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Damping factor function in AC electric arc models. Part 2: Damping factor function in universal electric arc models with moderate cooling

Abstract: Nonlinear function of the heat processes damping factor in thermal column plasma have been introduced to modified mathematical models of the AC electric arc. This way, universal arc models with moderate cooling and constant geometric sizes, using static characteristics, have been created. Similar method has been used to hybrid arc models with variable column length connecting Berger and Kulakov models. It has been proposed the introduction of nonlinear damping factor function to modified Voronin model using static characteristics. Thereby the extended range of current for application of these models has been achieved.

Keywords: electric arc, damping factor, time constant, Mayr model, Cassie model, Berger model, Kulakov model, Voronin model, hybrid model

Introduction

Most of today's arc and plasma-arc welding devices are designed as universal tools used for joining and cutting elements of various geometrical dimensions (thickness) and shapes made of various materials. Such processes require the use of diverse torches, electrodes, shielding gases, current pulses of various amplitudes, shape, packing degree, polarity and frequency. In variable operating conditions, the systems of automatic control ensure stable electric arc burning, high quality of technological processes, high production efficiency, low material consumption as well as minimum noxiousness to personnel, the environment etc. However, such versatility results in the occurrence of a wide range of state variables, the appearance of various non-linear static and dynamic characteristics of supply systems and electric discharge.

Increasing advancements in computer-aided methods for designing electrotechnological and electrothermal welding devices require more and more accurate arc discharge models, precisely reproducing the non-linearity and dynamics of thermal and electric processes. However, the earlier imperfections of the experimental determination of the dynamic characteristics of arc discharges are accompanied by imperfections in their mathematical modelling [1]. Despite emerging compromises between the required accuracy of the reproduction of physical processes and the ease of measurements, the simplicity of interpretation, the low complexity and short computational time, the most popular arc models still fail to ensure appropriate precision. This is largely due to the wide range of exciting current changes and external effects causing changes to column geometrical dimensions.

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This study presents universal electric arc models with moderate cooling of the extended current range of approximations of static and dynamic characteristics. The objective of this work required the application of modified mathematical models using static characteristics and hybrid models, matching the Mayr and Cassie models. The use of the non-linear damping factor function corresponding to experimentation results was suggested [1] in the models.

In modelling electric arc dynamic characteristics a damping function (or its specific value – time constant) along with other characteristics and dynamic parameters of a specific model constitute the whole complex of closely related quantities approximating the course of real physical processes with a pre-set accuracy. Due to random disorders, obtaining repeatable arc experimentation results with a pre-defined accuracy is very difficult. Even more problematic is matching several characteristics determined in various separately conducted experiments (e.g. static current-voltage characteristics, damping factor function) in a single model. This process cannot be carried out by means of simple analytical or even numerical methods but requires the use of complex procedures for the identification of electric arc mathematical model characteristics.

Damping function in electric arc models with moderate cooling using static characteristics

One of the most commonly known general static characteristics of an electric arc with moderate cooling is the dependence provided by Nottingham:

$$U_{stat}(l,I) = a_s + b_s l + \frac{c_s + d_s l}{I^n} \qquad (1)$$

where l – column length; I – direct current intensity; a_s – sum of near-cathode and near-anode voltage drop; b_s – gradient of deflected arc voltage, which in the case of arc burning coaxially with electrodes corresponds to electric field intensity E_{stat}. The shapes of typical static characteristics of electric field voltage and intensity are presented in Figure 1. In the case of strong currents it is possible to assume that static voltage U_{stat} does not depend on current.

$$U_{stat}(l) = a_s + b_s l \qquad (2)$$

According to formula (1), the generalised power static characteristic is described by the formula

$$P_{stat}=U_{stat}(l, I)\cdot I= \\ = (a_s + b_s l)\cdot I + (c_s + d_s l)\cdot I^{i-n} \qquad (3)$$

which in accordance with formula (2) in the case of string currents adopts a simpler form:

$$P_{stat}=U_{stat}(l)\cdot I=(a_s+b_s l)\cdot I \qquad (4)$$

If and arc length does not change (l = const), the static voltage-current characteristics is the following:

$$U_{stat}(I)=A+BI^{-n} \qquad (5)$$

where $A=a_s+b_s l$ = const; $B = c_s+d_s l$ = const. Similarly, the power of an arc is a non-linear current function:

$$P_{stat}(I)=U_{stat}\cdot I=f(I)=AI+BI^{1-n} \qquad (6)$$

and in the case of strong currents the dependence becomes linear

$$P_{stat}=U_{stat}\cdot I=I\cdot \text{const} \qquad (7)$$

A modified arc model by Mayr [2, 3] can be presented in a general conductance form

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_M(i(t))}\left[\frac{P_{kol}(t)}{P_{dys}(t)} - 1\right] \qquad (8)$$

where g – column conductance; i – alternating current intensity; P_{kol} – electric power supplied to plasma column; P_{dys} – power of energy dissipated from column; $\theta_M(i)$ – damping function corresponding to the time of relaxation of heat processes. Electric

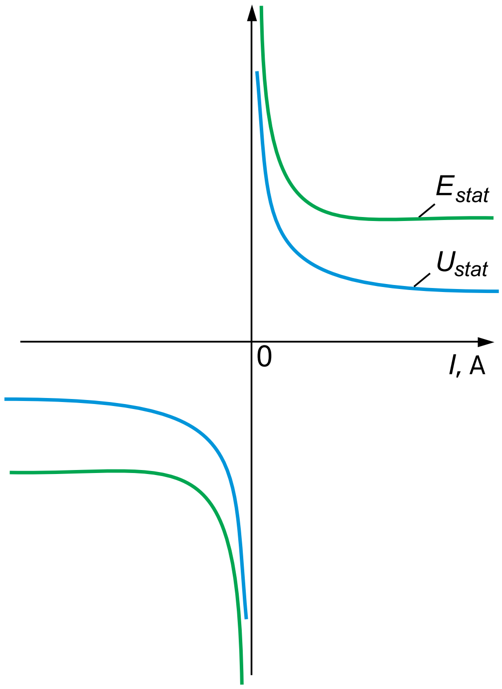

Fig. 1. Static characteristics of arc:
$U_{stat}(I)$ - voltage-current, $E_{stat}(I)$ – electric field intensity

power supplied to thermal plasma is expressed by the formula below:

$$P_{kol}(t) = u_{kol} i = \frac{i^2}{g} \tag{9}$$

where u_{kol} – voltage drop on arc column. In the classical Mayr model it is assumed that $P_{dys}(t)$ = const. In the range of stronger currents this condition cannot be met any longer and usually the Cassie model is used instead [3]. As heat dissipation processes react slowly to external disturbances, one can roughly assume that a power loss is principally determined by static characteristics [4], $P_{dys}(t) \approx P'_{stat}(i(t))$, i.e.

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_M(i)}\left[\frac{u_{kol} i}{P'_{stat}(i)} - 1\right] \tag{10}$$

where P'_{stat} – takes into consideration power losses only in column plasma, without near-electrode areas. The function of loss power can be approximated by means of dependences (6) or (7) allowing a generalised dynamic arc model to be obtained:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_M(i)}\left[\frac{u_{kol} i}{A'|i| + B|i|^{1-n}} - 1\right] \tag{11}$$

where $A' = b_s l$ = const. The power of losses in disequilibrium plasma $P_E = a_s\,|i|$ of very thin near-electrode areas is taken into consideration separately. More often applied are static voltage-current characteristics (1), (2) or (5), and then

$$P_{dys}(t) = U_{stat}(i) \cdot i = \frac{i^2}{G_{stat}(i)} \tag{12}$$

After substituting formula (12) to formula (10) it is possible to obtain the modified Mayr equations in the conductance form

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_M(i)}\left[\frac{G_{stat}(i)}{g} - 1\right] \tag{13}$$

The shapes of the typical static characteristics of power and column conductance are presented in Figure 2.

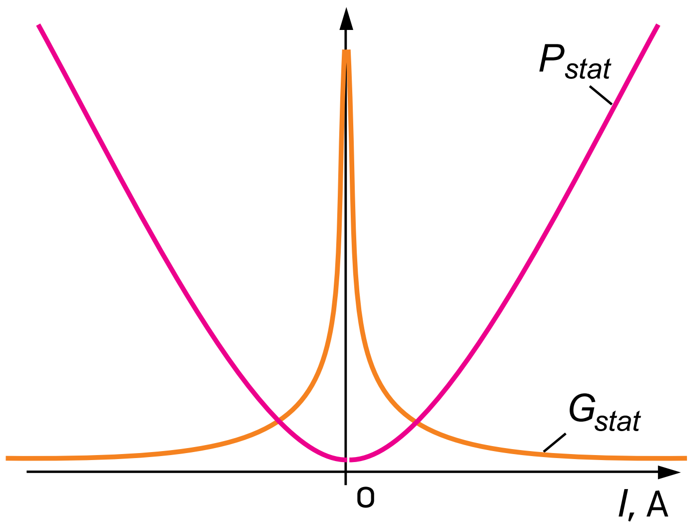

Fig. 2. Arc static characteristics:
$P_{stat}(I)$ - power-current, $Gstat(I)$ - conductance-current

At this moment, when conductance does not change in time, the static characteristics of an arc in this model has the following form:

$$U_{stat}(i) = \frac{P_{stat}(i)}{i} \tag{14}$$

$$G_{stat}(i) = \frac{i^2}{P_{stat}(i)} = \frac{i^2}{U_{stat}(i) \cdot i} \tag{15}$$

Thus, based on this it is possible to write model (13) in a conductance form

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_M(i)}\left[\frac{i}{g\cdot U'_{stat}(i)}-1\right] \tag{16}$$

which on the basis of formulas (5) and (10) leads to the generalised dependence

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_M(i)}\left[\frac{|i|}{g\cdot\left(A'+B|i|^{-n}\right)}-1\right] \tag{17}$$

In comparison with the classical Mayr model [2], models (11) and (17) can significantly more accurately reproduce the static and dynamic characteristics of an arc in a wider range of current changes. This results not only from the variation of a time constant obtaining the form of the damping function $\theta_M(i)$, but also from applying a more accurate function approximating static voltage--current or power-current characteristics.

Damping function in arc TWV hybrid model matching Mayr and Cassie models

In the TWV hybrid model of an arc [5] the fractions of currents flowing through two parallel non-linear conductances, corresponding to the Mayr and Cassie models, depend on their resultant value. Thus, it is possible to write

$$i(t)=u_{kol}g=u_{kol}\cdot g_M\exp\left(-\frac{i^2}{I_0^2}\right)+ +u_{kol}\cdot g_C\cdot\left(1-\exp\left(-\frac{i^2}{I_0^2}\right)\right) \tag{18}$$

where I_0 – limiting current between the Mayr and Cassie models. Hence we obtain

$$g(t)=g_M(t)\cdot\exp\left(-\frac{i^2}{I_0^2}\right)+g_C(t)\cdot\left(1-\exp\left(-\frac{i^2}{I_0^2}\right)\right) \tag{19}$$

The model selections conditions are the following:

- Mayr model

$$g(t)\approx g_M(t)=\frac{i_M^2}{P_M}-\theta_M\frac{dg_M}{dt},\ \text{if } i<I_0 \tag{20}$$

- Cassie model

$$g(t)\approx g_C(t)=\frac{u_{kol}i_C}{U_C^2}-\theta_C\frac{dg_C}{dt},\ \text{if } i>I_0 \tag{21}$$

where P_M – constant power of the Mayr model; θ_M – time constant of the Mayr model ($0<\theta_C<<\theta_M$); U_C – constant voltage of the Cassie model; θ_C – time constant of the Cassie model ($0<\theta_C<<10^{-3}$ s).

Figure 3 presents the shapes of the typical static characteristics of voltage and power. Adopted approximations in the form of the constant values of Cassie model voltage and Mayr model power are marked against their background of the aforesaid static characteristics. Both models also assume the constant values of a damping factor, which is presented in Figure 4 against the background of typical characteristics $\theta(i)$.

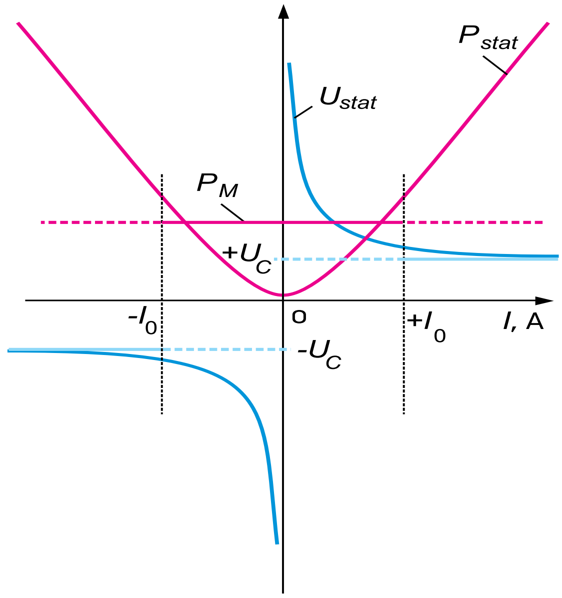

Fig. 3. Static characteristics and dynamic parameters of arc: $U_{stat}(I)$ - voltage-current, $P_{stat}(I)$ – power-current, $P_M(I)$=const – Mayr model power, $U_C(I)$ = const – Cassie model voltage, I_0 – limiting current between the Mayr and Cassie models

On the basis of formulas (18)-(21) a hybrid model is created [5]

$$g=G_{\min}+\left[1-\varepsilon(i)\right]\frac{u_{kol}i}{U_C^2}+\varepsilon(i)\frac{i^2}{P_M}-\theta_{MC}(i)\frac{dg}{dt} \tag{22}$$

where $\theta_{MC}(i)$ – damping factor function of TWV model.

In practical considerations it is usually as-

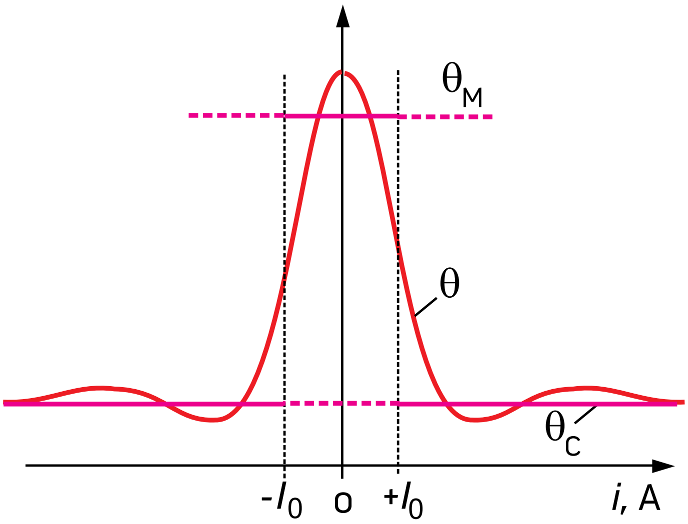

Fig. 4. Characteristics and dynamic parameters of arc:
$\theta(i)$ – characteristics of damping factor,
θ_M –Mayr model time constant,
θ_C –Cassie model time constant

sumed that $G_{min} = 0$ [5, 6]. Then, formula (22) can be written in a simplified form

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{MC}(i)}\left\{[1-\varepsilon(i)]\frac{u_{kol}i}{gU_C^2}+\varepsilon(i)\frac{u_{kol}i}{P_M}-1\right\} \quad (23)$$

where the designation of a tapering function was introduced

$$\varepsilon(i)=\exp\left(-\frac{i^2}{I_0^2}\right) \quad (24)$$

The form of this function may vary and depends on the type of an electrotechnological device and its operating conditions [7].

Damping function in variable length arc hybrid model matching Berger and Kulakov models

While considering the models of an electric arc with the variable length of a plasma column it is possible to modify separate Cassie and Mayr models retaining the constant value of damping factors (time constants) and current ranges preferred by them.

An increase in the geometrical dimensions of the plasma column of a high-current arc is accompanied by an increase in energy necessary for the generation of additional plasma volume. The assumed axial-cylindrical shape of a column and its tension by a length dl corresponds to an increase in thermal power

$$\frac{dQ_a}{dt}=\frac{dQ_a}{dl}\frac{dl}{dt}=q_l\frac{dl}{dt} \quad (25)$$

where q_l – arc energy linear density. Therefore, in simplifying conditions [8] thermal power required for the generation of additional plasma volume is roughly proportional to the rate of a length increase. This is accompanied by some relaxation times resulting from gas thermal inertia and additional cooling of a column. As the arc voltage grows along with an increase in a column length, in publication [8] the following approach to determining Cassie model voltage component was suggested (21):

$$u_c^2(l) = al \quad (26)$$

where the parameter a is almost constant in the wide range of current i changes. In turn, additional power $p_v(dl/dt)$ dissipated from the column is determined by $u_c^2 g$; due to dissipativity, the lack of even partial conservativity and the return of energy to the circuit leads to the following dependence [8]:

$$p_v\left(\frac{dl}{dt}\right)=\begin{cases} b\dfrac{dl}{dt} & \text{if } \dfrac{dl}{dt}>0 \\ 0 & \text{if } \dfrac{dl}{dt}\le 0\end{cases} \quad (27)$$

A modified Cassie equation with the variable value of voltage $U_C(t)=U_C\left(l,\frac{dl}{dt}\right)$ enables obtaining the conductance form of the Berger model [8]

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{CB}}\left(\frac{u_{kol}^2}{u_C^2(l)+\frac{1}{g}p_v\left(\frac{dl}{dt}\right)}-1\right) \quad (28)$$

where $p_v(dl/dt)$ - power needed for the generation of additional plasma volume; θ_{CB} – time constant of Cassie-Berger model.

Kulakov suggested the modification of a low-current arc model (16) taking into consideration column length changes. The first-degree Mayr-Kulakov model in the conductance form is the following [9]:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{MK}}\left[\frac{i}{g\cdot l\cdot E_{stat}(i)}-1\right]-\frac{1}{l}\frac{dl}{dt} \qquad (29)$$

where $E_{stat}(i)$ – non-linear static characteristics of electric field intensity; θ_{MK} – time constant of the Mayr-Kulakov model. One of the imperfections of approximation by means of this model is overlooking the impact of plasma physical properties on conductance dynamics during column length changes.

Using the dependence approximating the static voltage-current characteristics (1) it is possible to determine the static characteristics of electric field intensity

$$E_{stat}(I)=\frac{\partial U_{stat}(l,I)}{\partial l}=b_s+\frac{d_s}{I^n} \qquad (30)$$

Then, the Kulakov model has the following form:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{MK}}\left[\frac{|i|}{g\cdot l\cdot\left(b_s+d_s|i|^{-n}\right)}-1\right]-\frac{1}{l}\frac{dl}{dt} \qquad (31)$$

The arc column hybrid model, taking into consideration the changes of an arc length, matches models (28) and (29) in the manner (23) by means of an appropriate tapering function $\varepsilon(i)$. Therefore the model has the following form:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{BK}(i)}\left\{[1-\varepsilon(i)]\frac{u_{kol}^2}{u_C^2(l)+\frac{1}{g}p_v\left(\frac{dl}{dt}\right)}+\right. \qquad (32)$$

$$\left.+\varepsilon(i)\frac{u_{kol}}{g\cdot l\cdot E_{stat}(i)}-1\right\}-\varepsilon(i)\frac{1}{l}\frac{dl}{dt}$$

where $\theta_{BK}(i)$ – damping factor function of the hybrid Berger-Kulakov model.

If a relatively low rate of arc length changes is assumed ($dl/dt\approx 0$), equation (32) can be written as

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{BK}(i)}\left\{[1-\varepsilon(i)]\frac{u_{kol}^2}{u_C^2(l)}+\varepsilon(i)\frac{u_{kol}}{l\cdot E_{stat}(i)}-1\right\} \qquad (33)$$

After taking into consideration approximations (26) and (30) the equation is as follows:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{BK}(i)}\left\{[1-\varepsilon(i)]\frac{u_{kol}^2}{al}+\right. \qquad (34)$$

$$\left.+\varepsilon(i)\frac{|u_{kol}|}{l\cdot\left(b_s+d_s|i|^{-n}\right)}-1\right\}$$

The matching of the Berger and Kulakov models as well as using the non-linear function of a damping factor makes it possible to use the hybrid model for simulating processes in electrotechnological devices operating with a wide range of current and an arc discharge column length.

Damping function in Voronin model of arc with variable geometrical dimensions

The Voronin model makes it possible to take into consideration an external influence exerted on the length and diameter of a cylindrical column. In order to create such a model it is necessary to make a number of simplifying assumptions [9, 10]. The model basis is a simplified equation of the thermal balance of a column. It is assumed that the dissipated power is proportional to the side area of an arc:

$$P_{dysS}(l,S)=p_S l\sqrt{4\pi S} \qquad (35)$$

As a result, an arc model with variable geometrical dimensions $S(t)$ and $l(t)$ of the following general conductance form is obtained [10]:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_S(S)}\left(\frac{u_{kol}i}{P_{dysS}(l,S)}-1\right)+$$

$$-\frac{1}{l}\frac{dl}{dt}\left(1+\ln\frac{g\,l}{K_g S}\right)+ \qquad (36)$$

$$+\frac{1}{S}\frac{dS}{dt}\left(1-\ln\frac{g\,l}{K_g S}\right)$$

where a damping factor function is

$$\theta_S(S) = \frac{Q_0 S l}{P_{dysS}(l,S)} = \frac{Q_0}{p_S}\sqrt{\frac{S}{4\pi}} \tag{37}$$

and Q_0 – reference factor, J/m^3; K_g – coefficient of unitary conductance approximation, S/m; l – length of arc column, m; p_S – density of power dissipated by the side surface of a column, W/m^2; S – area of arc cross section, m^2. All three parameters Q_0, K_g, p_S are determined on the basis of experiments and are assumed to be constant quantities.

If the relative rate of arc length changes is low ($dl/dt \approx 0$), equation (36) can be written as

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_S(S)}\left(\frac{u_{kol} i}{P_{dysS}(l,S)} - 1\right) + \frac{1}{S}\frac{dS}{dt}\left(1 - \ln\frac{g\,l}{K_g S}\right) \tag{38}$$

Models (36) and (38), similarly as the Mayr model, reproduce arc characteristics in the low-current range relatively well. Therefore they can be used to calculate processes in devices with a relatively low temperature of the area surrounding a discharge. Such conditions occur in open arc welding or during melting of a charge at the initial stages of arc furnace operation. While considering a free or quasi-free arc, in which the area of column cross section is primarily determined by the module of current value, using equation (36) one obtains

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_S(i)}\left(\frac{u_{kol} i}{P_{dysS}(l,S(i))} - 1\right) + -\frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{g\,l}{K_g S(i)}\right) + \frac{1}{S(i)}\frac{dS}{di}\frac{di}{dt}\left(1 - \ln\frac{g\,l}{K_g S(i)}\right) \tag{39}$$

Adopted assumptions simplifying model (36) lead to almost a linear dependence of a damping function (37) on an arc column diameter $\theta(S(i)) \propto \sqrt{S(i)} \propto d(i)$. In turn, in theoretical deliberations [11] the relation $\theta(S(i)) \propto S(i)$ is assumed. However, experimental tests [5, 11] reveal almost a reverse tendency $\theta(i) \propto |i|^{-1}$, which should be recognised as a real one, especially due to the fact that simplified cylindrical arc models take into consideration only selected dominant heat processes, and entirely pass over gasodynamic and electromagnetic processes (e.g. effects of contraction and gas pumping by a column).

Figure 5 presents a typical shape of characteristics $d(i)$ and $S(i)$. One can see that there is no correlation of these quantities with a damping factor function. To some extent an increase in θ in areas where current passes through zero can be explained by the significant weakening of a contraction effect and momentary plasma expansion.

Damping function in Voronin modified arc model using static characteristics

While considering the case of a free or quasi-free welding arc, on the basis of experimentation and theoretical analysis [12] on can adopt a dependence related to an arc column diameter

$$d = k|i|^m \tag{40}$$

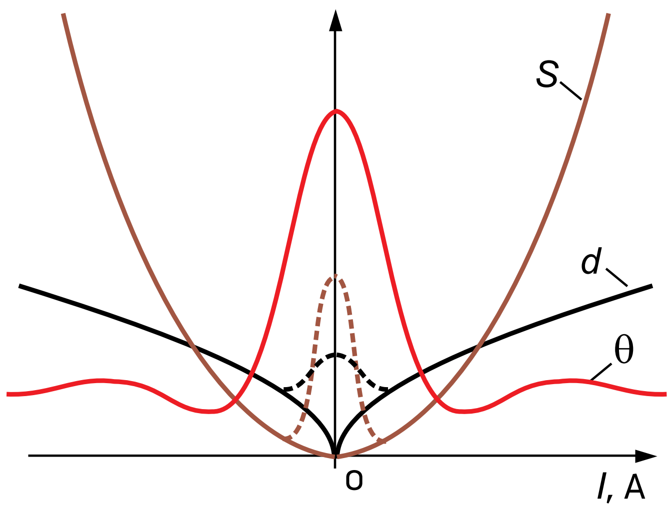

Fig. 5. Arc static and dynamic characteristics of: $\theta(I)$ – damping factor, $d(I)$ – column diameter, $S(I)$ – area of column cross section

As heat dissipation processes react slowly to external disturbances, just as previously it is possible to roughly assume that $P_{dys}(t) \approx P'_{stat}(i(t))$, which on the basis of formula (35) leads to the following dependence:

$$P_{dysS}(l, S(i)) = p_S l \sqrt{4\pi S(i)} = E_{stat}(i) \cdot li \qquad (41)$$

Thus, the intensity of a field is expressed by the following formula:

$$E_{stat}(i) = \pi p_s k|i|^{m-1} sign(i) = (b_s + d_s|i|^{-n}) \cdot sign(i) \qquad (42)$$

Hence, on the basis of formula (36) the following dependence is obtained:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_S(i)}\left(\frac{i}{gE_{stat}(i) \cdot l} - 1\right) + \\ - \frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{g\,l}{K_g S(i)}\right) + \\ + \frac{1}{S(i)}\frac{dS(i)}{di}\frac{di}{dt}\left(1 - \ln\frac{g\,l}{K_g S(i)}\right) \qquad (43)$$

If compared with the Kulakov model (29) this model significantly extends the possibility of approximating characteristics as it takes into consideration not only the dynamics of length changes but also the dynamics of column cross section changes.

Slow arc length changes ($dl/dt \approx 0$) correspond to the following equation:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_S(i)}\left(\frac{i}{gE_{stat}(i) \cdot l} - 1\right) + \\ + \frac{1}{S(i)}\frac{dS(i)}{di}\frac{di}{dt}\left(1 - \ln\frac{g\,l}{K_g S(i)}\right) \qquad (44)$$

If in formula (40) a typical value $m = ⅔$ [12] is assumed, using formula (42) the dependence $E_{stat}(i) \propto |i|^{-1/3}$ is obtained. This dependence corresponds to applied approximations of electric field intensity in a low-current arc with moderate cooling of a column [4]. In addition, formula (42) can be used to determine the basic model parameter, i.e. the surface density of thermally dissipated power:

$$p_S = \frac{b_s + d_s|i|^{-n}}{\pi k|i|^{m-1}} \qquad (45)$$

Concluding remarks

1. Introducing a damping function non-linearly dependent on current extends the possibilities of applying dynamic arc mathematical models using static characteristics.
2. Introducing a damping function non-linearly dependent on current strengthens the versatility of arc hybrid models using sub-models of a low and high-current arc.
3. The developed mathematical models include a wide range of changes of current and those of plasma column geometrical dimensions. As a result, they can be used to simulate processes in various arc electrotechnological and plasma devices (both welding and electrothermal ones) with moderate cooling of a discharge area.

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