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Approximations of arc voltage-current characteristics in electrotechnological devices

Abstract: The usefulness of different voltage-current characteristics has been defined for describing different properties (static, dynamic, energy, frequency etc.) of an arc in the electrical circuits of electrotechnological devices. The influence of different physical factors on the shape of characteristics has been analysed. The study presents the analytical forms of functions developed for approximating the static and commutative characteristics of an arc in AC plasma torches with a preset parameter in the form of a gas mass stream and/or a plasma column length.

Keywords: electric arc, static characteristics, dynamic characteristics, effective characteristics, commutative characteristics, AC plasma torch

Introduction

An electric arc constitutes a non-linear element both in DC and AC circuits. In electrical engineering such an element is described by means of various voltage-current characteristics namely the following [1]:

1. static ones, obtained during very slow changes of current excitation;
2. dynamic ones, described with momentary values, obtained during relatively fast changes of current excitation;
3. the first harmonics, described with the amplitudes (or root-mean-square values) of the first harmonics of current and voltage, obtained during relatively fast changes of current excitation;
4. effective (root-mean-square) ones, described with the root-mean-square values of current and voltage, obtained during relatively fast changes of current excitation;
5. mean ones, described with the mean half-period values of current and voltage of the source, obtained during relatively fast changes of current excitation;
6. incremental ones, described by small increments of current and voltage, obtained during slow changes of current excitation;
7. amplitude-phase ones, combining the amplitudes and phases of current harmonics with the amplitudes and phases of voltage harmonics, obtained during relatively fast changes of current excitation.

The families or evolutions of characteristics depending on a discrete or continuous change of a given parameter are determined experimentally and presented in a graphic form in relation to controlling and a more detailed description of the physical properties of an electric arc. The parameter used is usually the length of a column, the amplitude of a harmonic, the value of the component of a current excitation constant, the stream of mass (volume) or the pressure of gas washing around a column, the temperature of a gaseous medium,

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the chemical composition of a gaseous environment etc.

The selection of a specific kind of arc characteristics depends on the type of analysis of the physical properties of an electrotechnological device including the impact of technological conditions on the operation of a device, the type of a control system, the impact of a plasma device on a power supply network, the selected method of mathematical tests (analytical, numerical) etc. The possibility of the practical determination of a specific kind of current-voltage characteristics is determined by the properties and external characteristics of the source of supply as well as by the characteristics of the measuring equipment.

In the mathematical modelling of an arc the static and dynamic characteristics of the latter are of the greatest importance. In the case of a very low value of the arc time constant and due to the great complexity and ambiguity of dynamic characteristics it is sometimes necessary to use the simplification of the dynamics of a hysteresis loop to the form of a unique non-linear averaging characteristic, similarly as in the case of a commutative characteristic of ferromagnetic materials [2, 3].

As the characteristics of the first harmonics ignore a constant component, it is usually used in the character of the parameter of the family of curves.

Effective characteristics depend on voltage and current waveforms and while specifying them it is necessary to indicate conditions in which they were obtained. The frequently adopted independence of these characteristics from the excitation waveform curve does not always lead to true results.

Due to the very strong non-linearity of the characteristics of a DC electric current, the analytical calculations of control systems widely use incremental voltage-current characteristics. Small excitation current increments around the equilibrium point make it possible to use local linearization, which in turn enables the simplification of the analytical tests of stability conditions as well as offers the possibility of using an operational calculus to calculate transition states and steady states of a low change amplitude.

The analysis of non-linear circuits with an AC arc proves much more difficult. Only in the simplest cases of low-order systems it is possible to use very labour-consuming analytical methods for quantitative and qualitative tests in a low-dimensional state space [4]. The quantitative tests of high-order non-linear systems use the standard methods of numerical integration (usually tautness-stable algorithms), whereas qualitative tests require the use of specialist computer programmes [5]. The characteristics and the methods of the first harmonics have the features of the so-called harmonic linearization. Due to the significant simplification of models, they are willingly used in the analysis of non-linear control system with an electric arc (the descriptive function method). It should be noted that arc characteristics created by means of the first harmonics method remain non-linear and their shape can widely differ to that of e.g. static characteristics [1].

During the operation of a plasma torch the voltage-current characteristics of an arc evolve. Relatively fast changes include the start-up of a device when a temperature increase of structural elements affects the state of an electrode operation and a gas pressure increase inside a constrictor duct. The natural slow changes of characteristics may result from the erosion of electrodes or the change of their thermal states due to the formation of deposits from liquid in cooling ducts or the formation of chemical reaction products on electrodes.

Approximations of DC electric arc static characteristics

The static characteristics of an electric arc constitute the limit state of dynamic characteristics, in which the rate of conductance changes in time is very low $dg/dt \to 0$. It is only in such situations that the value of a damping factor function does not affect their shape. The lack of fast arc conductance changes may result from the absence of the changes of excitation current, plasma column length or plasma column diameter, conditions of heat exchange with the environment etc. in time.

In the case of a free arc, static characteristics can be described with the formula provided by Nottingham [6]

$$U_{stat}(I,l) = a + bl + \frac{c+dl}{I^n} \quad (1)$$

where *a, b, c, d, n* – constant approximation coefficients. The influence of the changes of a column length, *l*, on voltage, U_{stat}, is very similar to the influence of the changes of a gas stream, $\dot{m}_{gaz}$, washing an arc longitudinally or crosswise (Fig. 1) and a gas pressure, *p*. This encourages researchers to use this formula to describe the static characteristics of plasma torches with an internal arc usually operating with a defined arc length. What changes in them is the stream of mass and the chemical composition of a gas washing around a column. Experimental tests of such a DC device have led [7] to the establishment of a relationship between the voltage, U_{stat}, and current, *I*, of an arc.

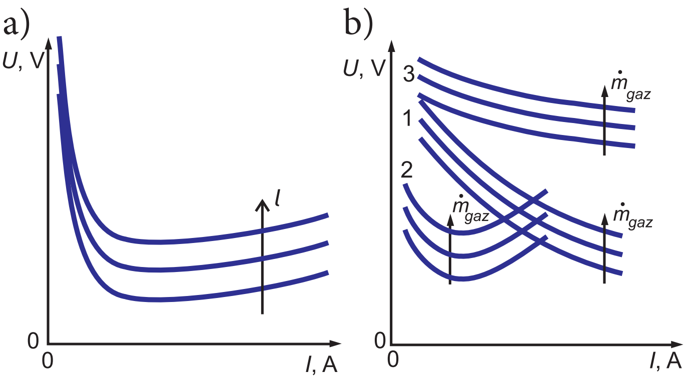

Fig. 1. Diagrams of arc static characteristics: a) with various length of column washed freely with convective gas flow; b) with various forced stream of washing gas and constant column length (1, 2, 3 – various types of plasma torches)

$$U_{stat}(I,\dot{m}_{Gaz}) = A + B\dot{m}_{gaz} + \frac{C + D\dot{m}_{gaz}}{I^n} \quad (2)$$

in which A, B, C and D – constant coefficients determined experimentally (A – depends on the sum of near-electrode voltage drops); *n* – exponent depending on an electrode material. Using a gas volume stream, $\dot{v}_{gaz}$, it is possible to obtain a dependence of a similar form

$$U_{stat}(I,\dot{v}_{gaz}) = A + B\dot{v}_{gaz} + \frac{C + D\dot{v}_{gaz}}{I^n} \quad (3)$$

where $\dot{v}_{gaz}$ - a gas volume stream in l/min (generally $\dot{v}_{gaz} = \dot{m}_{gaz} / \rho_{gaz}$). In the case of gaseous mixtures, e.g. composed of two ingredients *a* and *b*, the formula (3) can be written in the following form [7]

$$U_{stat}(I,\dot{v}_a,\dot{v}_b) = \\ = A + B\cdot(\dot{v}_a + \dot{v}_b) + \frac{C + D\cdot \dot{v}_a + \dot{v}_b}{I^n} \quad (4)$$

with an assumption that the chemical composition of the mixture does not significantly affect the values of the coefficients A, B, C and D.

Plasma torches with an external arc can operate both with a variable arc length, *l*, as well as with a variable plasma-creating gas stream. This causes the formulas for their static characteristics to be more complicated. Analysing the coefficients A, B, C, D, it was revealed [8] that starting from a gas mass flow of 0.7 g/s the coefficients are the functions of two variables $\dot{m}_{gaz}$ and *l* of the following form:

$$U_{stat}(I,l,\dot{m}_{gaz}) = a_0 + a_1\dot{m}_{gaz} + (b_0 + b_1\dot{m}_{gaz})\cdot l + \\ + \frac{c_0 + c_1\dot{m}_{gaz} + (d_0 + d_1\dot{m}_{gaz})\cdot l}{I^n} \quad (5)$$

where the coefficients A, B, C, D in the formulas (1)-(3) and a_0, a_1, b_0, b_1, c_0, c_1, d_0, d_1 in the formula (5) can be determined by means of the least squares method on the basis of experimental data. The intensity of an electric field in a column is expressed by the dependence

$$E_{stat}(I,\dot{m}_{gaz}) = \frac{\partial U_{stat}(I,l,\dot{m}_{gaz})}{\partial l} = b_0 + b_1\dot{m}_{gaz} + \frac{d_0 + d_1\dot{m}_{gaz}}{I^n} \quad (6)$$

The formulas (1)-(6) require the continual experimental determination of the set of coefficients (A, B, C, D) or (a_0, a_1, b_0, b_1, c_0, c_1, d_0, d_1) for the approximation of arc characteristics in a plasma torch of specific characteristic features such as the design (e.g. the shape of a constrictor and of a nozzle, the spatial arrangement of electrodes, the types of cathode and anode materials, geometrical dimensions), the type of a plasma-creating gas, the pressure of gas etc. Also, the ranges of current excitation, the ranges of external influence parameter changes (on a column length and a gas mass stream) can affect the values of approximation parameters. As can be seen in such a representation of characteristics, the sets of these parameters are neither explicit nor implicit analytical functions of plasma torch characteristic features. For this reason the analysis of their properties as well as their interpretations and practical application pose great difficulties.

Depending on the values of indicated parameters or whole characteristics and the designs of plasma torches, their voltage-current characteristics vary significantly. They may contain steeply-dropping, increasing, flat or very strongly non-linear fragments. It is possible to observe [9] that the shape of arc static characteristics is strongly determined by the design of a plasma torch (Fig. 2). For this reason it is possible to divide plasma torches into several basic types and carry out statistical generalisations on the basis of experimental tests of very many such devices of various power. On the basis of the similitude theory and dimensional analysis it is possible to obtain relatively simple formulas defining the families of voltage-current characteristics

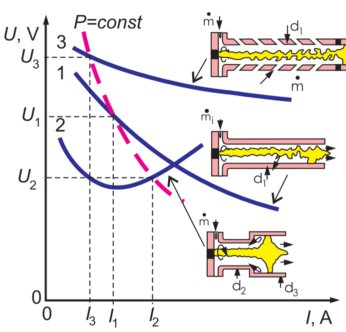

Fig. 2. Static characteristics of arc stabilised with gas flow in three types of plasma torch constrictor duct (1 – with self-adjusting arc length, 2 – with edge, 3 – with sectional constrictor);

correct in the limited ranges of the parameters and variables of the state of a plasma torch operating with a specific gas [10, 11].

A function approximating voltage-current characteristics is searched for in the form of the product of the powers of similarity numbers

$$\frac{Ud}{I} = A\cdot\left(\frac{I^2}{\dot m_{gaz} d}\right)^{n_1}\left(\frac{\dot m_{gaz}}{d}\right)^{n_2}(pd)^{n_3} \qquad (7)$$

where d – constrictor duct diameter; p – gas pressure. The values of the coefficients (A, n_1, n_2, n_3) depend on the range of variables and parameters (U, I, d, p, $\dot m_{gaz}$), the type of a plasma torch, the chemical composition of a gas and even on the polarity of electrodes [9].

The analysis of arc voltage-current characteristics (Fig. 2) reveals the possibility of obtaining the same power $P = UI$ using one of the selected structural diagrams taking into consideration arc current $I_3 < I_1 < I_2$ and voltage U_i ($i = 1, 2, 3$). The selection of a plasma torch design depends on many factors (required power, efficiency, life, the stability of stream parameters etc.) in a specific technological process. Important factors also include the simplicity of a plasma arch design, a plasma torch investment cost and the process of spare parts [11].

The thermal efficiency of a plasma torch, η_t, is defined by heat losses in structure cooling elements. For this reason, the geometrical shape of a constrictor is decisive in this case. On the basis of the generalisation of many experimental tests it was possible to obtain the following relationship describing the thermal efficiency of a plasma torch operating with an air stream [11]

$$\frac{1-\eta_t}{\eta_t} = 1{,}08\cdot 10^{-4}\left(\frac{I^2}{\dot m_{gaz} d}\right)^{0,27}\left(\frac{\dot m_{gaz}}{d}\right)^{-0,27}(pd)^{0,30}\left(\frac{l}{d}\right)^{0,50} \qquad (8)$$

The formula is correct in a wide range of current intensity $I = 50\div3600$ A, gas mass stream $\dot m_{gaz} = 1\cdot10^{-3}\div2.2$ kg/s, constrictor duct diameter $d = 1\cdot10^{-2}\div7.6\cdot10^{-2}$ m, pressure $p = 1\cdot10^5\div8\cdot10^5$ Pa. On the basis of experimental tests it was possible to establish, with an accuracy of ±10%, that the formula can be used for the calculation of one-chamber plasma torches with tubular output electrodes with an edge (in the latter case the assumed average arc length $\bar{l}$ is $\bar{l} = l/d + L/D$, where: l – constrictor duct length; D - electrode-nozzle diameter; L - electrode-nozzle length).

Approximations of AC electric arc dynamic characteristics

The usability of dependences describing the static characteristics of a DC plasma torch arc (1)-(7) for modelling an arc in AC plasma torches is significantly limited as such plasma torches usually differ in designs and electrode materials [12, 13], and therefore in characteristics. As AC plasma torches can operate with a DC arc, it is advisable to prefer experimental tests of such devices.

The degrees of the approximation of a plasma state to the equilibrium plasma of DC and AC arcs will be equal if the momentary values of the stream $\dot{m}_{gaz}$ and current I are equal as well. The condition of the quasi-stationarity of processes in an AC plasma torch duct is the following [15]:

$$\frac{ld^2 p_k}{\tau \dot{m}_{gaz} R_G \vartheta} << 1 \qquad (9)$$

where τ - half-period of current excitation vibration (in the case of the mains supply of 50 Hz it amounts to 10^{-2} s); l – constrictor duct length; p_k – gas pressure in a plasma torch chamber; R_T – gas constant; ϑ- average absolute gas temperature. In such a case, during the passage of gas through an arc, current will not change noticeably, i.e. in such a time, current can be recognised as being direct. In the range of frequencies close to the mains frequency and during operation with typical technological gases this condition is usually met and the state of arc plasma follows current changes in a circuit. A similarly pulsating gas pressure also follows current module changes. If the same value of a gas mass stream in an AC arc plasma torch and in the same plasma torch with a DC arc was applied, the dynamic characteristics of an arc are very similar to related static characteristics.

The degree of the fulfilment of the condition (9) and the assumptions [6, 14, 16] enabling the use of static characteristics for modelling an AC arc with intensive cooling depends on the design of a plasma torch, the gas stabilisation of a discharge, the type of a plasma-creating gas etc. [9].

The static characteristics of a moderately or intensively cooled DC arc are characterised by a very high flash-over voltage value. For this reason they need at least a single action of a discharge initiator in each work cycle. A similar effect occurs in an AC plasma torch. If combined with a very low damping factor value, models using such arc characteristics significantly impede obtaining the stability of the numerical integration method. Also the physical operation of a plasma torch is characterised by difficulties with obtaining discharge stability. Due to this fact such devices have a special design including electrodes for the generation of an auxiliary arc or are provided with special systems of stabilising generators in order to generate a high vibration frequency voltage.

If the frequency of a regulator is relatively low (from 0 to 20 kHz), the value of the amplitude of voltage flash-over between electrodes practically does not depend on the frequency and overlaps with the flash-over voltage excited by a DC source. If the frequency is higher, the flash-over voltage slightly decreases initially to obtain the minimum in an abscissa with a frequency of approximately 5 MHz. A further increase in frequency is accompanied by an increase in a flash-over voltage to reach an approximately 1.5 greater value if compared with the voltage during DC supply (Fig. 3).

A similar effect can be observed using an auxiliary DC arc (Fig. 4). An increase in DC intensity I_s causes an increase in gas enthalpy in a discharge area and maintains a high conductance value of a post-arc column. Using an

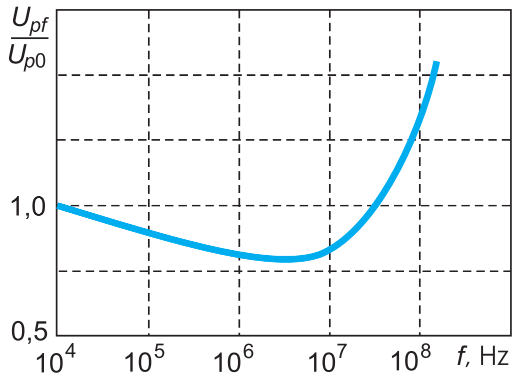

Fig. 3. Ratio of flash-over voltage U_{pf} of a pre-defined vibration frequency and the flash-over voltage U_{p0} coming from a DC source depending on the frequency *f* in the air.

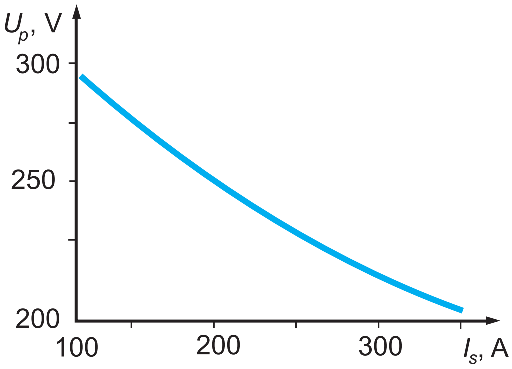

Fig. 4. Dependence between main arc ignition voltage and auxiliary arc DC intensity

appropriately high gas ionisation degree makes it possible to decrease a voltage value U_p [4, 17].

The main disadvantage of static characteristics is the lack of their relationship with an external excitation frequency, which strongly affects the location and the shape of dynamic characteristics. Dynamic characteristics strongly depend on the relation of the time constant θ of thermal processes in a column to the duration T of excitation current. The strongest non-linearity is typical of characteristics whose value θ/T is very low and which are the closest to static characteristics. Narrow dynamic loops surround a curve which is not yet a static characteristic, as in comparison with a static characteristic the curve can pass through the zero point (0, 0) relatively gently and obtains significantly lower ignition voltage extreme values with current different from zero. Otherwise, when θ/T is high, it is possible to observe the effect of the linearization of characteristics and a significant deviation from a static characteristic [18].

Difficulties in building the dynamic models of an AC electric arc relatively accurately describing voltage-current characteristics trigger undertaking numerous attempts [3, 19-25] of the direct approximation of these characteristics, ignoring even an approximated energy balance. In such situations an arc is treated as the ordinary non-linear inertialess resistance of an unambiguous characteristic with a proper criterion mechanism of changing over hysteresis loop branches. In doing so one uses selected points (first of all local extrema) on these loops and adjusts to them and concatenates pieces of known simple analytical functions (even linear ones). Such a method has the following numerous disadvantages:

1. no correlation of a loop with the shape of a current excitation waveform;
2. no correlation of a loop with the frequency (even the first harmonic) of a current excitation waveform;
3. impact of a damping factor function on arc stability in an electric circuit is ignored;
4. difficulties in representing random disturbances.

An example of such an approach is the approximation of a dynamic characteristic with hyperbolic and exponential functions:

– during the growth and the drop of a current module

$$f_h(i) = V_T \cdot (1 - \varepsilon) + \frac{C_{id}}{D_{id} + |i|} \tag{10}$$

– during the drop of a current module

$$f_e(i) = V_T \cdot (1 + \varepsilon) \cdot \left[1 - \exp\left(-\frac{|i|}{I_0} \right) \right] \tag{11}$$

where constants V_T – asymptotic value of the voltage of both these functions with an assumption ε = 0 and current i→∞; C_{id} – corresponds to arc power; D_{id} – corresponds to current; I_0 – constant corresponding to the steepness of an arc voltage module decrease; ε - constant parameter improving the intersection of the diagrams of the functions f_h and f_e ($0 < \varepsilon < 1$) with a limited current value. In addition, if the value of current is sufficiently high, it enables allowing for a slight overshooting directly before the recurrent passage of current through the zero value (Fig. 5). The resultant form of the curve is described by the following equations:

$$u(i) = \operatorname{sgn}(i) \times \begin{cases} f_h(i), \text{ if } \dfrac{d|i|}{dt} \geq 0; \\ f_h(i), \text{ if } \dfrac{d|i|}{dt} < 0 \text{ and } f_h(i) < f_e(i); \\ f_e(i), \text{ if } \dfrac{d|i|}{dt} < 0 \text{ and } f_h(i) \geq f_e(i) \end{cases} \quad (12)$$

The voltage of ignition results from the formula (10) and amounts to $U_p=f_h(0)=V_T \cdot (1-\varepsilon)+C_{id}/D_{id}$.

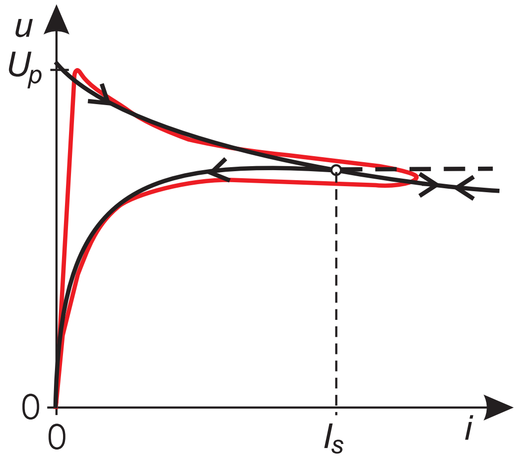

Fig. 5. Approximation of electric arc dynamic characteristic

Due to a great arc sensitivity to various excitations, influences and internal disturbances, its approximated description requires rather a set of dynamic characteristics. Such a possibility is offered mainly by mathematical models [5, 6, 14]. Even significant discrepancies between the dynamic characteristics of the deterministic model and the characteristics of a physical object are approved in comparison with unavoidable arc random disturbances.

Reference scientific-technical publications also present attempts [26, 27] of the approximated representation of arc dynamic characteristics by means of a circuit composed of the parallel connection of non-linear resistance of the averaged characteristic (close to commutative one) of an arc and a linear capacitor. Due to a relatively small butting face of electrodes and a significant distance between them, the capacity of such a system is negligibly small in comparison with other parasitic capacitances of a circuit. A capacitor connected in parallel should thus represent the inertness of the system connected with the thermal capacity of an arc. As an arc is a dissipative element and a capacitor is a conservative element, there are significant discrepancies between characteristics coming from physical experiments and those from numerical simulations. The introduction of the phase shift of waveforms between current and voltage by the capacitor (in this case a variable shift as resistance is variable) results in the lack of the simultaneous passage of these quantities through the zero value. An arc characteristic takes the shape of a deformed zero instead of taking the shape of a deformed octant. Such an approximation can be explained by the high level of random disturbances and the low operating accuracy of measuring and recording equipment.

In experimental tests and modelling an electric arc, particularly a short and low-voltage (welding) one, near-electrode voltage drops are of great importance. They can be determined on the basis of static and dynamic characteristics using appropriate measurement methods [9, 12].

Approximations of effective characteristics of AC electric current with intensive cooling

Effective characteristics are connected with root-mean-square values of deformed arc current and voltage waveforms. In accordance with the definition of the root-mean-square value, they are the characteristics of a DC equivalent arc with a virtual cathode and anode, equivalent to an AC arc as regards the conversion of electric energy in the same amount of heat. Due to the operation of integration, they contribute to a certain extent to damping high harmonics and averaging waveforms being analysed. Electrical engineers highly value the usability of such characteristics as they enable the selection of parameters and the effective characteristics of a supply source, measuring equipment, wattless power compensation systems etc. They make it possible to assess the amount of consumed electric energy and on this basis predict production cost indicators. Unfortunately, such characteristics are insufficient for automation specialists as they do not enable the assessment of the stability conditions or the stability margin of a dynamic system: a non-linear inert supply source – non-linear low-inert electric arc. Another significant difficulty in simulating processes in electrotechnological devices with an electric arc is cause by the significant tautness of a system of equations caused by the very low time constant of an arc intensively cooled in a plasma torch in relation to the long idle time or the high time constant of supply systems. Effective characteristics do not reveal such a problem. Even using such characteristics, electrical engineers do not obtain necessary information about the content of harmonics in the supply current waveform. The knowledge of such characteristics is necessary to assess the impact of devices on power networks and systems in order to select harmonic filtration systems.

From the point of view of the needs of a process engineer, the ordinary balancing of the components of thermal and electric energy is insufficient. The ability of various charges to absorb heat and losses of heat escaping through many ducts outside a device depend on the form of generated heat streams (conductive, convective, radiation). Thus, the distribution of energy dissipated into these components in an arc column and in the charge affecting area is very important. A further development of such an approach is replacing the general equivalent simplified DC arc with the detailed effective mathematical model of a two-layer arc [12]. On the basis of assumptions adopted for the model, it is possible to determine the individual components of heat streams in a plasma column and its surroundings. Using experimental data including the families of effective characteristics ($U(I, l)$, and more accurately of an electric field intensity $E(I)=dU/dl$ or deflected arc voltage gradient), it is possible to create a model of a two-layer arc, which is also a DC arc with a virtual cathode and anode. It is universal enough to enable converting and predicting the effective characteristics of a plasma torch during operation with parameters modified within the wide ranges of state constants, and even with other plasma-creating gases [12].

The use of an integrator in the calculation method results in certain averaging of the square of time courses. This causes significant ambiguity as various dynamic characteristics may correspond to the same effective characteristics of an arc. Root-mean-square values are particularly weak-sensitive to harmonics formed as a result of short “high-voltage” pulses during periodic arc ignitions. Also short periodic current stoppages can be covered by relatively long arc burning processes. The effective characteristics of a plasma torch arc have shapes close to hyperbolic, i.e. such as those presented in Figure 1 [9].

By analogy to DC plasma torches, it is possible to create a formula for the voltage-current characteristic of an arc in a one-phase plasma torch operating with air [9]

$$\frac{Ud}{I} = 2143 \cdot \left(\frac{I^2}{\dot{m}_{gaz} d} \right)^{-0,655} \left(\frac{\dot{m}_{gaz}}{d} \right)^{-0,345} (pd)^{0,20} \qquad (13)$$

The constant use of an arc discharge stabiliser (in the form of high-frequency current pulses or a DC source supplying an auxiliary arc) results in a significant ignition voltage decrease, a decrease in electric field intensity in a column and in the weakening of the non-linearity of the dynamic characteristics of a plasma torch. If such a plasma torch operates with an auxiliary arc it is possible to use the following simplified formula:

$$U = 2143 \cdot \left(\frac{\dot{m}_{gaz}}{I} \right)^{0,31} (pd)^{0,20} \qquad (14)$$

The non-stationarity caused by industrial frequency AC does not strongly affect work processes in plasma torches with the vertical stabilisation of an arc. In the case of "star-type" plasma torches the following formula is obtained [9]:

$$U = 732{,}5 \cdot \left(\frac{I^2}{\dot{m}_{gaz} d} \right)^{-0,16} \left(\frac{\dot{m}_{gaz}}{d} \right)^{0,16} (pd)^{0,34} \qquad (15)$$

In welding devices used today the frequency of arc current can be usually adjusted within a range from 20 to 200 Hz, and in some cases even up to 15 kHz. An increase in frequency causes an increase in arc constriction, greater concentration of energy over a smaller area of an electrode-detail, greater material penetration and improved quality and appearance of a narrower weld. In turn, lower frequency leads to the widening of an arc and that of a weld, increases the degradation ability of an oxide film and increases discharge stability at a high welding rate. The introduction of frequency changes is accompanied by current excitation shape changes. Obtaining better discharge stability (particularly in the case of low frequency) is favoured by the use of waveforms characterised by great steepness in areas where current passes through the zero value (rectangular waveform). The shape of the effective characteristics of an arc in TIG welding of aluminium is similar to that presented in Figure 1 [28]. Due to the required high efficiency of high-power electrothermal plasma devices, an arc is supplied with direct current or sinusoidal current of the mains frequency of 50 Hz.

Approximations of stabilised static and commutative characteristics of AC electric arc

Although the static characteristics of a discharge belong to basic data while determining the physical properties of arc and plasma devices, they are not always possible to obtain and not always an effort put into calculating them pays off. First of all, such problems refer to experimental tests of devices with an AC arc, in which one would like to use a DC excitation. The reasons for the difficulties can be the following:

1. lack of easily accessible DC sources (in tests of great-power furnaces and AC-EAF arc devices);
2. introduction of significant disturbances to a technological process (e.g.. to cutting or welding aluminium);
3. eliminating normal phenomena of a skin effect and a proximity effect in the power circuits (electrodes) of, in particular, high-current devices;
4. formation of another distribution of electrodynamic forces, and thus other displacements of arc spots and arc deflections in space;
5. triggering another thermal state of electrodes, and thus a possibility of the excessive erosion of electrodes or insufficient electron emission;
6. in the case of "short arcs" – the intensification of significant differences in the structure of plasma columns and in the distributions of thermal streams;
7. possibility of differences in the asymmetry of electrode or column DC and AC arcs due to the differences referred to in items 4, 5 and 6 above;
8. formation of dissimilar voltage spectra on arcs caused by differences in thermal and gasodynamic disturbances.

Such a great pile-up of difficulties in experimental tests may encourage a search for simplified manners of defining and determining characteristics representing the basic properties of an AC arc. One of them is by using a commutative characteristic.

The commutative characteristic of an electric arc should constitute the limit state of the family of dynamic characteristics, in which the value of a damping factor function is very low ($\theta(i) \to 0$) in comparison with the period of external excitation. In such a case, both an increase and decrease in the value of conductance takes place practically inertialessly in relation to current changes. Such a low damping factor function value may result either from the very intensive cooling of a column, or on the contrary, from its thermal insulation. The greatest value of this function [14, 15] corresponds to the intermediate case of moderate heat exchange

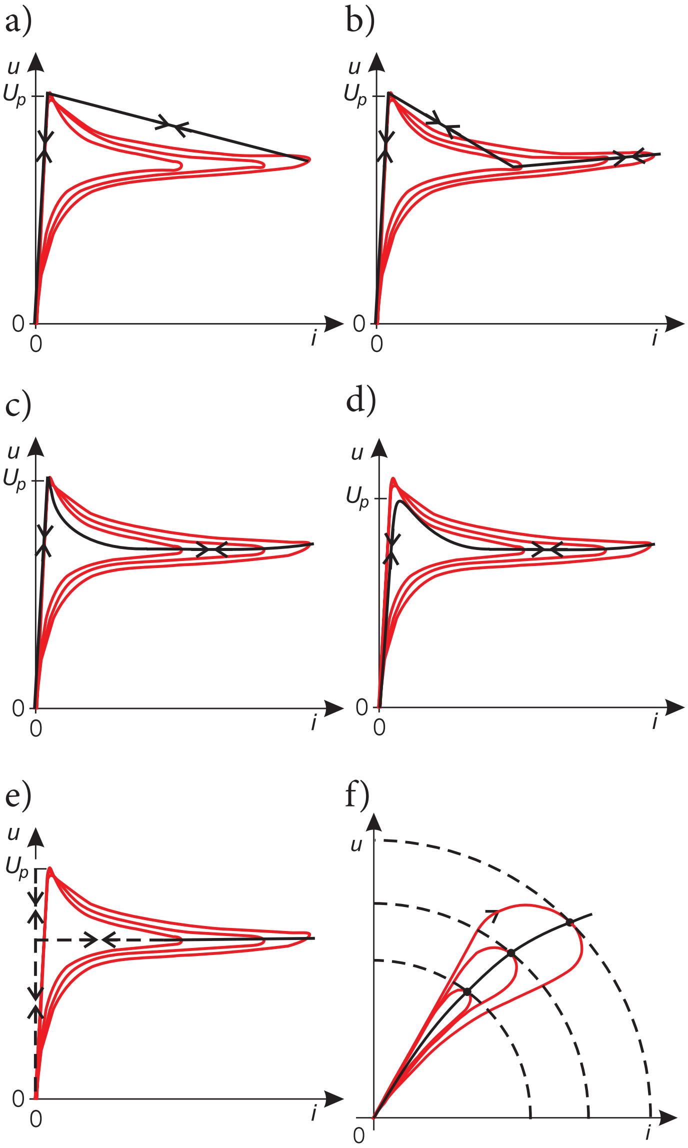

Fig. 6. Commutative curves of an electric arc: a) partly linear approximation (in sections) taking into account the extreme value of current amplitude; b) partly linear approximation (in sections) taking into account a section of negative differential resistance; c) approximation taking into account arc burning voltage stabilisation; d) approximation with smooth function; e) partly linear approximation (in sections) to qualitative analysis of the system; f) approximation of dynamic characteristics of arc supplied with current of heightened frequency ω, ($\omega\theta \geq 1$)

with the environment. The intensive cooling of an arc leads to the strong deformation of a commutative curve, whereas the thermal insulation conversely, i.e. to its linearization.

The manners of defining the commutative characteristic of an arc vary. In the publication [29] it is a curve broken on plane (i, u), composed of sections of straight lines connecting points of extreme current and voltage values and passing through the zero point of a coordinate system (see Fig. 6a, quarter I of the system). A similar approximation was used in the works [3, 19]. As distinguished from the commutative curve of magnetic materials [2], where its points are determined during the constant change of the amplitude of magnetic field intensity, in this case a single loop of the greatest current amplitude is used. The resistance element of such a non-linear characteristic enables obtaining satisfactory results of simulating processes in AC arc furnaces during tests of electromagnetic compatibility with a power network. The noticeable phenomenon of damped vibration excitation in some resonant circuits with an arc can be analysed after taking into account a negative differential resistance, which is possible owing to commutative curves presented in Figures 6a and 6b. A greater accuracy of the representation of the dynamic properties of an arc is obtained by expressing a commutative curve as a fragment of a hyperbola connecting points corresponding to extreme current and voltage values (Fig. 6c). In these three cases the curve of approximation is not a smooth function. The approximation presented in Figure 5d, possible to express with an uncomplicated analytical function, is free from such a disadvantage. In approximated analytical, particularly qualitative, tests, great usefulness can be obtained using a partly linear approximation (i.e. approximation linear in some sections), presented in Figure 6e. While analysing circuits with an arc and the heightened frequency of excitation current it is possible to use the commutative curve from Figure 6f.

The use of a commutative characteristic alone significantly simplifies the analysis of non-linear systems with an electric arc by decreasing the dimension of state space by ½ degree of freedom in comparison with simple dynamic models [4, 9, 30]. Even if a discharge pulse stabiliser does not work, almost in each case of the modelling of an electrotechnological device the serial inductance of a circuit is taken into account. This serial inductance usually enables obtaining the physical stability and the stability of numerical calculations of a circuit with a strongly non-linear arc characteristic.

Usually presented static characteristics do not allow for the possibility of the independent control of ignition voltage by means of an additional high-frequency generator or an additional DC arc. In order to take into account the operation of a stabiliser and obtain a smooth curve, it is necessary to use relatively complex approximating functions. Many variants of these functions such as exponential functions, Gaussian functions, hyperbolic secans, Lorentzian and thyristors functions were investigated by L. Marciniak during modelling a short-circuit arc [31-33]. The author also suggested estimating the coordinates of an ignition point (I_p, U_p) and associating the non-linear part of characteristics in the low-current range with the linear part in the high-current range. An exemplary exponential characteristic estimated in relation to the peak point is expressed as follows:

$$U(I)=U_p\cdot\left(\frac{I}{I_p}\right)\cdot\exp\left(-\left(\frac{I}{I_p}\right)+1\right) \tag{16}$$

Changes of an arc length trigger changes of ignition voltage and voltage in the high-current range

$$U_0(l)=U_{0_{min}}+u_0\cdot l \tag{17}$$

A longer arc has greater resistance

$$R_0(l)=R_{0_{min}}+r_0\cdot l \tag{18}$$

Associating these three dependences taking into account a tapering function gives a modified voltage-current characteristic:

$$\begin{aligned}U(I)=&\left(U_0+R_0I^{k_0}\right)\cdot\zeta(I)+\\&+\left[U_p-\left(U_0+R_0I^{k_0}\right)\cdot\zeta(I)\right]\times\\&\times\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right)\end{aligned} \tag{19}$$

where $k_1 > 1$ provides of possibility of selecting the steepness of a characteristic drop in the range of negative differential resistance. In the case when $k_1=1$ the formula provided by L. Marciniak is obtained.

If the arc length l increases, then in accordance with the equations (17) and (18), the quantities U_0 and R_0 increase also and so does the ignition voltage [34], approximated by means of the formula

$$U_p(l)=E_{p0}\cdot l^{k_2} \tag{20}$$

where k_2, E_{p0} – approximation coefficients dependent on the shape, temperature and material of electrodes as well as on the chemical composition of a gas ($0 < k_2 < 1$). In the range of small arc lengths in electrotechnological devices, one can adopt this dependence as linear $U_p(l)=E_{p0}\cdot l$. The quantity U_p also depends proportionally on the time which has passed since the moment at which current obtained the zero value.

The author of this article also suggests using a more flexible dependence, i.e. a sigmoidal unipolar function $\zeta''(I) = \{1+\exp[-k_3\cdot(I-I_p)]\}^{-1}$ in the character of a tapering function. In such a case the appropriate associating of analogical relations (16)-(19) also gives the family of characteristics. In both cases of using the tapering functions ζ' and ζ'' almost identical results were obtained. Improving the quality of approximation makes it possible to use a new tapering function, being the product of the aforesaid tapering functions $\zeta = \zeta'\times\zeta''$.

Figure 7 presents examples of calculated functions. Pre-set function parameters in Figure 7a were the following: U_0=30 V, R_0 = 0.03Ω, curve 1 - (U_p=150 V, I_p=3 A, k_0=1, k_1=2), curve 2 - (U_p = 175 V, I_p = 3 A, k_0 = 1,3, k_1 = 2.5), curve 3 - (U_p = 200 V, I_p = 3 A, k_0 = 1,5, k_1 = 2.5). Pre-set function parameters in Figure 7b were the following: U_0=40 V, R_0 = 0.02Ω, curve 1 - (U_p=150 V, I_p=4 A, k_0=1.5, k_1=1.5, k_3=0.5), curve 2 - (U_p = 175 V, I_p = 4 A, k_0 = 1.75, k_1 = 2, k_3=0.5), curve 3 - (U_p = 200 V, I_p = 4 A, k_0 = 1.2, k_1 = 2.5, k_3=0.5).

a) b)

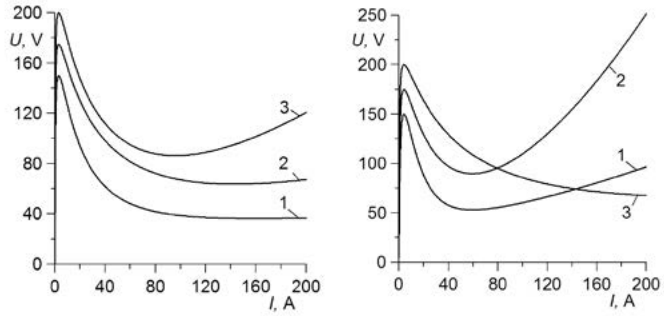

Fig. 7. Static characteristics of free arc expressed by formula (15): a) with the use of tapering function ζ'; b) with the use of tapering function ζ"

The voltage of arc re-ignition is a function of many variables $U_p(I, l, p, \vartheta, f)$ as this voltage is affected by the root-mean-square value of current, I, the duration of a currentless break, the distance between electrodes, l, pressure, p, absolute temperature, ϑ, frequency, f, and the chemical composition of a gas. Changes of the value and direction of an input gas stream trigger changes of a discharge area temperature [35] $\vartheta \propto \dot{m}_{gaz}^{-k_4}$, which as a result of the relation $U_p \propto \vartheta^{-1}$ leads to the following dependence:

$$U_p\left(\dot{m}_{gaz}\right) = U_{p0} \cdot \left(1 + u_{p4}\dot{m}_{gaz}^{k_4}\right) \qquad (21)$$

where U_{p0}, u_{p4}, k_4 – constant parameters of approximation (0< k_4 < 1). In turn, the changes of ignition current are much smaller and reveal a reverse tendency $I_p \propto \vartheta \propto \dot{m}_{gaz}^{-1}$. For this reason, it is possible to adopt an approximate of the constant value of current, I_p.

The impact of gas parameters on voltage, U_p, can be limited and the voltage can be controlled independently by using an additional discharge stabiliser. An increase in a gas stream corresponds to an increase in the closed-circuit voltage of a plasma torch demonstrated by an increase in static characteristics [36]. If the dependence (2) is taken into account, then using the equations (17) and (18) the following new modified parameters are obtained:

$$U_0\left(\dot{m}_{gaz}\right) = U_{0\min} + u_0'\dot{m}_{gaz} \qquad (22)$$

$$R_0\left(\dot{m}_{gaz}\right) = R_{0\min} + r_0'\dot{m}_{gaz} \qquad (23)$$

These parameters can be used to create the modified approximation of a voltage-current characteristic:

$$U\left(I, \dot{m}_{gaz}\right) = \left[U_0\left(\dot{m}_{gaz}\right) + R_0\left(\dot{m}_{gaz}\right)I^{k_0}\right] \cdot \zeta(I) + \left\{U_p\left(\dot{m}_{gaz}\right) - \left[U_0\left(\dot{m}_{gaz}\right) + R_0\left(\dot{m}_{gaz}\right)I^{k_0}\right] \cdot \zeta(I)\right\} \times \left(\frac{I}{I_p}\right)^{1/k_1} \cdot \exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1} + 1\right) \qquad (24)$$

If $\dot{m}_{gaz}$=0, the formula (24) changes into the formula (1) describing the characteristics of a constant-length free arc.

An example of using the formula (24) in the presentation of the static characteristics of a plasma torch is shown in Figure 8. The pre-set function parameters were as follows: U_{0min} = 40 V, R_{0min} = 0,01 Ω, u'_0 = 0.5 Vs/kg, r'_0 = 50 Ωs/kg, k_0 = 1.2, k_1 = 2.5, k_2 =0.5, curve 1 - (U_p = 150 V, I_p = 5 A, $\dot{m}_{gaz}$=0.001 kg/s), curve 2 - (U_p = 180 V, I_p = 5 A, $\dot{m}_{gaz}$=0.002 kg/s), curve 3 - (U_p = 200 V, I_p = 5 A, $\dot{m}_{gaz}$=0.003 kg/s).

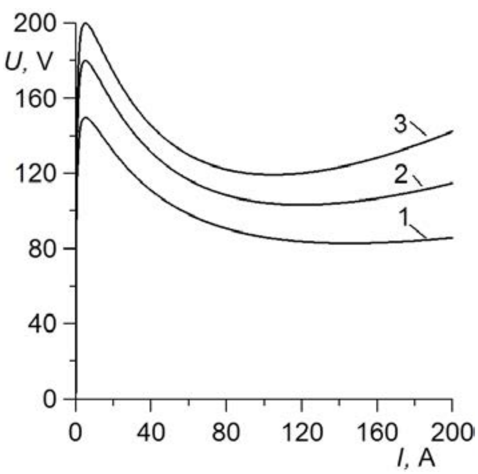

Fig. 8. Static characteristics of plasma torch with internal arc calculated using the formula (18)

In the case of a plasma torch with an external arc it is possible to act the same as while defining the approximation of a static characteristic with the formula (5). After taking into account the dependences (17) and (18) the following new modified parameters are obtained:

$$U_0(l,\dot{m}_{gaz}) = (a_0 + a_1\dot{m}_{gaz}) + (b_0 + b_1\dot{m}_{gaz})\cdot l \quad (25)$$

$$R_0(l,\dot{m}_{gaz}) = (c_0 + c_1\dot{m}_{gaz}) + (d_0 + d_1\dot{m}_{gaz})\cdot l \quad (26)$$

By means of these parameters it is possible to define a new static characteristic

$$\begin{aligned} U(I,l,\dot{m}_{gaz}) = {} & \left[U_0(l,\dot{m}_{gaz}) + R_0(l,\dot{m}_{gaz})I^{k_0}\right]\cdot\zeta(I) + \\ & + \left\{U_p(l,\dot{m}_{gaz}) - \left[U_0(l,\dot{m}_{gaz}) + \right.\right. \\ & \left.\left. + R_0(l,\dot{m}_{gaz})I^{k_0}\right]\cdot\zeta(I)\right\}\times \\ & \times\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right) \end{aligned} \quad (27)$$

In addition, the dependence of ignition voltage on a gas stream and a distance can be approximated using the following formula:

$$U_p(l,\dot{m}_{gaz}) = E_{p0}l\cdot\left(1 + e_{p5}\cdot\dot{m}_{gaz}^{k_5}\right) \quad (28)$$

where E_{p0}, e_{p5}, k_5 – constant parameters of approximation ($0 < k_5 < 1$). If $\dot{m}_{gaz}=0$, the formula (27) changes into the form (1) describing the characteristics of a variable-length free arc. If l = const., the formula (27) changes into the form (24) describing the characteristics of the arc of a plasma torch operating with a variable gas stream, $\dot{m}_{gaz}$.

An example of using the formula (27) for the presentation of the static characteristics of a plasma torch is shown in Figure 9. The pre-set function parameters were the following: $a_0 = 20$ V, $a_1 = 100$ Vs/kg, $b_0 = 1$ V/m, $b_1 = 100$ Vs/(kg·m), $c_0 = 0.05\ \Omega$, $c_1 = 10\ \Omega$s/kg, $d_0 = 0.1\ \Omega$/m, $d_1 = 100\ \Omega$s/(kg·m), $E_{p0} = 2000$ V/m, $e_{p5} = 50$ V, $I_p = 8$ A, $k_0 = 1.1$, $k_1 = 2$, $k_2 = 0.5$, $k_5 = 0.4$.

Most mathematical models of an electric arc use the function of electric field intensity. In such cases an appropriate approximation is described by the following formula:

$$\begin{aligned} E(I,l,\dot{m}_{gaz}) = {} & \frac{\partial U(I,l,\dot{m}_{gaz})}{\partial l} = \\ & = \left[\frac{\partial U_0(l,\dot{m}_{gaz})}{\partial l} + \frac{\partial R_0(l,\dot{m}_{gaz})}{\partial l}I^{k_0}\right]\cdot\zeta(I) + \\ & + \left\{\frac{\partial U_p(l,\dot{m}_{gaz})}{\partial l} - \left[\frac{\partial U_0(l,\dot{m}_{gaz})}{\partial l} + \frac{\partial R_0(l,\dot{m}_{gaz})}{\partial l}I^{k_0}\right]\right\}\times \\ & \times\zeta(I)\cdot\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right) \end{aligned} \quad (29)$$

where

$$\frac{\partial U_0(l,\dot{m}_{gaz})}{\partial l} = b_0 + b_1\dot{m}_{gaz};\quad \frac{\partial R_0(l,\dot{m}_{gaz})}{\partial l} = d_0 + d_1\dot{m}_{gaz};$$

$$\frac{\partial U_p(l,\dot{m}_{gaz})}{\partial l} = E_{p0}.$$

The presented non-linear functions (15), (18), (21) and (22) can approximate the voltage-current static characteristics of a DC arc or the commutative characteristic of an AC arc with intensive cooling. The section of a characteristic in the range ($-I_p$, $+I_p$) does not overlap with the actual static characteristics of a free arc or an intensively cooled arc. However, it provides the physically noticeable continuity of the dynamic characteristics of an AC arc, which is of particular importance in the case of a very low damping factor value. It can also be used to define the flash-over voltage of an inter-electrode gap. The voltage can be controlled independently by

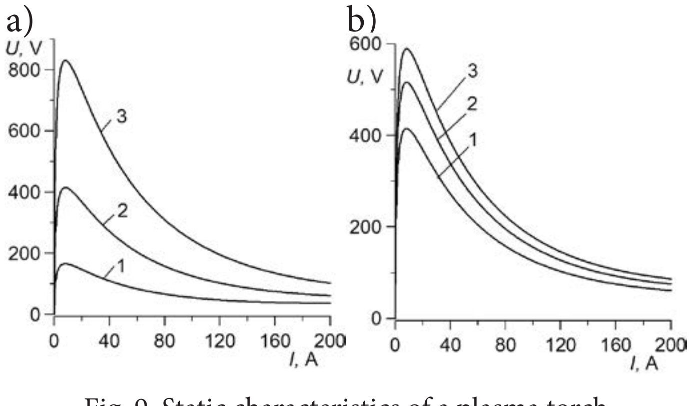

Fig. 9. Static characteristics of a plasma torch with external arc calculated using formula (27) with variable parameters:
a) arc length l
($\dot{m}_{gaz} = 1$ g/s, 1–$l = 0.02$ m, 2–$l = 0.05$ m, 3–$l = 0.1$ m);
b) gas mass stream $\dot{m}_{gaz}$
($l = 0.05$ m, 1–$\dot{m}_{gaz} = 1$ g/s, 2–$\dot{m}_{gaz} = 2$ g/s, 3–$\dot{m}_{gaz} = 3$ g/s)

means of an additional high-frequency generator or an auxiliary arc. In the case of the columnar symmetry of an arc [37], dynamic characteristics contain the whole current range, and therefore commutative characteristics should take into account current flow direction changes:

$$U_k\left(i,l,\dot{m}_{gaz}\right)=\begin{cases} U\left(i,l,\dot{m}_{gaz}\right), & \text{if } i\geq 0 \\ -U\left(\left|i\right|,l,\dot{m}_{gaz}\right), & \text{if } i<0 \end{cases} \quad (30)$$

Conclusions:

1. The multiplicity, complexity and non-stationarity of physicochemical processes in an electric arc column are manifested by the strong non-linearity, ambiguity and noise of voltage-current characteristics.
2. Various external effects (excitation, disturbances, controlling, stabilising) and the high sensitivity of an electric arc to such effects cause the great usability of the parametric forms of the approximation of the families of electric arc characteristics.
3. Due to the very difficult measurability of the non-electric physical parameters of thermal plasma, relatively easily and accurately determinable electric characteristics and appropriate approximations used with such characteristics constitute the basis for creating the mathematical models of an electric arc.
4. Due to the very low damping factor value of an intensively cooled arc and considerable requirements related to the accuracy of modelling arc plasma torches, the development of the precise approximations of static and dynamic electric characteristics is of particularly great importance.
5. Developed formulas used for the approximations of the static and commutative characteristics of an electric arc include a vast range of electrotechnological devices operating in relatively wide ranges of changes of current excitations and parameters.

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