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Development of Methods for the Assessment of Stresses in Welded Structure Elements.
Part 2. The Latest Methods

Abstract: Disadvantages of traditional methods for the determination of stresses in stress concentration zones of welded joints discussed in the first part of this overview required to be classified and inspired the development of technologically advanced solutions. The article contains an overview of works concerned with methods and approaches to the determination of maximum local stresses present in the above-named stress concentration zones. The article provides classification of methods in accordance with international documents and regulations, presents advantages and disadvantages of existing assessment methods concerning near-weld stresses as well as discusses further research trends.

Keywords: welded joints, weld geometry, stresses, concentration of stresses, computational methods

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The first part of this overview (*Biuletyn Instytutu Spawalnictwa no. 4/2017, pp. 59÷66*) indicated divergent results of numerous works focused on the determination of stresses near welds. This inspired the publication of a normative document by the International Institute of Welding (IIW), [1] presenting methods enabling the assessment of fatigue strength allowing for maximum stresses, in particular:

- in relation to nominal stress;
- using the hot spot method;
- using local notch stress and notch strain methods (in an effective concentrator);
- using methods involving stress intensity factors.

Differences between the above-named methods are conditioned by the use of various stress concentrators. The document also contains recommendations concerning the selection of one (out of four) assessment methods as well as recommendations concerning stress measurement methods and the application of the Finite Element Method (FEM).

The development of computer technique (in the late 1980s and early 1990s) was accompanied by the application of FEM to determine stresses and strains as well as the stress concentration factor (SCF) in structural elements. The latter has significantly affected the direction of further research in many countries until today. One of the features of tests performed in the 1990s was not the FEM application as such, but the application of the FEM in relation to particular structures. This was because of the fact that 3D modelling combined with the FEM

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made it possible to analyse the structural elements in whole.

In work [2] the FEM was used to obtain values of structural and local stresses in relation to assembly jigs used in ship structures and subjected to cyclic loads. The work also contained regression equations in relation to the SCF on butting faces of T-shaped welds fixing jigs to the plate. The work also involved an overview demonstrating that (in the above-named period) methods enabling the determination of local stresses (e.g. the hot spot method and the notch stress method) developed sufficiently to solve the problem related to the determination of maximum stresses. Usually, the method used in the aforesaid work has a lot in common with the approach based on the concentrator of the first and the second type, presented in work [3].

Works [4-6] involved FEM-based analyses of structures with various intersection angles described using hull elements and related to Y, X, K, T and type joints as well as cruciform joints of tubular structures. As a result, based on the analysis of regression it was possible to obtain new parametric equations enabling the determination of the SCF. The satisfactory convergence of calculation results related to obtained dependences with results of experiments performed on steel and acrylic models made it possible to recommend the use of the obtained formulas for calculations concerning tubular frames.

Work [7], based on publication [5] and databases of other works by the same authors involved the development of a method making it possible to forecast the value of ` and that of a bend angle in K-shaped tubular structures, where the remaining structure operation time was determined using the equations of cyclic crack resistance.

Work [8] involved the FEM-based obtainment of an SCF-related database used to develop a simple and effective methodology for the forecasting of SCF values concerned with various types of loads and related to welded joints of XX-shaped tubular structures. In addition, the work also involved the obtainment of equations and diagrams recommended for engineering practice in relation to the forecasting of SCF values.

It should be noted that works [4-8] were concerned with the definition of the structural SCF, which in work [3] was referred to as the first type concentrator. An entirely different approach to K-shaped tubular structures was adopted in work [9], where the authors used displacements in the effective concentrator to determine local stresses.

In the above-named period, in addition to the FEM, the effect of geometry on fatigue strength was frequently tested using experimental methods. In work [10] the author tested the effect of various geometry of joints and various types of loads on the rate of fatigue crack growth. It was ascertained that the worst indications were those of cruciform joints. Work [11] was concerned with welded structures with fillet welds, in relation to which the amplitude of local stresses in three concentration zones was expressed at the function of weld geometry, in particular of the toe radius. The issue was also referred to in technical recommendations [12].

As mentioned before, the SCF value affects fatigue service life, including the generation and growth of fatigue cracks. An increase in the potential of FEM-based tests was followed by a significant increase in the number of research works concerned with the above-named effect.

Work [13] was concerned with the investigation of the effect of the fillet weld geometry on the rate of fatigue crack growth in relation to various types of loads. The tests involved cases where cracks were initiated in the base plate or in the joined (welded) element. The tests resulted in the determination of a dependence concerning an increase in the nominal amplitude of threshold stress in relation to a toe radius of less than 0.5 mm.

Measurement results concerning actual profiles of welds in cruciform joints revealed the existence of diversified systems of profile angles and radiuses [14]. For this reason, the obtained data were subjected not only to FEM-based analysis but also to experimental tests. As a result, it was possible to obtain values of the SCF and the distribution of stresses along thicknesses, on the basis of which stress intensity factors (SIF) were calculated and their relation to the geometry of welds was identified.

To a significant extent, work [15] constitutes the continuation of work [14], in relation to T-joints. The work more profoundly investigated the effect of the toe radius on the SIF of a crack in the weld. Related tests involved the making pf specimens having various values of toe radiuses ρ, i.e. restricted within the range of 0.5 to 5.0 mm. The FEM and related values of ρ were used by the authors to determine the SIF in relation to modelled short cracks having a crack length-metal thickness (T) ratio restricted within the range of 0.001 to 0.1. It was ascertained that the above-named increase in the toe radius made it possible to reduce the SCF from 3.31 to 1.95 and increase the time of fatigue crack growth by 53%.

Tests concerning the effect of geometry on the SIF were also addressed in works [16-18]. Work [16] presented the mathematical dependence of the effect of geometry on the SIF taking into consideration effect factor M_k. The obtained mathematical model is used within the wide range of geometric parameters of butt welds. The model demonstrated the significantly greater effect of toe radius ρ and toe angle θ on the SIF and fatigue strength in comparison with the thickness of metal within a wide range of the ratio of the above-named parameters. However, the thickness of metal had a considerable effect at A/T ≤ 0.025, whereas the maximum effect of radius ρ was observed at a/T = 0.05 and that of angle θ – at a/T = 0.15.

Work [17] involved the obtainment of a simplified mathematical model of the effect of toe radius ρ and toe angle θ on the rate of fatigue crack growth, the development of fatigue curves and the formulation of practical recommendations useful in engineering practice and indicating significant prospects for methods of post-weld treatment of welds. The analysis of works [2-17], involving the use of FEM, revealed that the above-named method did not enable the assessment of fractions of external force factors in the total stress in the concentration zone.

One of characteristic features concerning welded joints is the fact that fatigue strength is affected not only by the SCF but also by other factors including the structure of metal and internal welding stresses. For this reason, many researchers prefer not to separate the above-named factors and treat their effect as common.

For instance, in work [18] the authors of publication [16, 17] used the method of linear crack mechanics to obtain an analytical model making it possible to assess the effect of internal stresses and weld geometry on the fatigue strength of butt joints. It was ascertained that a compressive internal stress of 62 MPa triggered using post-weld surface treatment provided effect comparable to that provided by heat treatment. In addition, the authors recommended that in order to increase the fatigue service life of welded joints, the toe radius be increased and the toe angle be decreased to less than 20°. Both effects (compressive stresses and improved geometry) could be triggered simultaneously by subjecting the weld to surface mechanical treatment [19].

In subsequent work [20] the same authors referred to a more general mathematical model making it possible to take into consideration geometric parameters of the joint along with internal stresses. In addition, geometric parameters were supplemented by misalignment and the eccentricity of the butt joint.

Work [21] involved the determination of the effect of stress concentration along with internal stresses, taking into consideration welding imperfections, in relation to fillet welds used in the production of railway tankers. The effect of stress concentration was assessed using strain gauges fixed along a weld. The test revealed that the end of the weld was a concentrator with $\alpha_\sigma = 3$ and had the greatest effect on the crack initiation process. In the above-named work, the quantitative assessment of the effect of stress concentration on fatigue strength involved the presentation of the idea of an equivalent crack based on the method of linear crack mechanics.

Work [22] investigated the effect of stress concentration combined with the metal microstructure in the heat affected zone on fatigue strength. It was ascertained that only in terms of the microstructure the fatigue strength in the HAZ was higher than that of the base material, yet in relation to welded joints the influence of both of the above-named factors produced the opposite effect. The work involved the obtainment of fatigue curves for various weld geometry as well as the dependence between the value of theoretical and effective SCF, the determination of which in relation to butt and fillet welds was discussed in work [23]. The conclusions formulated on the basis of overview [24] stated that the effect of stress concentration should not be excluded from analysis when considering various factors affecting fatigue service life. There are many approaches similar to the one presented above, yet their stage of development and the scope of application vary significantly. This fact impedes necessary validation and standardisation of the above-named approaches in relation to industrial applications. Similar conclusions were drawn by the author of subsequent overview [25]. The weld geometry is taken into consideration not only in calculations concerning structures subjected to changing loads but it also proves to affect service life related to non-elastic strains. For instance, work [26] investigated the effect of geometry on the fatigue strength of welded joints in pipelines, proposing the methodology based on the modified Neuber method and the FEM. The essence of the above-named methodology involves the determination of forces acting in the concentration zone (according to the Neuber method) and, afterwards, multiplying their value by the SCF of the weld.

Work [27] investigated the kinetics of butt joints strains and resulted in the formulation of the conclusion stating that pulsed load triggered free vibration in joints and, consequently, led to significant elastic-plastic strains (having a variable sign) of the interface material. The solution of a dynamic elastic-plastic problem and experimental test results [28] confirmed the effect of weld geometry parameters on the value of critical relative strain.

The issue concerned with the accurate determination of the SCF is strictly connected with the treatment of welds (during production) aimed to increase their fatigue strength. The comparison (presented in work [29]) of experimentally determined SCF values (using the polarisation-optical method) with calculation results concerning T-joint performed in accordance with formulas [30] revealed the dependence between a decrease in the SCF obtained through high-frequency hammering and an increase in the fatigue strength of welded joints.

It should be noted that works presented in post-soviet scientific publications had the same direction as works [1-18, 20-23]. For instance, work [31] was concerned with the fatigue service life of tubular T-joints. The authors proposed the idea concerned with the modelling of the above-named joints based on values of stresses in the weld area, i.e. including concentration. By using "reproductive" models, the authors obtained a computational-experimental method making it possible to assess the actual fatigue service life of structures and forecast the service life of structures. Work [32] involved tests concerning the effect of the concentration of structural stresses and strain along with internal stresses on fatigue strength. The research-related forecasting related to conditions of fatigue failure in concentrators of a small radius involved the development of a kinetic model of arterial crack formation. In addition to the rate of crack growth, the aforesaid model took into consideration an additional rate of failure connected with the amplitude of local strains, being the function of the SCF, determined using the following formulas [33, 34]:

$$\alpha_{\sigma} = 1 + A \cdot \rho^{-n} \quad n = 0,3 \div 0,67$$

where ρ – toe radius; A – parameter of joint macrogeometry, determined by dimensions and external forces preset when designing as well as by the welding technology.

It should be noted that the authors of work [32,] published in 2000, indicated nearly the entire lack of methodologies enabling the determination of the service life of structures exposed to loads in the low-cycle fatigue area as well as stated that taking into consideration all structural and technological factors was impeded by limitations of methodologies developed primarily by foreign classification societies. The year 2003 saw the publication of an updated normative document [35] which, to some extent, enabled the solution of the above-named problem. The document presented more than eighty structural elements having thicknesses of less than 5 mm, in relation to which, depending on the material (steel or aluminium alloy), it was possible to determine the ultimate fatigue strength according to FAT S (File Allocation Table), specifying the range of nominal stresses S in relation to base fatigue strength $N = 2 \cdot 10^6$ and the preset asymmetry coefficient. The value of the range of stresses $\Delta\sigma$ was substituted to the designation FAT instead of S. Some updates which entered regulations [35] are described in detail in work [36]. An important feature of document [35] concerning the concentration of stresses was the fact that the document regulated a number of manners used for the determination of corresponding computational stresses [1]. When selecting one of the above-presented methods enabling the evaluation of fatigue service life it is necessary to take into consideration the type of a stress concentrator corresponding to a specific level of concentration. In addition, each method provides its evaluation accuracy. For instance, the first method can be used for rough assessments where nominal stresses σ_{nom}, i.e. mean in cross-section, are relatively easy to determine. The second, more accurate, method makes it possible to take the structural SCF into consideration where the determination of nominal stresses is difficult because of structural complexity. The essence of this method consists in the linear or quadratic extrapolation of stresses calculated (using the FEM) at two or three check points, to the toe (hot spot) without taking into consideration the radius of the toe exposed to structural stresses σ_{hs}. The third and fourth methods are the most accurate in relation to welded joints as they take into consideration microgeometric parameters. In particular, the use of the third method, taking into consideration the toe radius, determines the SCF introducing a non-linear component to the distribution of stresses across the material thickness, where local maximum (peak) stresses σ_{peak} in the joint are determined as follows:

$$\sigma_{peak} = \sigma_{hs} \cdot \alpha_{\sigma}$$

Obviously, the precise determination of the SCF will make it possible to more reliably allow for the effect of stress gradient on fatigue strength, which is overtly expressed and, according to data [37], linear in nature. It should be noted that such a method of the determination of maximum stresses is relatively "young", yet it is intensely developed and widely applied globally when calculating stresses and designing structures exposed to cyclic loads. The first decade of the method development is described in detail in overview [38], whereas the practical aspects of its applications are described in publication [39].

Work [40] involved the successful use of a method of calculation according to notch stresses and aimed to identify reasons for the premature development of fatigue cracks in a T-joint of a steel tower structure. Taking into consideration the nature of the cracks it was possible to identify errors in the design. As a result, it became necessary to re-analyse all load parameters. It was demonstrated that the determination of the ultimate fatigue strength of the above-named joint required the division of data from FAT 225 by the value of the SCF near the welds.

The final version of the document by the IIW [35] was published in 2006. The year 2009 saw the publication of guidelines along with exemplary applications [41] and possible methods of the further development of the document. The analysis of results obtained in works [1-5, 7-29, 31-41] revealed the existence of analytical dependences only in relation to the determination of the structural SCF in tubular joints, whereas the SCF related to the weld geometry was determined in each case separately. In addition, similar to the previous approach, the mutual effect of concentrators on the face and root sides of the weld was not subjected to consideration.

Works attempting to fill up the above-presented gaps started to appear relatively late. Work [42] was concerned with testing the effect of weld convexity dimensions on stresses, yet works [43, 44] investigating the effect of the asymmetry of a welded joint on the value of local stresses and their distribution demonstrated that the mathematical model adopted in publication [42] did not correspond to actual load conditions affecting the modelled welded joint. The author of works [43, 44] indicated that the load application axis did not coincide with the symmetry axis of the cross-section in the convexity zone, thus leading to the generation of a bending moment reducing stresses on the weld face and increasing stresses on the weld root side.

Work [45] presents a methodology enabling the determination of local stresses in the root of a butt joint, based on the theory of offset sections [46]. The author obtained analytical expressions describing changes in stresses in the concentration zone, both on the contour and at the depth of a joint, in the form of functions of coordinates. The foregoing enabled the determination not only of the SCF but also of stresses at any point of a welded joint located in the concentration zone. The calculation results concerning the SCF and obtained using the formula proposed in the work satisfactorily coincided with the values provided by the authors of [47-49].

The analytical method enabling the determination of stresses in the concentration zone, developed by the author of publication [45], was used in work [50] in relation to the combined effect of tensile strength and bending moment on an overlap welded joint. The analytical dependences presented in the work make it possible to take into consideration both types of loads when determining the total SCF. The obtained results were consisted with the results obtained by authors using the FEM as well as with the results of work [51].

In spite of the active development of software for FEM calculations, there is a need for analytical dependences as the latest research [52, 53] demonstrated that the combination of analytical and numerical methods can prove beneficial. The authors of the above-named works proposed an effective methodology, which, first by using the FEM and the method of determining local stresses in the hot spot, enabled the determination of tensile and bending stresses, and next, by multiplying each of them by an appropriate SCF value, enabled the determination of the maximum local stress. According to the authors, without the above-named combination of calculation methods, the accurate evaluation of maximum local stresses requires the making of a 3D model of finite elements taking into consideration microgeometric features including the toe radius.

It is known that calculations with the detailed approximation of all structural elements and nodes, performed using 3D finite elements, are very difficult as they require the use of considerable computing powers and are conducted in exceptional cases [54]. In addition to that, scientific reference publications [55-58] contain very inconsistent information concerning the selection of types and sizes of elements. This significantly impedes the above-named selection and may reduce the accuracy of assessments.

When using the FEM when determining structural stresses at a hot spot, it is possible to apply a thick mesh with the automatic division into the minimum number of elements, the size of which amounts to 0.25 of the metal thickness. When calculating the SCF using the hot spot method, the authors recommend the use of analytical dependences found in scientific reference publications and selected in relation to an external force factor. The authors of publication [53] suggested the method of the distribution of stresses located across the thickness at the hot spot into tensile-compressive stresses (membrane) σ_m and bending stresses σ_b. For each of the above-named stresses, as in work [50], it is necessary to determine an appropriate SCF value and, next, to determine the maximum local stresses using the following formula:

$$\sigma_{\max} = \sigma_m \cdot \alpha_\sigma^m + \sigma_b \cdot \alpha_\sigma^b$$

The authors of works [52, 53] recommend the formulas presented in publication [30] as analytical dependences enabling the determination of the SCF. The above-named formulas have structures similar to formulas recommended in publications [59]. However, as mentioned before, according to the analytical data presented in publication [49], the aforesaid formulas have a limited application range and do not always enable the obtainment of a reliable result.

Obviously, the application range related to the approach presented in works [52, 53] is concerned with large-sized spatial structures. In relation to a given segment, the authors of publication [54] suggested an alternative methodology enabling the assessment of stresses and strains of welded joints in bridge spans. The aforementioned methodology contains notions of a fragment and a node of a structure. The analysis of the stresses and strains of a tested node involves the consecutive consideration of models with various types of finite elements, i.e. first the bar (in relation to the structure), next the hull (in relation to a fragment) and next a 3D model (in relation to a node). To ensure the transfer of data related to stresses and strains from one model to another, the authors used an absolutely rigid body.

Conclusions

1. Presently, the most popular methodologies used to assess stresses are those based on the FEM. However, the application of this method is connected with certain limitations including high laboriousness when creating a precise 3D model and its approximation using 3D finite elements. For this reason, the FEM-based analysis is performed to identify structural stresses by using the structural approximation method utilising hull elements with a thick mesh, where microgeometric parameters of a welded joint are taken into consideration using the SCF.

2. The SCF is determined either according to experimental dependences obtained as a result of the statistical development of experimental results involving actual elements or through analysis, i.e. by solving elastic theory problems or establishing equilibrium conditions present in the cross-section of stresses and external force factors affecting the welded joint. The analytical methods are more favourable as they make it possible to determine not only the SCF but also stresses triggered by all external force factors separately in the form of functions changing on the contour and at the depth of the welded joint.

3. The present-day computational dependences used for the determination of the SCF in butt welded joints are applied in relation to welds with one-sided or symmetric two-sided excess weld metal, whereas butt welded joints with asymmetric two-sided excess weld metal remain unnoticed. In addition to that, the usability area of existing dependences excludes their application to determine the SCF in welded joints of thin-walled elements. As a result, the authors deem it important to continue the development of activities in this this area as well as to develop analytical methods enabling the description of stresses in thin welded joints, including those having various shapes on the face and on the root side.

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