0{,}75I_g$ the functions χ_M, χ_C, δ_M, δ_C oscillate significantly, in spite of the fact that the $\delta_M(I_m) < \delta_C(I_m)$ is fulfilled. In the low-current ranges numerical instabilities are accompanied by process instabilities related to electric arc quenching.

Further modification of the TWV model consisted in the modification of the damping factor function

$$\theta(|i|) = \theta_0 + \theta_1 \cdot \exp(-\alpha' i^2) \quad (27).$$

The test results with the adopted value $\alpha' > 0.02\ A^{-2}$ and with the remaining parameters such as the previous ones are shown in Figure 5d. Their aim was to reduce the rate of fall of the damping factor characteristics in the area of the point with the abscissa $i = 0$ A. However, introducing this formula with a diagram closer to the real characteristics [2] also proved to have no significant influence on the current value Ig, which continued to be approximately 70 A. Only a slight reduction of the limiting current value ($I_g \cong 60$ A) was obtained following, in addition to the modification (24), the application of the new form of the tapering function (25). This effect is presented in Figure 4e.

Predicting the effect of the minimal damping factor value (corresponding to the Cassie model) on the accuracy of limiting current determination the tests were repeated setting the TWV model parameters: $\theta_0 = 1 \cdot 10^{-5}$ s, $\theta_1 = 1 \cdot 10^{-3}$ s, $\alpha = 0.02\ A^{-1}$, $U_C = 50$ V, $P_M = 90$ W, $I_0 = 5$ A. The simulation results are presented in Figure 4f. On this basis it is possible to state that the criterion conditions are fulfilled in the abscissa of approximately $I_g = 35$ A, which means that the limiting current value determined with this method is over seven times higher than the pre-set value Io. At the same time it is possible to observe the attenuation of oscillation in the low-current range.

The value of high-current damping factor function θ(i) affects the measure of deviation δ_C of the voltage spectrum on the column from the waveform spectrum described by means of the Cassie model in the idealised case $\theta_C = 0$ s. This effect can be expressed with the dependence $\delta_C \propto \theta_C$. For this reason it is possible to predict the modification of the formula (21) in the form of the function $I_g = f(I_0, \delta_C)$.

Determination of low-current arc model time constant

The analytical solution of the Mayr model of the low-current arc in the sinusoidal excitation circuit enables the determination of a time constant using the formula [12]

$$\theta_M = \frac{1}{4\omega}\left(\frac{1}{\chi_M} - \chi_M\right) \quad (28)$$

This dependence was subjected to simulation in MATLAB-Simulink software. For this purpose it was necessary to create a simple circuit with current excitation (f=50 Hz) and Mayr model (P_M= 100 W, θ_M= 10^{-3} s). The errors of the integration method and numerical roundings caused a time constant calculation error of approximately 5%. It was observed that the error decreased slightly along with an increase in the order of harmonics considered. Afterwards it was necessary to represent the negative effect of disturbance by adding Additive White Gaussian Noise (AWGN) to the sinusoidal excitation. The intensity of the noise effect was increased by changing the variance parameter σ^2 in the range between 0 and 5. Such noise is characterised by the normal distribution of disturbance samples, has a flat frequency spectrum and is additive with the prime signal. It was ascertained that the excitation amplitude increase and the noise variance increase lead to an increase in the time constant calculation error (e.g. $I = 3.5$ A, $\sigma^2 = 3$, $\delta_{\%} = 4.6\%$; $I = 12.5$ A, $\sigma^2 = 5$, $\delta_{\%} = 8.33\%$).

The next stage of research included testing the usability of the formula (28) for the approximated calculation of the time constant of the Mayr model making up the TWV hybrid model [1]. This formula can be applied only to the low-current range, in which the value of the time constant θ_M is close to the highest value of damping factor function $\theta_M \cong \theta(i \cong 0)$. A significant increase in current amplitude corresponds to a significant method error increase, which renders the formula (28) useless for the determination of a time constant. Therefore, the value of amplitude should meet the condition $I_m < I_0$. In addition, it was possible to observe a decrease in the accuracy of determining the time constant θ_M along with taking into consideration high-order harmonics.

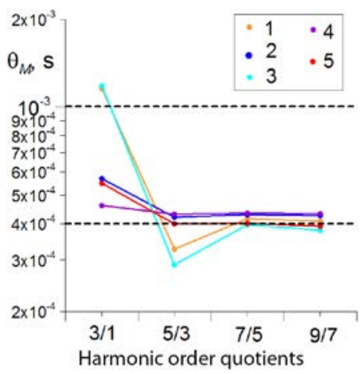

Fig. 5. Effect of the quotient of harmonic numbers $(2k+1)/(2k-1)$ on the determination of the time constant θ_M

Figure 5 presents the diagrams of the maximum TWV model damping factor function values, using the expression (28) for the Mayr model time constant. To this end it was necessary to use current excitations of low amplitude and hybrid model parameters presented in Table 1. It can be seen that the ratio of low-order harmonics (3/1) produces results close to θ_1. In turn, the ratios of high-order harmonic numbers produce damping factor values close to θ_0. Such a shape of diagrams in the high-current range calls for further research and analysis.

Table 1. Parameters of TWV hybrid model with sinusoidal current excitation with amplitude I_m

No.	P_M, W	U_C, V	I_0, A	α	θ_0, s	θ_1, s	I_m, A
1	30	30	5	10	$4 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	5
2	150	120	10	10	$4 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	5
3	150	120	10	10	$4 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	10
4	150	120	20	10	$4 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	5
5	150	120	20	10	$4 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	10

In addition, the simulation of processes in a circuit with excitation $I = 4.8$ A, $f = 50$ Hz and with the model of the following parameters: $P_M = 50$ W, $U_C = 10$ V, $I_0 = 5$ A, $\alpha = 3$, $\theta_0 = 4 \cdot 10^{-4}$ s, $\theta_1 = 1 \cdot 10^{-3}$ s was carried out. The changes of noise variances have an influence on the error of time constant determination according to the formula (28) ($\alpha^2 = 4$, $\delta_{\%} = 20.1\%$; $\sigma^2 = 8$, $\delta_{\%} = 19.9\%$).

The study also involved testing the Novikov-Shellhase model using the static characteristic $U_{stat}(I)$ and described with the following formula

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{NS}}\left(\frac{i}{g \cdot U_{stat}(i)} - 1\right) \qquad (29)$$

The static characteristic was approximated with the following formula

$$U_{stat}(I) = \frac{P_0}{I^m} \qquad (30)$$

where slight deviations of an exponent around the value of 1 were assumed. However, even if the parameter $m = 1$, this model does not correspond to the Mayr model [$P_M \neq U_{stat}(I) \cdot I$] as this model is not the specific case of the Mayr model. The adopted values $m = 0.8$ ($P_0 = 150$ W, $\theta_{NS} = 8 \cdot 10^{-4}$ s) corresponded to the time constant determination error amounting to approximately 12% and poorly increasing with the excitation amplitude. The changes of variance parameter σ^2 in the range between 0 and 5 caused a 4% increase in error, which then was also dependent on the amplitude of current excitation reaching the value of 12.5 A. When the exponent $m = 1.2$ was used, the error of time constant determination amounted to approximately 21% and was poorly dependent on the excitation amplitude. The use of the exponent $m = 1.5$ ($P_0 = 120$ W, $\theta_{NS} = 8 \cdot 10^{-4}$ s) leads to errors amounting to approximately 11%. Additionally introduced noise significantly affects the resultant value of the time constant determination error.

Conclusions:

1. The developed criterion for classifying electric arcs as low or high-current can facilitate the selection of adequate mathematical models and principles of controlling electrotechnological devices.
2. The time constant of the Mayr model approximating real arc waveforms can be determined experimentally on the basis of these voltage spectral analysis data which were used to classify this arc as low-current.
3. The accuracy of determining the TWV model constituent time constant corresponding to the Mayr model decreases along with an increase in the order of arc voltage harmonics used for the calculation of this constant.
4. Disturbances in voltage waveforms have a significant effect on the arc classification accuracy and on the determination of a low-current arc time constant.

References:

1. Tseng K.J., Wang Y., Vilathgamuwa D.M.: Development of a dynamic model of electric arc for power electrics simulations. IEEE Industrial Applications Conference, IAS'96, 1996, vol. 4, p. 2173-2180.
2. Kalasek V.: Measurements of time constants on cascade D.C. arc in nitrogen. Technische Hogeschool Eindhoven, TH-Report 71-E-18. 1971.
3. Edels H., Fenlon F.H.: Theory of a filled-tube thermal arc column. Brit. J. Appl. Phys. 1965, vol. 16, p. 219-230.
4. Sawicki A.: Approximations of arc voltage-current characteristics in electrotechnological devices, http://bulletin.is.gliwice.pl/PDF/2013/06/05_Sawicki_Approximations_of_arc_voltage-current_characteristics_in_electrotechnological_devices.pdf
5. Filkenburg W., Maecker H.: Elektrische Bogen und thermisches Plasma. Handbuch der Physik. S. Flugge (ed.), Springer, Berlin, 1956.
6. Пентегов И.В., Сидорец В.Н.: Сравнительный анализ моделей динамической сварочной дуги. Автомат. Сварка, 1989, № 2, с. 33-36.
7. Sawicki A., Świtoń Ł., Sosiński R.: Próba wykorzystania modeli Cassiego i hybrydowego Cassiego-Mayra do symulowania procesów w obwodach z lampami rtęciowymi. Śląskie Wiadomości Elektryczne, 2010, no. 1, pp 4-9.
8. Junwen Dai, Ruixiang Hao, Xiaojie You, Hu Sun, Xianjin Huang, Yan Li: Modeling of plasma arc for the high power arc heater in MATLAB. 5th IEEE Conference on Industrial Electronics and Applications. 2010, p. 463-468.
9. Sawicki A.: Damping Factor Function in AC Electrical Arc Models. Part 1: Heat Process Relaxation Phenomena, their Approximations and Measurement. http://bulletin.is.gliwice.pl/PDF/2013/02/05_Sawicki_Damping_Factor_Function_in_AC_Electrical_Arc_Models_Part1.pdf
10. Sawicki A.: Damping factor function in AC electric arc models. Part 2: Damping factor function in universal electric arc models with moderate cooling. http://bulletin.is.gliwice.pl/PDF/2013/03/03_Sawicki_Damping_Factor_Function_in_AC_Electrical_Arc_Models_Part2.pdf
11. Sawicki A.: Funkcje wagowe w modelach hybrydowych łuku elektrycznego. Śląskie Wiadomości Elektryczne, 2012, no. 5, pp. 15-19.
12. Мякишев В.М., Жеваев М.С., Шишков Е.М.: Способ определения постоянной времени сварочной дуги. Электротехника, 2009, № 02, с. 20-22. (Myakishev V.M., Zhevaev M.S., Shishkov E.M.: A Method for Determining the Time Constant of a Welding Arc. Russian Electrical Engineering, 2009, Vol. 80, No. 2, pp. 78–80.)