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Modified Habedank and TWV hybrid models of a variable length arc for simulating processes in electrical devices

Introduction

The multiplicity, complexity, and problematic measurability of the characteristics of processes in arc discharges entails the necessity of applying various methods of mathematical description and quantitative analyses. Numerous physical processes including electromagnetic, thermal, gasodynamic, and acoustic as well as the mechanical processes take place in the plasma and electrodes. The complexity of such processes results from significant nonlinearities of static and dynamic characteristics, collectiveness, and interaction of plasma components as well as from the very short relaxation times of the elementary processes. Difficulties in measurability are caused by high temperatures, strong heat (and light) emission, significant gas flow rates, high quantity gradients in state variables, very high chemical reaction rates, occasional difficulties in accessing sensors (including optical ones) to a discharge area because of its small size or the fact of being closed. Due to this and depending on the needs in designing the power-supply and control systems of electrotechnological devices, one adopts various simplifying assumptions leading to various degrees of approximation of physicochemical phenomena using mathematical models. The simplest and most commonly applied models include those of Cassie-Berger and Mayr-Kulakov. However, approximations obtained using these models are often considered to be very rough in relation to needs connected with designing and building various measuring, supply, and control systems; this being caused by a necessity to apply various electric excitations and various conditions of arc burning in devices. For this reason, various modifications of equations and new mathematical models of arcs have been proposed.

Most of the existing arc models (i.e. also those by Cassie-Berger and Mayr-Kulakov) take into consideration only one manner of heat transfer, either conduction or convection, regarding each of them as dominant in a variety of technological conditions. The Cassie-Berger model provides more satisfactory results in cases when strong currents are required, whereas the Mayr-Kulakov model is preferred when weak currents are preferred. Such an approach to modelling is justified by an experimentally confirmed assumption, according to which there is a boundary between a column of thermal plasma and a turbulent gas flow around it [1].

The extension of the applicability area of widely known simplified Cassie-Berger and Mayr-Kulakov models required the development of several associations. One of them is the series connection of two resistances corresponding to the nonlinear Cassie-Berger and Mayr-Kulakov models suggested by Habedank. This combined model, however, lacks appropriate physical interpretation of phenomena present in the column. A more rational solution to the issue of generalisation of description of processes taking place in the arc

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in a wide range of currents was proposed in publication [2], in the form of a parallel connection of conductances corresponding to the nonlinear Cassie-Berger and Mayr-Kulakov models, whose participation is determined by appropriate tapering functions of current.

The basic methods of controlling welding and electrothermal (arc and plasma-arc) devices include modifications of excitation source current as well as modifications of arc (length) voltage. Most of the simple and hybrid dynamic models applied so far, however, treat the arc as an element of the constant length of a plasma column. Moreover, there are even some cases when taking into consideration the modifications of the arc length only in relation to a single model (e.g. Cassie-Berger or Mayr-Kulakov) does not ensure that the approximation of power characteristics within a wide range of work current amplitudes can be obtained.

Cassie-Berger and Mayr-Kulakov models of arc with constant plasma column length

Dynamic models of an electric arc column are created on the basis of the power balance equation

$$P_{kol} = u_{kol} i = P_{dys} + \frac{dQ}{dt} \tag{1}$$

where P_{kol} – power supplied to the column, P_{dys} – thermal power dissipated from the column, Q –thermal plasma enthalpy, u_{kol}, i – voltage and current of the arc column. The effect of strong nonlinearity of the models results from column conductance variability, which is a composite function in the form of $g(t) = F_g(Q(t))$.

Popular electric arc models by Cassie-Berger and Mayr-Kulakov take advantage of two different simplifying assumptions [3]:

- Mayr-Kulakov model: T(t,(x,y,z)) = variab., arc power dissipated through conduction

$S(i)$=const; $\sigma(i)$=var; $P_S(i)$=const; $g(i) = K_1 \cdot \exp\left(\frac{Q_V(\sigma(i))}{Q_0}\right)$

- Cassie-Berger model: T(t,(x,y,z))= const., arc power dissipated through convection,

$S(i)$=var; $\sigma(i)$=const; $P_S(i) \sim Q(i) \sim g(i)$ =var; $g(i) = K_2 \cdot \frac{Q_V(i)}{Q_0}$

where T – temperature, K; x,y,z – system coordinates, m; S – cross-section area, m^2, σ- plasma conductivity, S/m; P_S – dissipated power, W; Q_V – plasma enthalpy volumetric density, J/m^3; Q_0 – constant reference coefficient, J/m^3; K_1 – constant coefficient of approximation with exponential function, S/m; K_2 – approximation coefficient, S. As the Mayr-Kulakov model makes it possible to obtain the best approximation in cases when weak currents are required and the Cassie-Berger model in the case of strong currents, it is the latter model that is of basic importance in simulating electromagnetic processes in industrial welding and electrothermal devices. Transitory processes in the areas of the transition of current through zero values are significant for ensuring the stability of arc burning and appropriate start and stop characteristics. In addition, the processes are decisive for the proper operation of commutation devices.

After adopting appropriate simplifying assumptions and transformations [3] from formula (1), one obtains the already known Cassie-Berger models

- in the conductance form

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_C}\left(\frac{u_{kol}^2}{U_C^2} - 1\right) \tag{2}$$

- in the resistance form

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_C}\left(1-\frac{u_{kol}^2}{U_C^2}\right) \tag{3}$$

where Θ_C – time constant of the model, U_C – model voltage, g = 1/r – conductance and resistance of the arc column.

Similarly, on the basis of the power balance equation (1) and after adopting appropriate simplifying assumptions and transformations [3], one can obtain the Mayr-Kulakov models in the conductance form

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_M}\left(\frac{u_{kol}i}{P_M}-1\right) \tag{4}$$

or in the resistance form

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_M}\left(1-\frac{u_{kol}i}{P_M}\right) \tag{5}$$

where Θ_M – time constant of the model, P_M – power of Mayr-Kulakov model.

The Mayr-Kulakov arc model can be transformed into another, general conductance form

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{Ms}}\left|\frac{P_{kol}(t)}{P_{dys}(t)}-1\right| \tag{6}$$

or the resistance form

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_{Ms}}\left|1-\frac{P_{kol}(t)}{P_{dys}(t)}\right| \tag{7}$$

where Θ_{Ms} – corresponds to relaxation time of thermal process, and the supplied electric power amounts to

$$P_{kol}(t)=u_{kol}i=\frac{i^2}{g}=i^2r \tag{8}$$

As the processes of heat dissipation slowly respond to external disturbance, one can assume that the power of losses is basically determined by static characteristics [4] i.e.

$$P_{dys}(t)=U_{stat}(i)\cdot i=\frac{i^2}{G_{stat}(i)}=i^2R_{stat}(i) \tag{9}$$

where $U_{stat}(i)$ – static voltage-current characteristics, $G_{stat}(i)$ – static nonlinear conductance, $R_{stat}(i)$ – static nonlinear resistance.

After substituting (8) and (9) to (6) and (7) one obtains a generalised Mayr-Kulakov equation in the conductance form

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{G_{stat}(i)}{g}-1\right] \tag{10}$$

or in the resistance form

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_{Ms}}\left[1-\frac{r}{R_{stat}(i)}\right] \tag{11}$$

When conductance does not change in time, the static characteristics of the arc in this model are as follows:

$$U_{stat}(i)=\frac{P_M}{i} \tag{12}$$

$$G_{stat}(i)=\frac{i^2}{P_M}=\frac{i^2}{U_{stat}(i)\cdot i}=\frac{1}{R_{stat}(i)} \tag{13}$$

Therefore, on the basis of these formulas one can write models (6) and (7) in the conductance form

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{i}{g\cdot U_{stat}(i)}-1\right] \tag{14}$$

or in the resistance form

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_{Ms}}\left[1-\frac{ri}{U_{stat}(i)}\right] \tag{15}$$

$U_{stat}(i) = - U_{stat}(-i)$.

The application of the appropriate approximation of static characteristic $U_{stat}(i)$ offers more precise determination of arc dynamic characteristics if compared with hyperbolic static characteristic, pre-set only by one constant Mayr-Kulakov power value. Such an approach extends somewhat the range of the model applicability to include stronger currents, when a characteristic is no longer drooping [5] but becomes flat and, in the case of stronger currents, is even rising.

Combined models of arc with constant column length

The series connection of the Cassie-Berger and Mayr-Kulakov models makes it possible to obtain the Habedank model, where substitute conductance fulfils the dependence

$$\frac{1}{g} = \frac{1}{g_M} + \frac{1}{g_C} \tag{16}$$

and resistance is

$$r = r_M + r_C \tag{17}$$

As the same current flows through both elements and after taking into consideration that $g_C = i / u_C$, $g_M = i / u_M$ and $g = i / u$ it can be stated that

$$u_C = u\frac{g}{g_C} = u\frac{r_C}{r} \tag{18}$$

and

$$u_M = u\frac{g}{g_M} = u\frac{r_M}{r} \tag{19}$$

Then on the basis of (18) and (19), one can express the Habedank model in the conductance form:

$$\frac{1}{g_C}\frac{dg_C}{dt} = \frac{1}{\theta_C}\left[\frac{u^2}{U_C^2}\left(\frac{g}{g_C}\right)^2 - 1\right] \tag{20}$$

$$\frac{1}{g_M}\frac{dg_M}{dt} = \frac{1}{\theta_M}\left[\frac{u^2 g}{P_M}\frac{g}{g_M} - 1\right] \tag{21}$$

The resistance form of the formulas is as follows:

$$\frac{1}{r_C}\frac{dr_C}{dt} = \frac{1}{\theta_C}\left[1 - \frac{u^2}{U_C^2}\left(\frac{r_C}{r}\right)^2\right] \tag{22}$$

$$\frac{1}{r_M}\frac{dr_M}{dt} = \frac{1}{\theta_M}\left[1 - \frac{u^2}{rP_M}\frac{r_M}{r}\right] \tag{23}$$

If one now implements the simplified Mayr-Kulakov model taking into consideration the virtual static characteristics of the arc component $U_{Mstat}(i)$, instead of (21) one receives

$$\frac{1}{g_M}\frac{dg_M}{dt} = \frac{1}{\theta_{Ms}}\left[\frac{u^2 g}{i \cdot U_{Mstat}(i)}\frac{g}{g_M} - 1\right] \tag{24}$$

and after reduction

$$\frac{1}{g_M}\frac{dg_M}{dt} = \frac{1}{\theta_{Ms}}\left[\frac{u}{U_{Mstat}(i)}\frac{g}{g_M} - 1\right] \tag{25}$$

Similarly, instead of (23) the resistance form of the model will be

$$\frac{1}{r_M}\frac{dr_M}{dt} = \frac{1}{\theta_{Ms}}\left[1 - \frac{u^2}{r \cdot i \cdot U_{Mstat}(i)}\frac{r_M}{r}\right] \tag{26}$$

and after reduction

$$\frac{1}{r_M}\frac{dr_M}{dt} = \frac{1}{\theta_{Ms}}\left[1 - \frac{u}{U_{Mstat}(i)}\frac{r_M}{r}\right] \tag{27}$$

where

$U_{Mstat}(i) = U_{stat}(i) - U_0\, sign(i),\ U_C = f(U_0)$.

The voltage of U_0 corresponds to ranges of strong arc currents.

The Habedank model (20)-(23) is sometimes used to simulate commutation processes in electric circuits with high voltage electrical devices; its expansion being the series connection of as many as three models (1 – Cassie-Berger, 2 – Mayr-Kulakov) [7]. Known as KEMA, the model was even implemented as a blackbox in simulation programmes [8, 9].

In the TWV hybrid arc model [2], the values of currents flowing through two parallel nonlinear conductances, corresponding to the Mayr-Kulakov and Cassie-Berger models, depend on their resultant value and therefore can be presented as follows:

$$i(t) = u_{kol} g = u_{kol} \cdot g_M \exp\left(-\frac{i^2}{I_0^2}\right) + u_{kol} \cdot g_C \cdot \left(1 - \exp\left(-\frac{i^2}{I_0^2}\right)\right) \tag{28}$$

Hence one receives

$$g(t) = g_M(t) \cdot \exp\left(-\frac{i^2}{I_0^2}\right) + g_C(t) \cdot \left(1 - \exp\left(-\frac{i^2}{I_0^2}\right)\right) \tag{29}$$

The conditions of the selection of models are as follows:

- Mayr-Kulakov model

$$g(t) \approx g_M(t) = \frac{i_M^2}{P_M} - \theta_M \frac{dg_M}{dt}, \quad \text{if} \quad i < I_0 \tag{30}$$

- Cassie-Berger model

$$g(t) \approx g_C(t) = \frac{u_{kol} i_C}{U_C^2} - \theta_C \frac{dg_C}{dt}, \quad \text{if} \quad i > I_0 \tag{31}$$

In welding [10, 11] and furnace [2] arcs, the value of limiting current I0 is approximately 5 A. In the case of the application of the TWV model for the approximation of the characteristics of high-pressure arc lamps, the value of I0 is almost 10 times lower [12].

The hybrid model of the arc column in the conductance form is [2]

$$g_{kol}(t) = G_{\min} + \left[1 - \exp\left(-\frac{i^2}{I_0^2}\right)\right]\frac{u_{kol} i}{U_C^2} + \left[\exp\left(-\frac{i^2}{I_0^2}\right)\right]\frac{i^2}{P_M} - \theta(|i|)\frac{dg_{kol}}{dt} \tag{32}$$

where $G_{\min}$ – constant conductance dependent on the distance between electrodes, their shape and arrangement as well as on the gas and the temperature of the environment in currentless moments; I_0 - transition current between the Cassie-Berger and Mayr-Kulakov models. In a general case, the suppression function Θ depends on current i

$$\theta = \theta_0 + \theta_1 \exp(-\alpha|i|) \tag{33}$$

If the current is relatively low, one can assume that $\Theta \approx \Theta_1$, and when current is high one can assume that $\Theta \approx \Theta_0$. If $\Theta \to 0$, the static characteristic results from adopted assumptions related to the participation of individual constituent models:

- if $|i| < I_0$ and $dg/dt = 0$, then $u = P_M/i$;
- if $|i| > I_0$ and $dg/dt = 0$, then $u = U_C \operatorname{sign}(i)$.

In practical considerations [2] one usually assumes that $\Theta(|i|) = \text{const}$ and $G_{\min} = 0$. Then, formula (32) can be simplified to

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta}\left\{[1 - \varepsilon(i)]\frac{u_{kol} i}{gU_C^2} + \varepsilon(i)\frac{u_{kol} i}{P_M} - 1\right\} \tag{34}$$

with the designation

$$\varepsilon(i) = \exp\left(-\frac{i^2}{I_0^2}\right) \tag{35}$$

The creation of the resistance form of the hybrid model, analogous to the conductance TWV, is difficult for computer recording and implementation. For this reason, the resistance form is not subject to consideration.

The TWV model is successfully used in simulations of stationary processes in welding and electrothermal devices as well as in systems with discharge light sources [2, 10-12].

If here, like previously, one introduces the Mayr-Kulakov model, taking into consideration the virtual static characteristic of the arc component Ustat(iM), instead of (34) one receives

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta}\left\{[1 - \varepsilon(i)]\frac{u_{kol} i}{gU_C^2} + \varepsilon(i)\frac{u_{kol} i}{i_M \cdot U_{stat}(i_M)} - 1\right\} \tag{36}$$

At time intervals when current $|i|$ values are low, the Mayr-Kulakov conductance constituent plays an important role and the dependence $i_M \approx i$ takes place. Thus, one can approximately write that $U_{stat}(i_M) = U_{stat}(i)$ and then

$$\frac{dg}{dt} = \frac{1}{\theta}\left\{[1 - \varepsilon(i)]\frac{u_{kol} i}{gU_C^2} + \varepsilon(i)\frac{u_{kol}}{U_{stat}(i)} - 1\right\} \tag{37}$$

Representation of the disturbance of arc column length with modified Cassie-Berger and Mayr-Kulakov

An increase in the geometric size of the plasma arc column is accompanied by an increase in energy necessary to generate an additional volume of plasma. The adopted assumption of the axial-cylindrical shape of the column and its stretching by length dl corresponds to an increase in thermal power

$$\frac{dQ_a}{dt}=\frac{dQ_a}{dl}\frac{dl}{dt}=q_l\frac{dl}{dt} \tag{38}$$

where q_l – arc energy linear density. Hence in simplifying conditions, the thermal power necessary to generate the additional volume of plasma is approximately proportional to the length of the increment rate. The phenomenon is accompanied by relaxation times resulting from gas thermal inertia and additional cooling of the column. Modified equations (2) and (3), with a variable value of the Cassie-Berger voltage, $U_C(t)=U_C\left(l,\frac{dl}{dt}\right)$ give the conductance form [13, 14]

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_C}\left(\frac{u_{kol}^2}{u_C^2(l)+\frac{1}{g}p_v\left(\frac{dl}{dt}\right)}-1\right) \tag{39}$$

The resistance form is

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_C}\left(1-\frac{u_{kol}^2}{u_C^2(l)+rp_v\left(\frac{dl}{dt}\right)}\right) \tag{40}$$

where $p_v(dl/dt)$ - power necessary to generate additional volume of plasma.

Kulakov proposed a modification of model (14) taking into consideration the modification of the column length. Model I of the order in the conductance form is [15]

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{i}{g\cdot l\cdot E_{stat}(i)}-1\right]-\frac{1}{l}\frac{dl}{dt} \tag{41}$$

where $E_{stat}(i)$ – static characteristic of electric field intensity. The resistance form of the model is described by the following formula:

$$\frac{1}{r}\frac{dr}{dt}=\frac{1}{\theta_{Ms}}\left[1-\frac{ir}{l\cdot E_{stat}(i)}\right]+\frac{1}{l}\frac{dl}{dt} \tag{42}$$

Representation of disturbance of arc column length in modified Habedank and TWV hybrid models

The series connection of nonlinear conductances (16) corresponding to the Cassie-Berger and Mayr-Kulakov models makes it possible to obtain the modified Habedank model. The conductance form is expressed by the following formulas:

$$\frac{1}{g_C}\frac{dg_C}{dt}=\frac{1}{\theta_C}\left[\frac{u^2}{u_C^2(l)+\frac{1}{g_C}p_v\left(\frac{dl}{dt}\right)}\left(\frac{g}{g_C}\right)^2-1\right] \tag{43}$$

$$\frac{1}{g_M}\frac{dg_M}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{i}{g_M\cdot l\cdot E_{Mstat}(i)}\frac{g}{g_M}-1\right]-\frac{1}{l}\frac{dl}{dt} \tag{44}$$

If one takes into consideration the series connection of resistances (17), the form of the model will be

$$\frac{1}{r_C}\frac{dr_C}{dt}=\frac{1}{\theta_C}\left[1-\frac{u^2}{u_C^2(l)+r_C\cdot p_v\left(\frac{dl}{dt}\right)}\left(\frac{r_C}{r}\right)^2\right] \tag{45}$$

$$\frac{1}{r_M}\frac{dr_M}{dt}=\frac{1}{\theta_{Ms}}\left[1-\frac{ir_M}{l\cdot E_{Mstat}(i)}\frac{r_M}{r}\right]+\frac{1}{l}\frac{dl}{dt} \tag{46}$$

where $E_{Mstat}(i)$ – virtual characteristic of electric field intensity.

The hybrid model of the arc column, taking into consideration its length changes, associates, by means of an appropriate tapering function $\mathcal{E}(i)$, models (39) and (41) in the manner (34). Thus, its form is as follows:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta}\left\{[1-\varepsilon(i)]\frac{u_{kol}^2}{u_C^2(l)+\frac{1}{g}p_v\left(\frac{dl}{dt}\right)}+\varepsilon(i)\frac{i}{g\cdot l\cdot E_{stat}(i_M)}-1\right\}-\varepsilon(i)\frac{1}{l}\frac{dl}{dt} \tag{47}$$

Similarly as previously (37), one can write the approximate equation

$$\frac{1}{g}\frac{dg}{dt}= \frac{1}{\theta}\left\{[1-\varepsilon(i)]\frac{u_{kol}^{2}}{u_C^2(l)+\frac{1}{g}p_v\left(\frac{dl}{dt}\right)}+\varepsilon(i)\frac{u_{kol}}{l\cdot E_{stat}(i)}-1\right\} -\varepsilon(i)\frac{1}{l}\frac{dl}{dt} \qquad (48)$$

For simulations of processes in the circuits of electro-technological devices in which the electrode travel rate is relatively low (dl/dt≈0), formulas for the modified Habedank model are reduced to the conductance form

$$\frac{1}{g_C}\frac{dg_C}{dt}=\frac{1}{\theta_C}\left[\frac{u^2}{u_C^2(l)}\left(\frac{g}{g_C}\right)^2-1\right] \qquad (49)$$

$$\frac{1}{g_M}\frac{dg_M}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{i}{g_M\cdot l\cdot E_{Mstat}(i)}\frac{g}{g_M}-1\right] \qquad (50)$$

Similarly, the simplified resistance form of this model is as follows:

$$\frac{1}{r_C}\frac{dr_C}{dt}=\frac{1}{\theta_C}\left[1-\frac{u^2}{u_C^2(l)}\left(\frac{r_C}{r}\right)^2\right] \qquad (51)$$

$$\frac{1}{r_M}\frac{dr_M}{dt}=\frac{1}{\theta_{Ms}}\left[1-\frac{ir_M}{l\cdot E_{Mstat}(i)}\frac{r_M}{r}\right] \qquad (52)$$

The simplified hybrid model of the arc column taking into consideration relatively slow changes of the length of the arc, takes the following form:

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta}\left\{[1-\varepsilon(i)]\frac{u_{kol}^2}{u_C^2(l)}+\varepsilon(i)\frac{u_{kol}}{l\cdot E_{stat}(i)}-1\right\} \qquad (53)$$

Obtained dependences (43)-(53) are relatively simple mathematical models approximating very complex physical processes taking place in high-pressure electric arcs supplied with direct or alternating current and disturbed by factors affecting the length of the plasma arc column.

The macro-models of arcs and simulations of courses in circuits implemented in the programme MATLAB-Simulink can be a subject of a separate article.

Conclusions

1. The combined Habedank and TWV models extend the possibilities of simulating processes in electric arcs of electro-technological and electrical power engineering devices, yet only in cases in which the length of the plasma arc column is constant.

2. The Cassie-Berger model extends the possibilities of simulating processes in variable length electric arcs of electro-technological and electrical power engineering devices, but only in case of those with relatively high intensity of the plasma arc current.

3. The Mayr-Kulakov model extends the possibilities of simulating processes in variable length electric arcs of electro-technological and electrical power engineering devices, yet only in case of those with relatively low intensity of the plasma arc current.

4. The modified Habedank and TWV hybrid models extend the possibilities of simulating processes in electric arcs of electro-technological and electrical power engineering devices in which the length of the plasma arc column is changeable and the range of variability of electric current intensity is wide.

5. The combined series Habedank model of the electric arc, usually preferred in simulating processes in high-voltage devices, can after the implementation of necessary modification take into account the variable length of the plasma arc column.

6. The combined parallel TWV model of the electric arc, usually preferred in simulating processes in low-voltage devices, can after the

implementation of necessary modification take into account the variable length of the plasma arc column.

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