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# Testing the Effect of Column Length Disturbances on the Method used for Determining the Pentegov Model Describing the Electric Arc of the Ayrton Type Static Characteristics

**Abstract:** The research involved the determination of physical factors affecting the generation of arc column length disturbances in electrotechnological devices. The article contains a justification related to the selection of the Pentegov model for representing the dynamic characteristics of a sinusoidal current powered arc. The research also included the modification of the Ayrton formula approximating static voltage-current characteristics by taking into consideration arc column length random disturbances as well as the development of an arc macro-model and simulations of processes in a simple circuit using Matlab-Simulink software. The research demonstrated the efficiency of the three measurement method for determining the Pentegov model parameters in conditions arc column length disturbances.

**Keywords:** electric arc, Pentegov model, Ayrton formula, time constant

## Introduction

The strong electric arc sensitivity to various external effects is one of major obstacles hampering the obtainment of appropriate discharge stability, maintaining required technological process parameter ranges and ensuring the high quality of electrotechnological device control and diagnostics. In spite of using various measures and methods of arc stabilisation in AC plasma torches, the phenomena of turbulent gas flow, the effect of external magnetic fields on a plasma column, disturbances generated in grids supplying power or plasma-creating gas affect arc parameters, predominantly column length changes. Such changes also take place in free arcs due to intensive turbulent gas flows, consumable electrode material drop transitions or electrode vibration accompanying CMT welding. The representation of length changes is relatively simple in the Pentegov model [1] by appropriately modifying a static voltage-current characteristic. It is in the Pentegov model where this characteristic can be of any shape, which allows taking into consideration various physical properties of gases, gas pressure values, types of material, shapes of electrodes etc.

There are numerous reasons [2] why electrotechnologists try to develop and use various electric arc simplified mathematical models. In spite of these simplifications, one of the difficulties related to the practical application of such models is a quite complex "online" determination of model characteristics and parameters, often requiring the use of specialist measurement equipment and introducing additional undesirable disturbances [3].

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Depending on the design and the plasma device operation principle as well as on the power level of the technology used, it is sometimes possible to determine the family of AC plasma torch static characteristics in a full or limited range of direct current excitation. However, quite frequently due to high technological requirements, the necessity of ensuring the long, active life of a device, the lack of a proper supply source, the lack of appropriate measuring equipment and of dedicated control system as well as due to a pursuit of avoiding failures, only AC arc devices are subjects of investigations. In such a situation by using the known Mayr and Cassie simple arc models, it is possible to determine [4, 5] only very rough approximations of static characteristics. However, the Pentegov model enables a more accurate representation of dynamic processes in a circuit with an AC arc and, at the same time, offers a more accurate approximation of static voltage-current characteristics. In this model an arc is treated as a non-linear, inert and two-clamp element of an electric circuit. The model approximates arc dynamic characteristics without going into the details of physical processes taking place in plasma and on electrodes. For this reason, it is not necessary to distinguish processes in column equilibrium plasma and those in the non-equilibrium plasma of near-electrode areas. The effect of near-electrode voltage drops on arc modelling depends on numerous factors and can be negligible in the case of long arcs.

## Pentegov Electric Arc Model Creation Postulates

The input object for considerations is a real electric arc with thermal inertness and related electric inertness. In such an arc due to a stepped change of current $i(t)$, a stepped voltage impulse and a quasi-exponential waveform of column resistance changes are formed. In the Pentegov model a real arc is replaced with a hypothetical arc which also fulfils an energy balance equation. Unlike in the case of an original arc, this model is electrically inertialess and provides the possibility of stepped resistance changes. For this reason, the resistance of an arc is determined using fixation lagging current $i_\theta(t)$. This current changes with a specified time constant $\theta$ and constitutes a real current $i(t)$ representation. All arc isoenergetic states are characterised by one variable referred to as arc state current $i_\theta(t)$. This variable enables the determination of arc model parameters and arc model dynamic characteristics [1].

The inertia of an electric arc is represented by a first-order differential linear equation binding the state current square with the real arc current square

$$\theta \frac{di_\theta^2}{dt} + i_\theta^2 = i^2 \tag{1}$$

The advantage of the Pentegov model is the possibility of using any approximation of static voltage-current characteristics and a constant damping factor function value (so-called time constant) at the same time. Other popular simple models use only a very simplified approximation form of current – voltage dependence. In turn, hybrid models utilise non-linear (yet close to real) damping factor functions [6]. Using the possibilities offered by the Pentegov model it is possible to consider an arc whose family of static characteristics can be described using the dependence provided by Ayrton

$$U_{st}(I, L_a) = a + bL_a + \frac{c + dL_a}{I} = U_C + \frac{P_M}{I} \tag{2}$$

where $L_a$ – constant arc length; $a, b, c, d, U_C, P_M$ – constant approximation coefficients. Such a mathematical model is, to some extent, the generalisation of the known Mayr and Cassie models [2].

In practice, due to various previously mentioned external effects the arc column length $L_a(t)$ undergoes periodic or random changes.

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This can be taken into consideration while describing the position of points on a static characteristic

$$U_{st}(I, L_a(t)) = a + bL_a(t) + \frac{c + dL_a(t)}{I} = \\ = a + bL_0 \cdot \left(1 + \frac{\lambda(t)}{100}\right) + \frac{c + dL_0 \cdot \left(1 + \frac{\lambda(t)}{100}\right)}{I} \quad (3)$$

where $L_0$ – determined distance between electrodes; $\lambda(t)$ – column length disturbances expressed in %.

## Determining the Parameters of a Selected Pentegov Model Variant of an Electric Arc

If in a circuit with an arc of a static characteristic (2) there is variable sinusoidal current excitation of pulsation $\omega$

$$i = I_m \cos\left(\omega t + \frac{\varphi}{2}\right) \quad (4)$$

and the state current can be described by the dependence

$$i_\theta^2 = I_{sk}^2 \left(1 + \cos\varphi \cos 2\omega t\right) \quad (5)$$

where $I_{sk} = I_m / \sqrt{2}$ – root-mean-square current, the value of phase angle $\varphi$ is related to the arc time constant

$$\sin\varphi = \frac{2\omega\theta}{\sqrt{1 + (2\omega\theta)^2}} \quad (6)$$

The remaining model parameters can be determined using a method of three measurements with three various excitation sinusoidal root-mean-square current values. On the basis of these values it is possible to determine root-mean-square currents and voltages as well as arc resistance average values ($U_{sk}1$, $I_{sk}1$, $R_1$), ($U_{sk}2$, $I_{sk}2$, $R_2$), ($U_{sk3}$, $I_{sk3}$, $R_3$). Then, model parameters searched for are defined by the following formulas [7]:

$$P_M = \frac{W_P}{W} \quad (7)$$

$$U_C^2 = \frac{W_U}{W} \quad (8)$$

where WP, WC and W determinants are defined as follows:

$$W = \begin{vmatrix} 2\left(R_1 I_{sk1}^2 - R_2 I_{sk2}^2\right), & I_{sk1}^2 - I_{sk2}^2 \\ 2\left(R_2 I_{sk2}^2 - R_3 I_{sk3}^2\right), & I_{sk2}^2 - I_{sk3}^2 \end{vmatrix} \quad (9)$$

$$W_P = \begin{vmatrix} \left(U_{sk1} I_{sk1}\right)^2 - \left(U_{sk2} I_{sk2}\right)^2, & I_{sk1}^2 - I_{sk2}^2 \\ \left(U_{sk2} I_{sk2}\right)^2 - \left(U_{sk3} I_{sk3}\right)^2, & I_{sk2}^2 - I_{sk3}^2 \end{vmatrix} \quad (10)$$

$$W_U = \begin{vmatrix} 2\left(R_1 I_{sk1}^2 - R_2 I_{sk2}^2\right), & \left(U_{sk1} I_{sk1}\right)^2 - \left(U_{sk2} I_{sk2}\right)^2 \\ 2\left(R_2 I_{sk2}^2 - R_3 I_{sk3}^2\right), & \left(U_{sk2} I_{sk2}\right)^2 - \left(U_{sk3} I_{sk3}\right)^2 \end{vmatrix} \quad (11)$$

In turn, the arc time constant can be determined using the formula

$$\theta = \frac{1}{2\omega \sqrt{\left[\frac{2RI_{sk}^2}{P_M} + \left(\frac{U_C I_{sk}}{P_M}\right)^2 - \left(\frac{U_{sk} I_{sk}}{P_M}\right)^2\right]^2 - 1}}$$

where associated values (Isk, Usk, R) can come from one measurement selected out of the three measurements.

## Testing the Efficiency of the Method of Three Measurements in Determining the Pentegov Model Parameters in Arc Length Disturbance Conditions

The publication [8] presents a representation of sinusoidal current disturbances in a circuit with an arc by properly modulating the source of supply current. To this end it was necessary to use a noise generator having a maximum amplitude $\Delta i$ defined in percentage in relation to the amplitude of the supply source current (±5%). On this basis it was possible to demonstrate the efficiency of the method of three measurements in determining the Pentegov model parameters.

Depending on the design of the plasma torch and the technology performed by means of this plasma torch the column length random disturbances can be generated. The intensity of these disturbances can disrupt the diagnostics of the arc and that of the electric device. In order to investigate generated effects it was necessary to use a random generator connected in a cascade manner with a first order inert module

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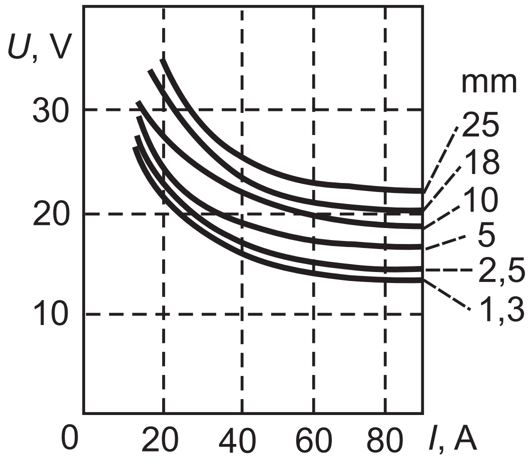


Fig. 1. Family of static characteristics of an arc in argon

($T$ = 0.35 ms) having a maximum amplitude $\lambda$ defined in percentage in relation to the distance between electrodes. The arc length was disturbed with a signal having a timing frequency of 300 Hz.

The input data was the family of voltage-current static characteristics of an arc burning in argon between tungsten electrodes 18 mm away from each other (Fig. 1) [9]. The characteristics were approximated using the Ayrton formula, which enabled the determination of parameters:

Table 1. Accuracy of determining the Pentegov model parameters based on the simulation of processes in a circuit with an electric arc

| $I_m$ A | $I_{sk}$ A | $U_{sk}$ V | $R_{sr}$ Ω | $\delta_P$ % | $\delta_U$ % | $\delta_\theta$ % | $\lambda$ % |
|---|---|---|---|---|---|---|---|
| 58 | 41.01197 | 26.02237 | 0.84012 | | | | 0.0 |
| 60 | 42.42619 | 25.69288 | 0.801088 | 0.008061 | 0.071157 | 0.025939 | |
| 62 | 43.8404 | 25.38492 | 0.765262 | | | | |
| 73 | 51.6186 | 23.98459 | 0.611332 | | | | |
| 75 | 53.03282 | 23.77646 | 0.589422 | 0.008307 | 0.002644 | 0.022297 | |
| 77 | 54.44703 | 23.5793 | 0.568934 | | | | |
| 58 | 41.01198 | 26.01585 | 0.839948 | | | | 0.1 |
| 60 | 42.42619 | 25.68619 | 0.800916 | 0.037958 | 0.001674 | 0.021718 | |
| 62 | 43.84041 | 25.37808 | 0.765091 | | | | |
| 73 | 51.6186 | 23.98889 | 0.611451 | | | | |
| 75 | 53.03282 | 23.78067 | 0.589535 | 0.037013 | 0.001725 | 0.022793 | |
| 77 | 54.44703 | 23.58341 | 0.569042 | | | | |
| 58 | 41.01197 | 26.06329 | 0.841597 | | | | 1.0 |
| 60 | 42.42619 | 25.73221 | 0.802463 | 0.445804 | 0.040984 | 0.027533 | |
| 62 | 43.84041 | 25.42278 | 0.766546 | | | | |
| 73 | 51.6186 | 24.02762 | 0.612524 | | | | |
| 75 | 53.03282 | 23.8185 | 0.590556 | 0.444801 | 0.041028 | 0.028579 | |
| 77 | 54.44703 | 23.6204 | 0.570014 | | | | |
| 58 | 41.01197 | 26.27525 | 0.848926 | | | | 5.0 |
| 60 | 42.42619 | 25.93782 | 0.80934 | 2.263977 | 0.216087 | 0.097209 | |
| 62 | 43.84041 | 25.62246 | 0.773013 | | | | |
| 73 | 51.6186 | 24.20066 | 0.617295 | | | | |
| 75 | 53.03282 | 23.98755 | 0.595094 | 2.263413 | 0.216223 | 0.099058 | |
| 77 | 54.44703 | 23.78566 | 0.574337 | | | | |
| 58 | 41.01198 | 26.54266 | 0.858086 | | | | 10.0 |
| 60 | 42.4262 | 26.19721 | 0.817936 | 4.548997 | 0.435992 | 0.275703 | |
| 62 | 43.84041 | 25.87435 | 0.781096 | | | | |
| 73 | 51.61858 | 24.41868 | 0.623256 | | | | |
| 75 | 53.03279 | 24.20052 | 0.600764 | 4.547507 | 0.435838 | 0.277535 | |
| 77 | 54.44701 | 23.99384 | 0.579737 | | | | |

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$a$ = 12 V; $b$ = 0.241 V/mm; $c$ = -494.996 W; $d$ = 44.519 W/mm. On the basis of these parameters it was possible to determine the model parameters: $U_C$ = 16,338 V, $P_M$ = 306,346 W. The adopted Pentegov model time constant value amounted to $\theta = 1{\cdot}10^{-3}$ s.

Afterwards Matlab-Simulink software was used to create the macro-models of a circuit and of an electric arc. A current source of the parameters $I_m \approx (58 \div 77)$ A and $f$ = 50 Hz connected in series with a reactor of resistance R = 0.1 Ω and inductance $L$ = 1 mH was used as excitation. In order to satisfy the assumptions of the methods of three measurements it was necessary to set three current amplitude values (58; 60, 62) A and (73; 75; 77) A performing three series of simulations with various arc length disturbance.

Table 1 contains the results of the numerical tests performed. If $\lambda$ = 0%, errors $\delta_P$, $\delta_U$ and $\delta_0$ of determining the model parameter values are very low and do not exceed 0.1%. In fact, they are numerical calculation errors and to some extent represent interference in a circuit with an arc and in a digital measurement system. Afterwards, the level of arc length disturbances was increased by introducing an additional signal from a noise generator. Increasing these disturbances leads to an increase in measurement uncertainty. High values $\lambda$>1% correspond to uncertainty of several per cent. Particularly poorly sensitive to arc length disturbances is the determination of value UC.

## Conclusions:

1. The use of the Pentegov electric arc mathematical model significantly extends the possibilities of engineering testing of electrotechnological devices with discharges in various physical conditions.

2. The use of the Pentegov electric arc mathematical model significantly simplifies the representation of any discharge static and dynamic characteristics.

3. The method of three measurements enables the relatively accurate determination of the Pentegov mathematical model parameters in the conditions of limited electric arc length disturbances.

4. The method of three measurements enables the determination of approximation coefficients of static characteristics using the Ayrton type formula in spite of arc energising only with sinusoidal current.

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