Antoni Sawicki

Hybrid Models of Electric Arc with Improved Dynamic Characteristics in Wide Ranges of Changes in Bipolar Current Value

Abstract: The article discusses the limitations of the Mayr and Cassie linear models and of the Mayr generalized model accompanying the precise mapping of electric arc column within a wide range of changes in bipolar current. The research involved the development of new non-linear hybrid models of arc by the association of linear and non-linear models, the creation of arc hybrid macromodels and the verification (utilising the Matlab-Simulink software programme) of the efficiency of dynamic voltage-current characteristics through simulations of processes in a simple circuit with electric arc.

Keywords: electric arc, Mayr model, Cassie model, generalised Mayr model, hybrid model

DOI: 10.17729/ebis.2017.2/5

Introduction

The mathematical Mayr model of electric arc is used to represent processes occurring in circuits characterised by low current values. In cases of bipolar current, the extreme values of voltage in the neighbourhood of current passage through zero are not directly determined by a static characteristic but also result from the value of the time constant of the model. The above-named constant aims to relatively accurately represent waveforms also within the range of relatively high current values. The adoption of the linear model of electric arc constitutes significant simplification in view of the fact that experimental tests revealed the strong non-linearity of the damping function depending on current [1, 2], which, in turn, implies the limited possibilities of the approximation of arc discharge by simple linear models of arc. In addition, the use of only low values of the time constant might favour the instability of computational processes.

One of the solutions to the problem of the accurate representation of the waveforms of electric quantities within the entire range of changes in low bipolar current is the generalised Mayr model [3]. However, similar to the ordinary Mayr model, the generalised Mayr model is incapable of accurately representing processes within the range of high current values. For this reason, this study recommends the application of the same procedure as in the case of the ordinary Mayr model, where, because of the limited application-related possibilities, the modification of the model, i.e. the hybrid model, was developed (by the parallel association with the Cassie model) [4].

dr hab. inż. Antoni Sawicki (PhD (DSc) hab. Eng.). Professor at Częstochowa University of Technology, Faculty of Electric Engineering

This publication aims to create and test the properties of mathematical hybrid arc models formed through the association of the generalised Mayr model with the Cassie models (where the non-linear damping function was used). The further expansion of the approximating possibilities of the hybrid model takes into consideration the power dissipated by arc column radiation.

The initial stage of experimental tests and of the modelling of electric arc involved the determination of the values of near-electrode voltage drops, usually performed using various direct or indirect methods [5]. Knowing the values of the above-named drops, it is possible to eliminate their effect on plasma column characteristics. To this end, it is possible to use an appropriate electronic compensator [6]. In terms of the computer-based version of the measurement system, it is convenient to use the computational method of the compensation of near-electrode voltage drops and to obtain a momentary voltage drop on the column of arc alone.

Mayr and Cassie Models and the Generalised Mayr Model of the Electric Arc Column

The Mayr model of the electric arc column can be described using the well-known linear differential equation

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M} \quad (1),$$

where g – column conductance, S; P_M – Mayr constant power, W; θ_M – time constant of the Mayr model expressed in millisecond fractions. The model corresponds to the static voltage-current characteristic

$$U_{col} = \frac{P_M}{I} \quad (2).$$

The effect of processes in the column on the waveform of voltage in the neighbourhood of the passage of current through zero values can be taken into consideration using equation [3] in the following form

$$\theta_W \frac{dg}{dt} + g = \frac{i^2 + I_W^2}{P_W} = \frac{i^2}{P_W} + G_W \quad (3),$$

where G_W – characteristic conductance of the generalised Mayr model, S; P_W – constant power of the generalised Mayr model, W; I_W – constant component of the current of the generalised Mayr model, A. The above-named model corresponds to the following static characteristic

$$U_{col} = \frac{P_W I}{I^2 + P_W G_W} = \frac{P_W I}{I^2 + I_W^2} \quad (4).$$

In terms of arc having a quasi-flat static characteristic, dynamic processes are relatively well approximated by the Cassie model describing (using a differential equation) time changes of the conductance of plasma g

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2} \quad (5),$$

where U_C – constant value of the Cassie model voltage; θ_C – time constant of the Cassie model. In the case of excitation triggered by direct current, the model corresponds to the static characteristic

$$U_{col} = \pm U_c \quad (6).$$

Equation (5) can be transformed into the most frequently used non-linear form

$$2\theta_C \frac{dg}{dt} + g = \frac{i^2}{g U_C^2} \quad (7).$$

Considerations concerning some high-current electric arcs or arcs in high-pressure gases take into account the rise in voltage-current characteristics within the range of high current values [7, 8]. Then, the modified Cassie model of such arc has the form of the following non-linear differential equation

$$2\theta_C \frac{dg}{dt} + g = \frac{i^2}{g U_C^2} - \frac{P_{rad}(i)}{U_C^2} \quad (8).$$

The advantage of linear models over non-linear models results from the relatively easier development of the effective spectral and integral methods of the experimental determination of parameters [6, 9]. The above-named methods [10] were developed only in some cases of non-linear models.

Generalised Mayr-Cassie Hybrid Model of Electric Arc

The creation of the hybrid model required the parallel combination of two elements having conductances described by the Mayr (1) and Cassie (7) models

$$g_M = g = \frac{i^2}{P_M} - \theta_M \frac{dg}{dt}, \text{ if } |\mathrm{i}| < I_0 \quad (9),$$

$$g_C = g = \frac{i^2}{gU_C^2} - 2\theta_C \frac{dg}{dt}, \text{ if } |\mathrm{i}| \geq I_0 \quad (10),$$

where I_0 – value of current corresponding to the fuzzy changeover between the models activated by tapering function ε depending on current [4]

$$g = \varepsilon(i) g_M + [1-\varepsilon(i)] g_C \quad (11).$$

Depending on the type of electric current and the required accuracy of approximation, function ε can take various analytical forms [11]. Because of the significantly varying values of the time constants of the models and due to the physical non-linearity of the damping function, it is necessary to introduce its approximations leading to the obtainment of a resultant formula for the hybrid model

$$g = G_{\min} + \varepsilon(i)\frac{i^2}{P_M} + \\ + [1-\varepsilon(i)]\frac{i^2}{gU_C^2} - \theta(g)\frac{dg}{dt} \quad (12),$$

where G_{min} – minimum value of conductance, in the generalised Mayr model corresponding to conductance G_W. In publication [4], the damping function depends on current $\theta(|i|)$.

Taking into account the generalised Mayr model (3), useful within the limited range of current changes

$$g_W = g = \frac{i^2 + I_W^2}{P_W} - \theta_W \frac{dg}{dt} = \\ = \frac{i^2}{P_W} + G_W - \theta_W \frac{dg}{dt}, \text{ if } |i| < I_0 \quad (13),$$

in the association with the conductance of arc described by the Cassie model (10)

$$g = \varepsilon(i) g_W + [1-\varepsilon(i)] g_C \quad (14),$$

the generalised Mayr-Cassie hybrid model is obtained in the following form

$$g = \varepsilon(i)\frac{i^2 + I_W^2}{P_W} + \\ + [1-\varepsilon(i)]\frac{i^2}{gU_C^2} - \theta(g)\frac{dg}{dt} \quad (15)$$

or

$$g = \varepsilon(i)\cdot\left(\frac{i^2}{P_W} + G_W\right) + \\ + [1-\varepsilon(i)]\frac{i^2}{gU_C^2} - \theta(g)\frac{dg}{dt} \quad (16).$$

The hybrid arc model associating the Mayr sub-model (1) and the modified Cassie sub-model (8) has the following form

$$g = \varepsilon(i)\frac{i^2}{P_M} + \\ + [1-\varepsilon(i)]\cdot\left[\frac{i^2}{gU_C^2} - \frac{P_{rad}(i)}{U_C^2}\right] - \theta(g)\frac{dg}{dt} \quad (17).$$

In turn, the hybrid arc model associating the generalised Mayr sub-model (3) and the modified Cassie sub-model (8) can be presented in the following form

$$g = \varepsilon(i)\frac{i^2 + I_W^2}{P_W} + \\ + [1-\varepsilon(i)]\cdot\left[\frac{i^2}{gU_C^2} - \frac{P_{rad}(i)}{U_C^2}\right] - \theta(g)\frac{dg}{dt} \quad (18)$$

or

$$g = \varepsilon(i)\cdot\left(\frac{i^2}{P_W} + G_W\right) + \\ + [1-\varepsilon(i)]\cdot\left[\frac{i^2}{gU_C^2} - \frac{P_{rad}(i)}{U_C^2}\right] - \theta(g)\frac{dg}{dt} \quad (19).$$

Simulation Tests of Electric Arc Hybrid Models

To test the possibilities concerning the use of hybrid models when representing the dynamic voltage-current characteristics of arc, it was necessary to build their macromodels using the Matlab-Simulink software programme. The macromodels were included in a simple electric circuit containing the source of sinusoidal current having amplitude I_m = 300 A and frequency f = 50 Hz. In addition, the model of arc took into consideration near-electrode voltage drops (U_{AK} = 16 V), assuming the use of identical electrodes (cathode and anode), corresponding to symmetric static and dynamic characteristics. The damping function could be preset in the non-linear form depending on current [4]

$$\theta(i) = \theta_{i0} + \theta_{i1}\exp(-\alpha_i|i|) \approx \begin{cases} \theta_1, & \text{if } |i| \text{ is low} \\ \theta_0, & \text{if } |i| \text{ is large} \end{cases} \quad (20),$$

where α > 0, $\theta_1 >> \theta_0$ – constant approximation coefficients. However, according to Pentegov and Sidoretz [12, 13], more physically justified is its dependence on the conductance of column $g\frac{dQ}{di_\theta^2} = \theta(g)$, where Q – plasma enthalpy. Such functions are used in non-linear models of arc [10]. In the case under consideration, the similar approximation of the damping function could be used

$$\theta(g) = \theta_{g0} + \theta_{g1}\exp(-\alpha_g g) \quad (21).$$

The assumption involved the tapering function expressed using the Gaussian curve

$$\varepsilon_0(i) = \exp\left(-\frac{i^2}{I_0^2}\right) \quad (22).$$

Figure 1a presents the results of simulations of processes in the circuit with the generalised Mayr-Cassie hybrid model of arc (15) with the damping function having the form of formula (20), while Figure 1b reffers to the damping function (21). The selected values of the model and circuit parameters corresponded to the relatively weak effect of the value of additional variable I_W on the shape of the characteristics, which, in turn, affected the selection of the form and parameters of the tapering and damping function.

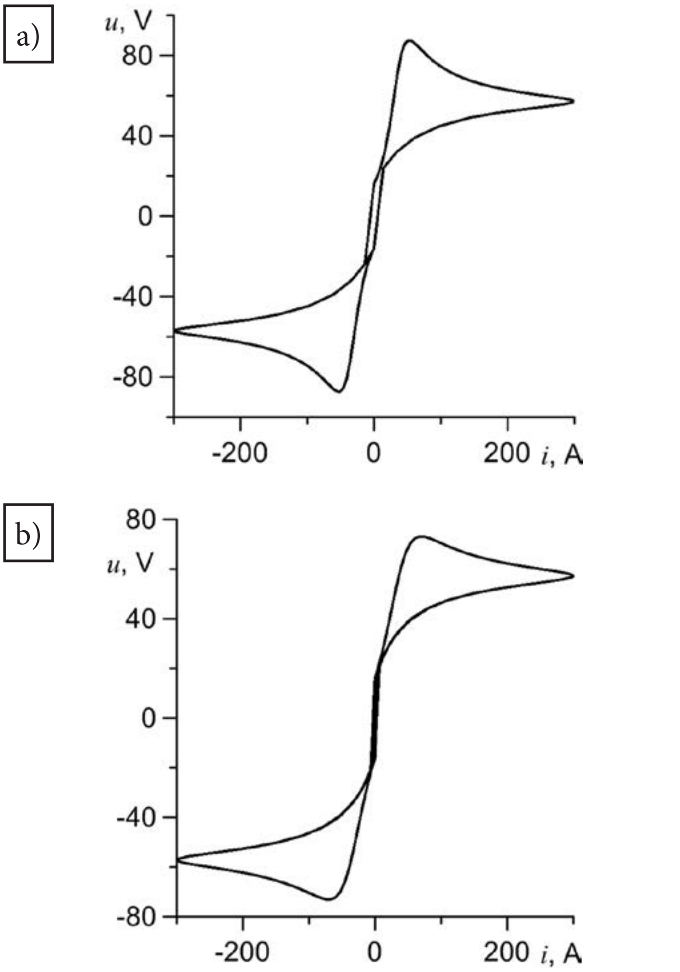

Fig. 1. Dynamic characteristic of electric arc described by the generalised Mayr-Cassie hybrid model (P_W = 160 W; I_W = 1,5 A; U_C = 40 V; I_0 = 4 A):
a) with the damping function depending on current (20) ($\theta_{i0} = 1\cdot10^{-4}$ s; $\theta_{i1} = 5\cdot10^{-4}$ s; α_i = 0.001 A^{-1});
b) with the damping function depending on conductance (21) ($\theta_{g0} = 1\cdot10^{-4}$ s; $\theta_{g1} = 5\cdot10^{-4}$ s; α_g = 0.02 S^{-1})

Figure 2a presents the results of simulations of processes in the circuit with the generalised Mayr-Cassie hybrid model of increased radiation arc (18) and with the damping function having the form of formula (20). The characteristic of radiation-dissipated power [14] can be approximated using the following dependence

$$P_{rad}(i)=a_1|i|+a_2i^2 \quad (23),$$

where constant coefficients are a_1 in V; a_2 in Ω. The selection of the values of the above-named coefficients enables the easy representation of the voltage waveform in the high-current range. In turn, Figure 2b presents the results of simulations of processes in the circuit with the generalised Mayr-Cassie hybrid model of increased radiation arc (18) and with the damping function described by the dependence (21).

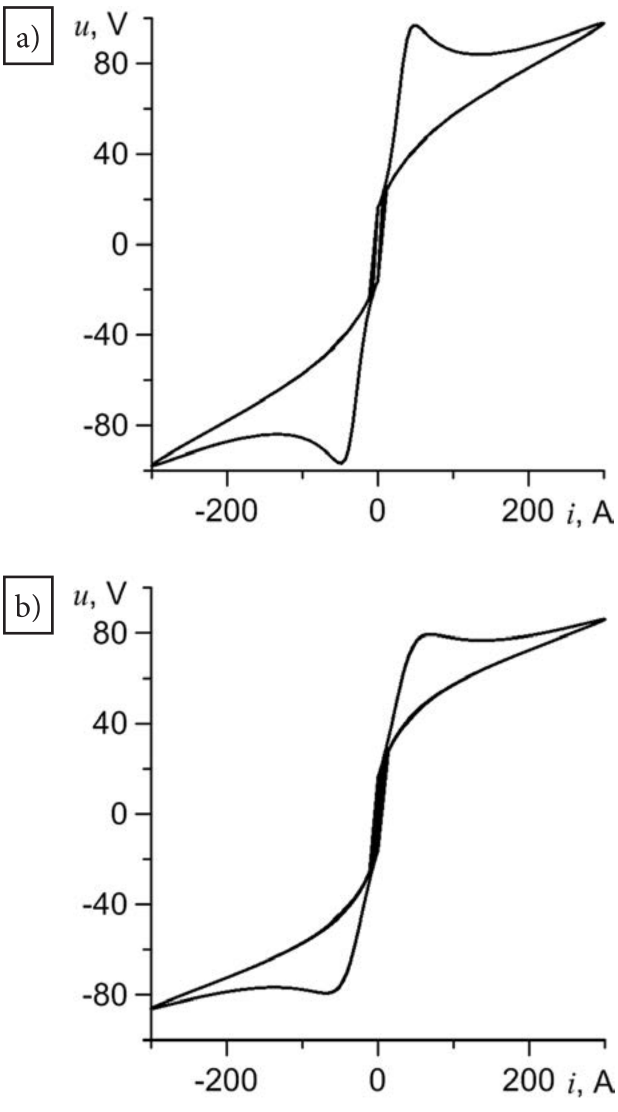

Fig. 2. Dynamic characteristic of electric arc described by the generalised Mayr-Cassie hybrid model (P_W = 160 W; I_W = 1,5 A; U_C = 40 V; I_0 = 4 A; a_1= 1 V):
a) with the damping function depending on current (20) (θ_{i0} = 1·10^{-4} s; θ_{i1} = 5·10^{-4} s; α_i = 0.001 A^{-1}; a_2=0.2 Ω);
b) with the damping function depending on conductance (21) (θ_{g0} = 1·10^{-4} s; θ_{g1} = 5·10^{-4} s; α_g = 0.02 S^{-1}; a_2=0.15 Ω)

Conclusions

1. The newly created arc hybrid models associate relatively simple unidimensional models, each of which satisfies the requirements concerning the accuracy of representation within the restricted range of excitation current. Such features are used in the hybrid model making it possible to control the activity of each model.

2. Although new arc hybrid models are relatively complex, they enable the shaping of dynamic voltage-current characteristics in a relatively flexible manner.

3. The use of combined models can improve the accuracy of arc representation but at the same time it impedes the physical interpretation of simulation results and the experimental identification of model parameters.

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