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Influence of Arc Ignition Voltage on Vibrations Generated in Autonomous RLC Circuits Part. 2. Oscillations in Circuits with Arc of Undefined and Specified Ignition Voltage Values

Wpływ napięcia zapłonu łuku na drgania generowane w obwodach autonomicznych RLC Cz. 2. Drgania w obwodach z łukiem o wartościach napięcia zapłonu nieokreślonej i określonej

Abstract: The determination of parameters and initial conditions responsible for the formation of chaotic vibrations necessitated the considering of mathematical models of simple circuits with electric arc of unspecified ignition voltage. The use of numerical calculations made it possible to identify ranges of circuit element parameters, where at least one of the Lyapunov exponents of chaotic attractors had a positive value. The subsequent stage of the research work discussed in the article involved the reduction of arc ignition voltage until all exponents reached negative values. The above-named situation corresponds to the disappearance of chaotic vibrations in the circuit. The tests revealed low relative current values of the arc model causing the aforementioned bifurcation.

Key words: electric arc, arc ignition voltage, Lyapunov exponents

Streszczenie: W celu ustalenia parametrów oraz warunków początkowych powstawania drgań chaotycznych, rozpatrzono modele matematyczne prostych obwodów z łukiem elektrycznym o nieokreślonej wartości napięcia zapłonu. Korzystając z obliczeń numerycznych, uzyskano przedziały wartości parametrów elementów obwodu, w których przynajmniej jeden z wykładników Lapunowa atraktorów chaotycznych ma wartość dodatnią. Następnie zmniejszano wartość napięcia zapłonu łuku aż wszystkie wykładniki uzyskały ujemne wartości. Taka sytuacja odpowiada zanikowi drgań chaotycznych w obwodzie. W badaniach stwierdzono małe wartości względne prądu modelu łuku powodującego taką bifurkację.

Słowa kluczowe: łuk elektryczny, napięcie zapłonu łuku, wykładniki Lapunowa

1. Introduction

The first part of the article [1] presents eight simple diagrams of electric circuits of autonomous welding systems with electric arc. Mathematical models corresponding to the above-named systems are characterized by non-linearity and the state space dimension increased to 3. As a result, the systems can generate various types of periodic and aperiodic oscillations. At the same time, there are significant difficulties as regards the analysis of steady and transition states. Appropriate numerical methods [2, 3] enable the performance of quantitative and qualitative tests of such systems. Taking into account parameter changes makes it possible to examine types of bifurcation of periodic solutions as well as to determine ranges corresponding to them. Particular effort is required to investigate a strange attractor triggering the formation of chaotic oscillations.

In terms of technological usability, the most favourable operating state of the system may correspond to the stable equilibrium point. However, variable conditions of welding operations require changes in electric circuit parameters. The knowledge of conditions responsible for the formation of other states (periodic, chaotic, etc.) is therefore needed to ensure the stable implementation of technological processes.

The use of very simplified mathematical models of arc facilitates the performance of analytical and numerical tests of circuits. However, the aforesaid approach could contribute to ignoring the effect of various physical and chemical factors on arc burning conditions (particularly sensitive to ignition voltage) [4, 5]. The assumption of the infinitely great value of such voltage may contribute to the discrepancy of numerical and experimental calculations.

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2. Mathematical models of circuits with electric arc of unspecified ignition voltage

In order to investigate the effect of arc ignition voltage on the formation of oscillations in circuits of simple welding systems it was necessary to perform simulations in accordance with assumption that $I_M = 0$ A. The approximation of current-voltage characteristic (19) enabled the use of dependences (1)–(12) [2] to obtain simplified forms of non-dimensional differential equations [2].

Schematic diagrams in Figures 1a and 1b [1] correspond to the simplified system of equations:

$$\frac{dx}{d\tau} = \frac{1}{\overline{L}}\left(y - xz^{\frac{n-1}{2}}\right) \tag{28}$$

$$\frac{dy}{d\tau} = \frac{1}{\overline{R}\overline{C}}\left(1 + \overline{R} - \overline{R}x - y\right) \tag{29}$$

$$\frac{dz}{d\tau} = x^2 - z \tag{30}$$

Schematic diagrams in Figures 1c and 1d [1] correspond to the simplified system of equations:

$$\frac{dx}{d\tau} = \frac{1}{\overline{L}}\left(1 + \overline{R} - \overline{R}x - y\right) \tag{31}$$

$$\frac{dy}{d\tau} = \frac{1}{\overline{C}}\left(x - yz^{\frac{1-n}{2}}\right) \tag{32}$$

$$\frac{dz}{d\tau} = y^2 z^{1-n} - z \tag{33}$$

Schematic diagrams in Figures 1e and 1f [1] correspond to the simplified system of equations:

$$\frac{dx}{d\tau} = \frac{1}{\overline{L}}\left(\frac{1 + \overline{R} - \overline{R}x}{\overline{R} + z^{\frac{n-1}{2}}}\, z^{\frac{1-n}{2}} - y\right) \tag{34}$$

$$\frac{dy}{d\tau} = \frac{1}{\overline{C}}\, x \tag{35}$$

$$\frac{dz}{d\tau} = \left(\frac{1 + \overline{R} - \overline{R}x}{\overline{R} + z^{\frac{n-1}{2}}}\right)^2 - z \tag{36}$$

Schematic diagrams in Figures 1g and 1h [1] correspond to the simplified system of equations:

$$\frac{dx}{d\tau} = \frac{1}{\overline{L}}\, y \tag{37}$$

$$\frac{dy}{d\tau} = \frac{1}{\overline{C}}\left(\frac{1 + \overline{R} - y}{\overline{R} + z^{\frac{n-1}{2}}} - x\right) \tag{38}$$

$$\frac{dz}{d\tau} = \left(\frac{1 + \overline{R} - y}{\overline{R} + z^{\frac{n-1}{2}}}\right)^2 - z \tag{39}$$

3. Numerical tests of dynamic states in circuits with electric arc of unspecified ignition voltage

Each of mathematical models (28)–(39) of the circuits presented in Figure 1 [1] was assigned a set of characteristics obtained using numerical calculation methods. The Author decided that, in the first instance, the most favourable approach would involve the calculation of the spectrum of Lapunov exponents of periodic solutions in relation to selected parameters. Changes in the parameter of circuit $\overline{L}$ or $\overline{C}$ with the simplified model of arc ($I_M = 0$ A) made it possible to determine an interesting range of the presence of a strange attractor (Figures 2, 4, 6 and 8). It can be seen that the schematic diagrams from Figures 1a-1f [1] correspond to areas of changes of parameters with positive values of greatest Lapunov exponents λ. In cases of small positive values of exponents it was necessary to create additional diagrams $\lambda+$, excluding negative values. The selection of electric circuit parameters was followed by the performance of calculations of dynamic characteristics, the shapes of which were presented in the three-dimensional state space (Figures 3, 5 and 7).

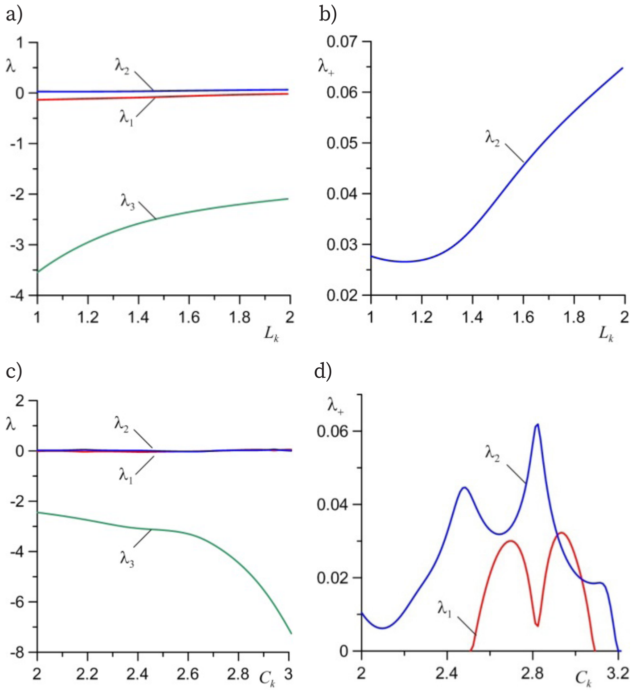

Fig. 2. Correlation between the spectrum of Lapunov exponents and parameters of the elements of the circuit presented in Figures 1a and 1b [1] ($\overline{R} = 15$, $n = -1/3$): a) and b) as functions of the variable inductance of choking coil $\overline{L}$ ($\overline{C} = 3.04$) and c) and d) as functions of the variable capacity of capacitor $\overline{C}$ ($\overline{L} = 1$)

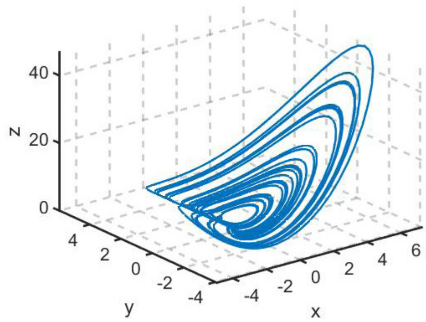

Fig. 3. Dynamic characteristic of the circuit (Fig. 1a) in the three-dimensional state space ($\overline{R} = 15$, $\overline{L} = 1.8$, $\overline{C} = 3$, $n = -1/3$)

Qualitative tests performed in [2, 3] revealed that the system from Figure 1a [1] was characterised by the presence of oscillations corresponding to bifurcations of period doubling. In relation to circuit parameters it was possible to observe strange attractors of "screw-like" or "strip-like" type.

The positive value of Lapunov exponent (Fig. 4) indicates the presence of chaotic oscillations in the system. The strange attractor may adopt the form of a closed strip (Fig. 5).

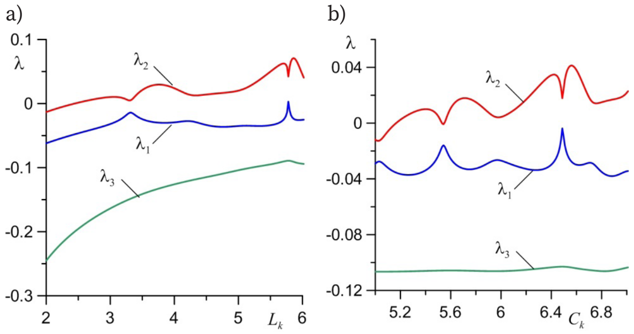

Fig. 4. Correlation between the spectrum of Lapunov exponents and parameters of the elements of the circuit presented in Figures 1c and 1d [1] ($\overline{R}$ = 0,8, n = -1/5): a) as functions of the variable inductance of choking coil $\overline{L}$ ($\overline{C}$ = 6.982), and b) as functions of the variable capacity of capacitor $\overline{C}$ ($\overline{L}$ = 5)

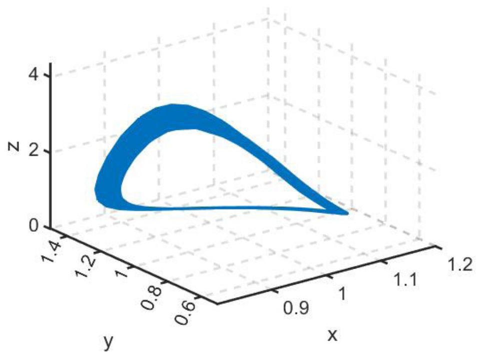

Fig. 5. Dynamic characteristic of the circuit (Fig. 1c) in the three-dimensional state space ($\overline{R}$ = 0.8, $\overline{L}$ = 5, $\overline{C}$ = 7, n = -0.2)

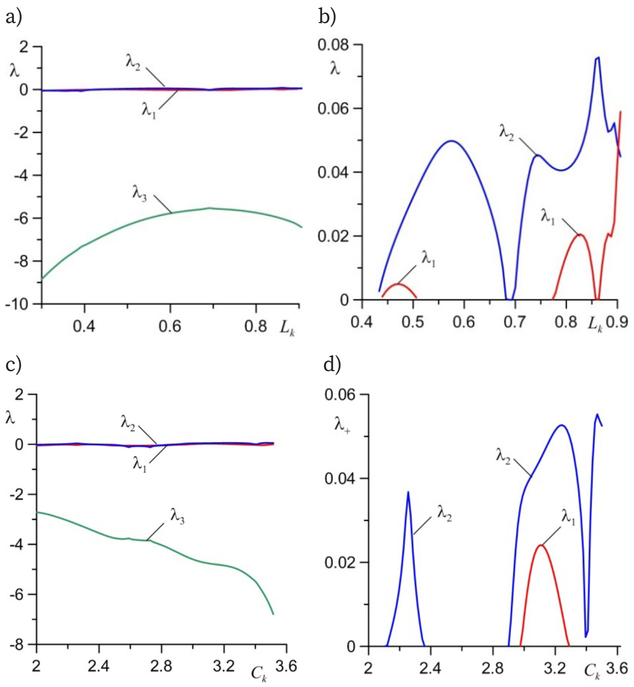

Fig. 6. Correlation between the spectrum of Lapunov exponents and parameters of the elements of the circuit presented in Figures 1e and 1f [1] ($\overline{R}$ = 15, n = -1/3) and b) as functions of the variable inductance of choking coil $\overline{L}$ ($\overline{C}$ = 3.41)) and c) and d) as functions of the variable capacity of capacitor $\overline{C}$ ($\overline{L}$ = 0.7)

The qualitative tests of the system presented in Figures 1g and 1h were performed in [2]. The tests revealed the lack of Hopf bifurcation in the system. All the phase trajectories approached the stable point of equilibrium. In such a situation, the Lapunov exponents presented on Fig. 8, had only negative values.

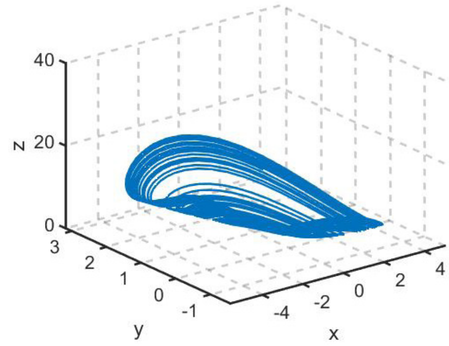

Fig. 7. Dynamic characteristic of the circuit (Fig. 1e) in the three-dimensional state space ($\overline{R}$ = 15, $\overline{L}$ = 0.9, $\overline{C}$ = 3.2, n = -1/3)

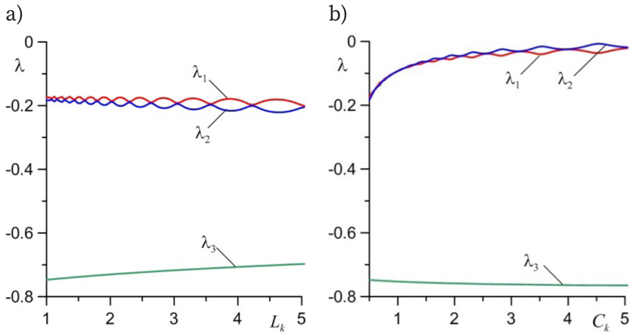

Fig. 8. Correlation between the spectrum of Lapunov exponents and parameters of the elements of the circuit presented in Figures 1g and 1h [1] ($\overline{R}$ = 5, n = -1/3)): a) as functions of the variable inductance of choking coil $\overline{L}$ ($\overline{C}$ = 0,5)and b) as functions of the variable capacity of capacitor $\overline{C}$ ($\overline{L}$ = 1)

4. The effect of ignition voltage on the presence of chaotic oscillations in circuits with electric arc

The assumption of the unspecified value of arc ignition voltage is the rough approximation of the actual state. Physically, there are factors decreasing its value, which leads to the reduction of the range of parameters of chaotic solutions. In the tests, the diagrams presented in Figures 2, 4 and 6 were used to initially select parameters of systems (28) – (36) ensuring the presence of strange attractors. The foregoing corresponded to current $\overline{I}_M$ = 0. Afterwards, the value of current was increased, recording positive values of

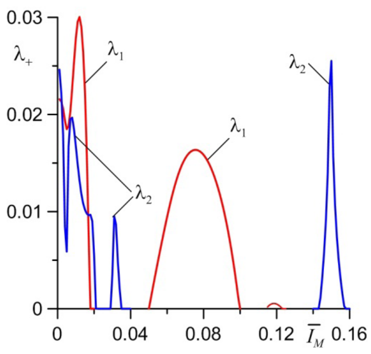

Fig. 9. Correlation between the spectrum of positive values of Lapunov exponents λ_+ and non-dimensional current $\overline{I}_M$ in the circuit presented in Fig. 1a [1] ($\overline{R}$ = 15, $\overline{L}$ = 1.8, $\overline{C}$ = 3, m = 1/3)

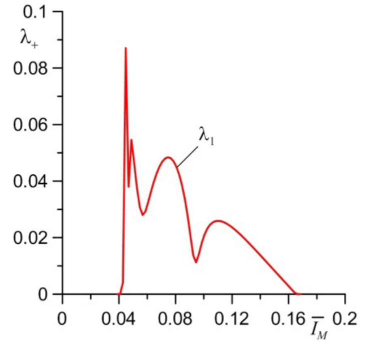

Fig. 10. Correlation between the spectrum of positive values of Lapunov exponents λ_+ and non-dimensional current $\bar{I}_M$ in the circuit presented in Fig. 1b [1] ($\bar{R}$ = 0.8, $\bar{L}$ = 5, $\bar{C}$ = 7, m = 1/5)

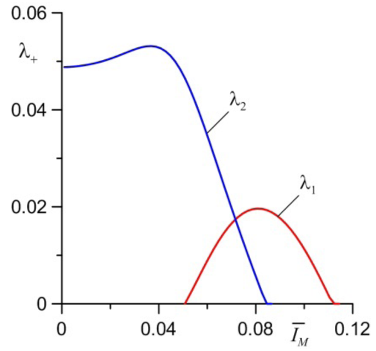

Fig. 11. Correlation between the spectrum of positive values of Lapunov exponents λ_+ and non-dimensional current $\bar{I}_M$ in the circuit presented in Fig. 1c [1] ($\bar{R}$ = 15, $\bar{L}$ = 0,9, $\bar{C}$ = 3,2, m = 1/3)

Lapunov exponents (Figures 9–11) in the circuits described by means of equations (1)–(9). It could be observed that even a little value of current could trigger the fading of chaotic oscillations. It could be concluded that the measures and methods used in welding engineering for stabilising arc could prevent the excitation of chaotic oscillations.

5. Conclusions

1. The inclusion of the finite value of electric arc ignition voltage led to a relatively insignificant increase in the complexity of mathematical models of circuits with electric arc.
2. Increasing the value of preset current $\bar{I}_M$ led to the reduction of electric arc ignition voltage. The foregoing resulted in the reduction of areas with positive values of Lapunov exponents and thus, the reduced generability of chaotic oscillations in circuits with arc.
3. As a result of the reduction of ignition voltage of arc, its static characteristic became similar to the characteristic of high-current arc, which, in turn, led to the loss of the negative value of dynamic resistance and, consequently, to the impossibility of generating self-excited oscillations in the circuit.

REFERENCES

[1] Sawicki A.: Wpływ napięcia zapłonu łuku na drgania generowane w obwodach autonomicznych RLC. Cz. 1. Modele matematyczne obwodów i kryteria klasyfikacji ich rozwiązań. Materials Science and Welding Technologies, 2024, no. 1, pp. 57-60.

[2] Sydorets V.N., Pentegov I.V.: Deterministic Chaos in Nonlinear Circuits With Electric Arc, Kiev, Ukraine: Svarka, 2013.

[3] Sydorets V.N.: The Bifurcations and Chaotic Oscillations in Electric Circuits with Arc. W. Mitkowski and J. Kacprzyk (eds.): Model. Dyn., [w:] Processes & Sys., SCI 180, pp. 2–42. Springer-Verlag Berlin Heidelberg 2009.

[4] Sawicki A.: Wykorzystanie charakterystyk statycznych o określonym napięciu zapłonu do modelowania łuków w szerokim zakresie zmian wymuszenia prądowego. Biuletyn Instytutu Spawalnictwa, 2019, vol. 63, no. 1, pp. 29-33.

[5] Sawicki A.: Electric arc models with non-zero residual conductance and with increased energy dissipation. Archives of Electrical Engineering, 2021, vol. 70, no. 4, pp. 819-834. DOI: 10.24425/aee.2021.138263.