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Effect of Arc Ignition Voltage on Oscillations Generated in Autonomous RLC Circuits. Part 1: Mathematical Models of Circuits and Criteria for Classifying Their Solutions

Abstract: The article discusses reasons for the occurrence of non-linear oscillation in selected welding systems. Special attention was paid to the effect of auxiliary systems on the reduction of arc ignition voltage and, consequently, self-excited oscillation. The study involved the development of mathematical models of simple autonomous circuits with electric arc of specific ignition voltage. The article also describes selected criteria for enabling the identification of non-linear oscillation types. Particular attention was paid to difficulties accompanying the identification of conditions triggering the formation of deterministic chaos.

Key words: electric arc, arc ignition voltage, chaotic oscillation criteria

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1. Introduction

Electric vibrations in nonlinear welding systems may result from periodic excitations triggered by power sources. The aforesaid states take place during AC welding performed using welding transformers or electronic converters (e.g. in TIG welding processes). The generation of vibrations can also be favoured by various external factors affecting the welding circuit, e.g. variable magnetic fields. There are welding technologies using the periodically deflected arc column in the variable magnetic field [1]. In cases involving the welding of aluminium alloys performed using consumable electrodes, the transverse magnetic field is responsible for the widening of the bath and the reduction of its depth as well as for the reduction of porosity and granularity of the metal. Periodic disturbance in the plasma column length can also be induced by mechanical factors, resulting in the transfer of small portions of electrode metal onto the workpiece and the formation of thin layers of the weld; such a solution is used in welding and surfacing technologies [2]. Systems with such external disturbances are classified as non-autonomous or parametric. Non-linearity may result from the dynamic characteristics of arc or those of the power supply system.

When analysing autonomous systems, the effects of external variable influences are usually ignored. In such circuits, oscillation can be triggered as a result of the negative value of dynamic arc resistance or the positive feedback in the system with arc. The transition from two-dimensional systems to three-dimensional ones leads to significant qualitative changes in their behaviour. The primary property of such systems is the possible appearance of deterministic chaos. The transition from typical deterministic states (equilibrium, periodic oscillation, nearly periodic oscillations) to chaos takes place through universal phenomena, e.g. the doubling of the oscillation period. Because of the nonlinearity of the circuit and the multidimensionality of the state space, tests concerning the bifurcation of such systems (even in the simplest cases) are very complex. Therefore, the simplest arc models are usually assumed in them [3-5]. However, the above-presented assumptions do not always allow the modelling (with satisfactory accuracy) of processes taking place in circuits with electric arc. In some arc and plasma devices, the initiation and stabilisation of arc discharge require the application of appropriate auxiliary systems ensuring the reduction of ignition voltage. As a result, both static and dynamic current-voltage characteristics of arc undergo transformations [6, 7]. Such changes can trigger various types of self-excited oscillation in circuits with arcs. Taking into account ignition voltage increases the complexity of a function approximating the static model characteristic, which significantly complicates the use of analytical methods in further research.

2. Autonomous RLC circuits with electric arc and their mathematical models

Publication [3] presents eight simplest circuits containing electric arc, linear elements *R, L* and *C* and the excitation source characterised by constant voltage. Under appropriate conditions, periodic or chaotic oscillation can be generated in the aforesaid circuits. Selected diagrams of the circuits are presented in Figure 1. The adopted initial state variables were current i in inductor L, voltage u in capacitor C and state current i_0 in arc. Work [3] also presents four sets of systems of non-dimensional differential equations. Such a reduction of the number of equations by half in relation to the number of circuits resulted from the possibility of transformation between pairs of corresponding systems of three differential equations. To this end, it was necessary to use the replacement of variable $u \Leftrightarrow E - u$. The non-dimensional form of differential equations reduced the number of required parameters and made the equations more universal.

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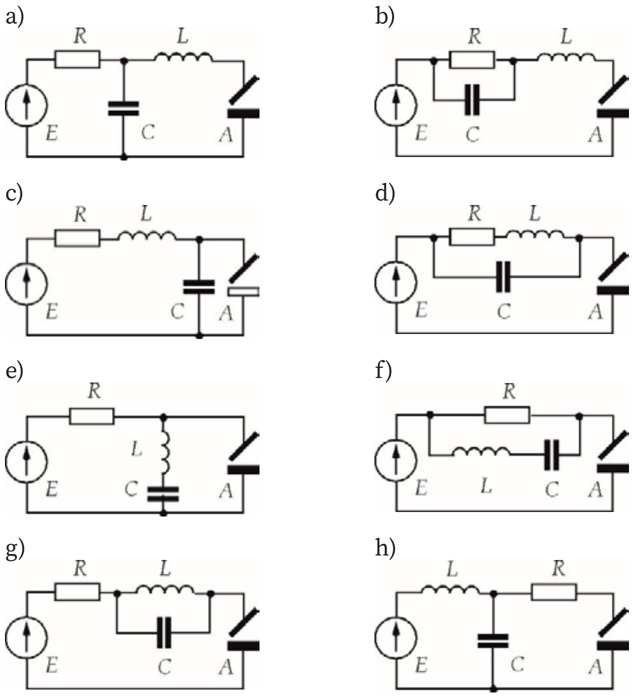

Fig. 1. Electric circuits with arc [3] (A – system (electrode – arc – welded material)

This article presents four mathematical models of circuits with the generalised nonlinear static current-voltage characteristic of arc $U_{arc}(i)$:

- diagrams presented in Figures 1a and 1b correspond to the system of three differential equations

$$\frac{dx}{d\tau} = \frac{1}{\bar{L}}\left(y - xf_{arc}(z)\right) \tag{1}$$

$$\frac{dy}{d\tau} = \frac{1}{\bar{R}\bar{C}} = \left(1 + \bar{R} - \bar{R}x - y\right) \tag{2}$$

$$\frac{dz}{d\tau} = x^2 - z \tag{3}$$

- diagrams presented in Figures 1c and 1d correspond to the system of three differential equations

$$\frac{dx}{d\tau} = \frac{1}{\bar{L}}\left(1 + \bar{R} - \bar{R}x - y\right) \tag{4}$$

$$\frac{dy}{d\tau} = \frac{1}{\bar{C}}\left[x - y(f_{arc}(z))^{-1}\right] \tag{5}$$

$$\frac{dz}{d\tau} = (f_{arc}(z))^{-2}y^2 - z \tag{6}$$

- diagrams presented in Figures 1e and 1f correspond to the system of three differential equations

$$\frac{dx}{d\tau} = \frac{1}{\bar{L}}\left[\frac{(1 + \bar{R} - \bar{R}x)}{\bar{R} + f_{arc}(z)} f_{arc}(z) - y\right] \tag{7}$$

$$\frac{dy}{d\tau} = \frac{1}{\bar{C}}\, x \tag{8}$$

$$\frac{dx}{d\tau} = \frac{1}{\bar{L}}\left[\frac{1 + \bar{R} - \bar{R}x}{\bar{R} + f_{arc}(z)}\right]^2 - z \tag{9}$$

- diagrams presented in Figures 1g and 1h correspond to the system of three differential equations

$$\frac{dx}{d\tau} = \frac{1}{\bar{L}}\, y \tag{10}$$

$$\frac{dy}{d\tau} = \frac{1}{\bar{C}}\left[\frac{1 + \bar{R} - y}{\bar{R} + f_{arc}(z)} - x\right] \tag{11}$$

$$\frac{dz}{d\tau} = \left[\frac{1 + \bar{R} - y}{\bar{R} + f_{arc}(z)}\right]^2 - z \tag{12}$$

The non-dimensional parameters of equations are described by the following formulas:

$$\bar{R} = \frac{RI_0}{U_0}, \quad \bar{C} = \frac{CI_0}{\theta I_0}, \quad \bar{L} = \frac{LI_0}{\theta U_0}, \tag{13}$$

whereas the non-dimensional parameters of time and state variables are the following:

$$\tau = \frac{T}{\theta}, \quad x = \frac{i}{I_0}, \quad y = \frac{u}{U_0}, \quad z = \frac{i_\theta^2}{I_0^2}, \tag{14}$$

where θ – time constant of the Pentegov model of arc.

The non-dimensional form of the static current-voltage is expressed as follows:

$$f_{arc}(z) = \frac{U_{arc}(I_0\sqrt{z})}{U_0\sqrt{z}} \tag{15}$$

Monograph [3] and publications [5, 8] assume a relatively simple form of the static current-voltage characteristic of the model, approximating experimental data with the following power function:

$$U_{arc}(i) = U_0\left(\frac{i}{I_0}\right)^n \tag{16}$$

where $n < 0$.

Publications [6, 7, and 9] describe a relatively simple form of the static current-voltage characteristic of the model, yet taking into consideration the finite value of arc ignition voltage:

$$U_{arc}(i) = U_0\left(\frac{I_0 i}{i^2 + I_M^2}\right)^m \tag{17}$$

The coordinates of extreme point S(I, U) of characteristic (17) are as follows:

$$I = I_M, \quad U = \frac{U_0 I_0^n}{(2I_M)^n} \tag{18}$$

which corresponds to arc ignition voltage. If the current adopted in formula (17) is $I_M = 0$ A, the following characteristic is obtained:

$$U_{arc}(i) = U_0\left(\frac{I_0}{i}\right)^m \tag{19}$$

The above-presented characteristic is identical (16) to the assumption that $m = -n$.
Taking into account correlation (17) in equations (1)–(12) enables the introduction of the following non-linear function:

$$f_{arc}(z) = \frac{U_{arc}(I_0\sqrt{z})}{U_0\sqrt{z}} = \frac{z^{\frac{m-1}{2}}}{(z + \bar{I}_M^2)^m} \tag{20}$$

where non-dimensional current is as follows:

$$\bar{I}_M = I_M / I_0 \tag{21}$$

3. Selected criteria for assessing non-linear system solutions

As mentioned above, the direct application of analytical methods aimed at finding solutions of systems of differential equations with nonlinear characteristics (16) or (17) and determining conditions necessary for the existence of stable periodic oscillation is very difficult. The reasons for such a situation include the relatively high dimension of the state space of the system of equations and the strong nonlinearity of the arc model. Only in some cases of relatively simple systems it is possible to determine the conditions responsible for the generation of various oscillations using the analytical method of Melnikov [10]. A relatively detailed qualitative analysis of the systems of equations with characteristic (16) is presented in publication [3].

The adoption of rational values of circuit parameters can help obtain a non-rigid system of differential equations, and thus facilitate numerical integration processes. However, even the partial investigation of these systems in the areas of parameters of selected ranges entails very laborious calculations and requires the development of an extensive set of diagrams which are difficult to present in an article of limited volume. Periodic and unstable non-periodic oscillations usually involve waveforms in time, phase portraits and point-based modelling. The availability of computer software makes it relatively easy to determine functions of autocorrelation and the spectral power density of mathematical model solutions.

One of the characteristics of dynamic chaos is the sensitivity of its behaviour to assumed initial states. This property reflects the exponential divergence of initially almost infinitely close trajectories. Not all non-periodic and non-harmonic oscillations constitute deterministic chaos. For this reason, it was necessary to develop various chaos criteria, some of which were classified as qualitative, whereas some as quantitative [11, 12].

Numerical tests of systems generating chaotic oscillation require verification using the following criteria:

a) process ergodicity,
b) exponential decrease of the autocorrelation function to zero,
c) continuous spectrum of the power density function with the initial pedestal,
d) positive values of the maximum characteristic Lyapunov exponent.

The qualitative criteria include the extraordinary sensitivity of solution behaviour to changes in initial conditions. Qualitative tests of dynamic systems enable the building of certain mathematical representations from phase portrait images, on the basis of which it is possible to conclude whether a given phase trajectory approaches the point of equilibrium, limit cycle or a strange attractor.

The study involved one of the trajectories $z = z(t)$ of a dynamic system with constant parameter values. On the waveform in time, it was necessary to identify the maximum points of the above-named trajectory and create Lorenz transformation, constituting the dependence of the next maximum on the previous one:

$$z_{n+1} = f(z_n) \tag{22}$$

The shape of the above-named dependence might correspond to a discrete representation known (e.g. from related scientific publications literature) to have chaotic states. In terms of a periodic waveform, it should be a single point.

A more universal tool is Poincaré transformation. In the three-dimensional phase space, it is necessary to select a two-dimensional plane, the arrangement of which affects the form of transformation. The setting of the initial condition (x_0, y_0, z_0) is followed by the identification of a trajectory intersecting the plane at the point having coordinates $(\bar{x}_n, \bar{y}_n)$. The aforesaid intersection takes place many times. Finally, a set of points $(\bar{x}_{n+1}, \bar{y}_{n+1})$, $(\bar{x}_{n+2}, \bar{y}_{n+2})$, ... is obtained. The above-presented points create Poincaré transformation. Chaos is created in the system if a compact cloud is formed from the points of intersection of the trajectory with the plane. However, if one or several points are created, the nature of oscillation is periodic.

Autocorrelation is a measure concerning the relationship between the current and past values of time series. The aforesaid measure determines which past values of series are the most useful for predicting future values. If $x(t)$ is the value of the trajectory of the dynamic system at moment in time t and $x(t + \tau)$ is the value of moment $t + \tau$, the form of the autocorrelation function is the following:

$$C(\tau) = \lim_{T\to\infty} \int_0^T x(t)\cdot x(t+\tau)dt \tag{23}$$

It is even and has a maximum value in state $\tau = 0$. Correlation and autocorrelation functions are used to detect similar or repeating (periodic) structures in signals. In cases of periodic signals, it is a periodic function of the same period. One of the properties of such a function is a rapid decrease in value $C(\tau)$ in the case of the presence of chaotic oscillation.

Knowing values $x(t)$, it is possible to create signal power distribution in relation to pulsation ω. To this end, it is necessary to create an image by decomposing the signal in relation to frequency using the Fourier transform:

$$x(\omega) = \lim_{T\to\infty} \int_0^T x(t) e^{i\omega t} dt \tag{24}$$

The spectrum of the power of signal $P(\omega) = |x(\omega)|^2$ is the distribution of its energy in relation to frequency. In cases of chaotic signals, the so-called pedestal within the low-frequency range is formed on the diagram of such a function. In the aforesaid situation, local minima of function $P(\omega)$ do not have values close to zero. The function itself is continuous, with numerous and very close extrema and decreasing within the high-frequency range. In contrast to the above-named function, function $P(\omega)$ of the periodic signal consists of distinct and sharp peaks of decreasing height, between which the values of the function are close to zero. The foregoing also constitutes a qualitative criterion.

On one hand, the system can show deterministic chaos, whereas on the other it may be dissipative along with decreasing phase volume. Based on the foregoing, Smale created a qualitative criterion in the form of a horseshoe, whose presence indicates the existence of homoclinic orbits.

Quantitative criteria making it possible to verify the creation of chaos include Lyapunov exponents. The greatest Lyapunov exponent characterises the degree of the exponential divergence of close trajectories. If ε_0 is the initial distance between two points in the state space at moment t_0, $\varepsilon(t)$ is the distance between the points after time Δt. The greatest Lyapunov exponent can be determined using the following formula [13]:

$$\lambda \cong \frac{1}{\Delta t} \ln \frac{\varepsilon(t)}{\varepsilon(t_0)} \tag{25}$$

The more precise determination of the exponent is obtained using the following formula:

$$\lambda = \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \ln \frac{\varepsilon(t)}{\varepsilon(t_0)} \tag{26}$$

The greatest Lyapunov exponent characterises the degree of the exponential divergence of initially close trajectories. If the exponent is positive, the system is sensitive to the disturbance of initial conditions and could be characterised by chaotic behaviour.

A more precise behaviour of the system is represented by the spectrum of Lyapunov exponents. In such a situation, the rate of the divergence or convergence of trajectories along various coordinate directions is taken into account. The system of n-dimensional space has n Lyapunov exponents. If a small n-dimensional sphere of radius ε is assumed initially, after time Δt it is possible to obtain an n-dimensional ellipsoid with principal semi-axes designated as $\varepsilon_1, \varepsilon_2, .., \varepsilon_n$. The individual exponents are expressed by the following formula:

$$\lambda_i \cong \frac{1}{\Delta t} \ln \frac{\varepsilon_i(t)}{\varepsilon(t_0)} \tag{27}$$

Based on the spectrum of Lyapunov exponents it is possible to classify the attractors of the dynamic system. If the autonomous system in the three-dimensional space has exponents with the following signs [11]:

- <-,-,-> – is the attractive point of equilibrium;
- <0,-,-> – is the limit cycle,
- <0,0,-> – is the two-dimensional torus,
- <+,0,-> – is the strange attractor.

The most problematic could be the identification of Lyapunov exponents as it requires the analytical determination of the Jacobian matrix composed of derivatives of functions being the right-hand sides of differential equations. Using the analytical capabilities of the MATHEMATICA programme could significantly facilitate the identification of the above-named exponents.

Quantitative criteria of deterministic chaos are related to fractals [11] and include the fractal dimension and Alfréd Rényi›s spectrum of generalised dimensions.

The use of only one (even very strong) criterion in order to qualitatively determine the stability of mathematical model solutions could be risky. For instance, the possibility of precisely determining the signs of Lyapunov exponents is affected by errors in the numerical integration method. The foregoing is particularly important if exponent values are close to zero. For this reason, some authors suggest the use of artificial intelligence for this purpose [13].

4. Conclusions

1. The operation of auxiliary (ignition and stabilising) systems affects the value of electric arc ignition voltage, which should be taken into account when modelling and simulating the operating states of electrotechnical equipment.
2. Taking into account the voltage of arc ignition increases the complexity of mathematical models of circuits with electric arc.
3. The operation of complex autonomous systems with electric arc can entail the generation of various types of nonlinear oscillations. For this reason, the identification of the aforesaid oscillations requires the use of several qualitative and quantitative criteria.

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