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Non-Linear and Parametric Mathematic Models of Electric Arcs

Abstract: The article describes factors affecting changes of physical parameters of plasma in a column and in near-electrode areas of arc in electrotechnological devices. The article also presents the effect of chemical composition and of temperature on the non-linearity of selected characteristics of plasma-creating gases. In addition, the study concerns the effect of the heterogeneity of electrode materials on the non-linearity and parametrisation of near-electrode voltage drops, taken into consideration in the approximation of static characteristics of an arc column. The article also presents selected linear and non-linear models of electric arc columns. Research-related simulation results, in the form of non-linear dynamic characteristics of electric arcs, were highly consistent with experimental data.

Keywords: electric arc, Pentegov model, Schwarz-Avdonin model, non-linear model

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Introduction

Electrotechnological arc devices are characterised by very wide ranges of changes in numerous physical parameters, i.e. current, temperature, gas flow rates, rates of arc column and electrode spots travel etc. Such changes are particularly visible when devices are being activated and deactivated and as reactions to disturbances in supply and control systems, e.g. changes of arc column length and gas pressure. In some electrotechnological devices, the above named changes should be either short or slight, due to adverse disturbances of emission of electrodes from the surfaces of cathodes made of refractory materials. Wide ranges of arc current changes reveal non-linear phenomena caused by changes in plasma and electrode temperature as well as by changes in the concentration of charges in the working area. The intensification or attenuation of non-linear effects can be influenced by intentional external effects in the form of disturbances of column geometrical dimensions, conditions of heat exchange with the environment [1] etc. The effect of numerous non-linearities of plasma physical characteristics can result in non-linearities of arc mathematical models.

Non-Linearities of Gas and Plasma Characteristics

Depending on their design and technological purpose, arc and plasma devices are characterised by various compositions of gas-vapour environments. On one hand, the type of working environment affects the possibilities of electrode material selection, whereas on the other hand, the selection of electrode materials affects

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the chemical composition and physical characteristics of plasma. The final effect includes chemical processes in the charge (metal bath or weld pool) such as oxidation, nitrogenation, carburisation etc.

The use of electrodes in the solid, liquid or even gaseous form (so-called plasma cathodes [2]) leads to the evaporation or sublimation of materials. The use of cathodes saturated with materials reducing electronic work function and facilitating the ionisation of gases can be accompanied by significant changes in the physical properties of the gas environment. The effects of near-electrode magnetic "pinch" lead to the suction of electrode material vapours along with plasma-forming gases and their pumping by the column. In high-current arcs, the suction effect primarily influences the material of a cathode. A cathode stream formed in this manner can have a positive operating effect, stabilising discharges. In short arcs, when reaching the anode surface, the stream affects it chemically with various technological results.

In turn, the selection of the gaseous environment not only affects the properties of plasma but also the active life of electrodes. In oxidising conditions only some electrodes with a stationary cathodic spot (e.g. made of graphite, zirconium, and hafnium) or with a non-stationary cathodic spot (e.g. made of steel or copper) can be used. The use of a diaphragm, aimed to protect a cathode against oxidation and a significant change of the chemical composition of the plasma torch gas environment, slightly affects arc plasma properties. Changes are primarily concerned with the properties of a gas stream at the outlet of a plasma torch nozzle.

Only in some cases is it possible to use a gaseous environment supporting the operation of electrodes. The use of graphite electrodes in a hydrocarbon environment can lead to the regeneration of electrodes, significantly extending their active life.

The chemical composition of plasma-forming gases variously affects the physical characteristics of plasma [3, 4]. Examples of thermo-physical characteristics of selected mixtures of gases with metal vapours are presented in Figure 1. An increase in aluminium vapour concentration in air δ_{Al} increases the conductivity of plasma σ, but only in the temperature range below 10,000 K (Fig. 1a). Near the above named temperature, the function also becomes more non-linear. The presence of metals in plasma is responsible for the linearization of the heat conductivity coefficient λ.

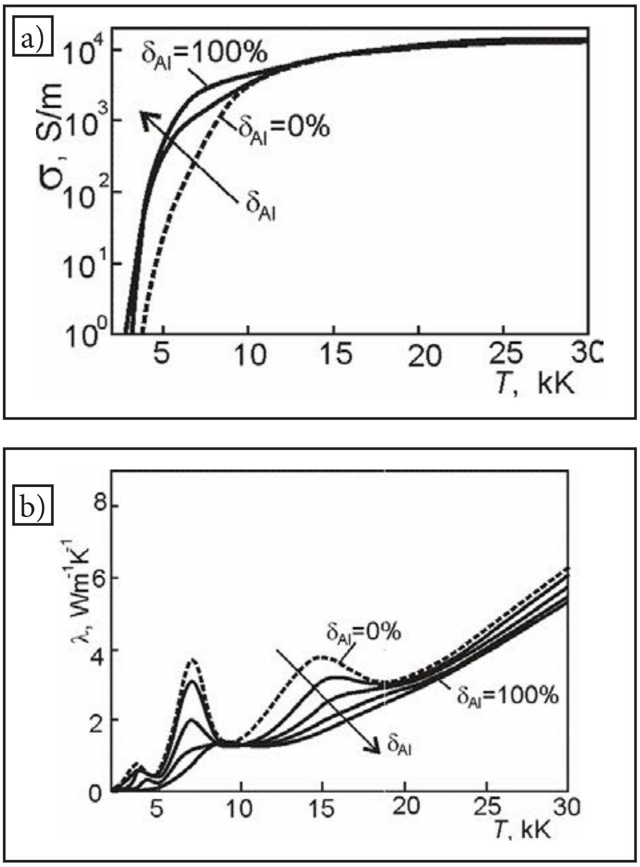

Fig. 1. Selected physical characteristics of mixtures of gases and metal vapours under a pressure of 1 bar
a) conductivity; b) heat conductivity coefficients (metals – full lines, air – dashed lines)

On the basis of experimental test results [5], it can be stated that in the range of low arc currents (below 50 A), an increase in the average temperature of plasma accompanied by an increase in arc current is steeper (Fig. 2) than in the current range above 50 A. Hence it can be concluded that the non-linearity of gas physical characteristics (Fig. 1) could only have a stronger influence on arc models in the low-current range. This conclusion can also refer to AC arcs. On this basis, temperature is used as a state variable when modelling arc lamps [6].

The shape of arc static characteristics is strongly influenced by gas ionisation energy. The greatest energy characterises atoms of inert gases (e.g. He – 24.58 eV, Ne – 21.56 eV, Ar – 15.76 eV). In turn, the lowest ionisation energy characterises alkaline metals (e.g. Na – 5.14 eV, K – 4.34 eV, Cs – 3.89 eV). The most commonly used metals are characterised by the medium ionisation energy (Al – 5.98 eV, Fe – 7.87 eV). The carbon-related ionisation energy amounts to 11.24 eV. The low ionisation energy of the gas-vapour environment decreases arc ignition voltage, which reduces the non-linearity of its characteristics in the low-current range.

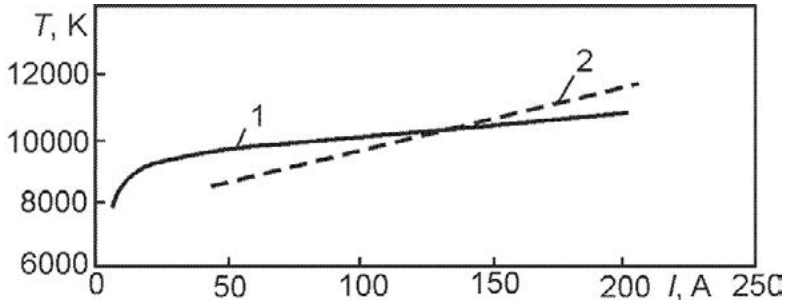

Fig. 2. Dependence of plasma temperature on TIG welding arc current (1, 2 – approximations of various data by various authors)

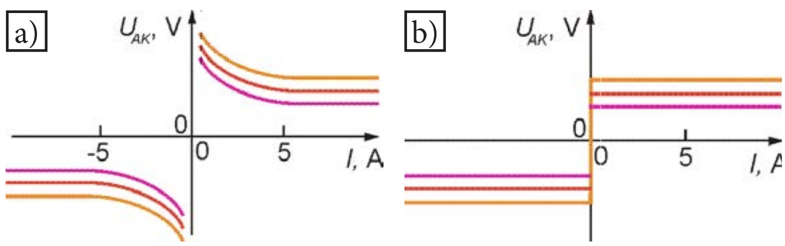

Fig. 3. Approximations of near-electrode voltage drop characteristics a) for low-current arcs and b) high-current arcs

Effect of Materials on the Non-Linearity and Parametrisation of Near-Electrode Voltage Drops

The non-uniform structure of electric arc requires detailed consideration of processes in at least two thin near-electrode areas and in an arc column. It is usually assumed that these three areas are characterised by a uniformity of plasma characteristics. The negligible inertness of processes in near-electrode areas justifies a very simple modelling of cathodic and anodic voltage drops, using approximately polarised voltage drops, which is equivalent to the presence of strongly non-linear serial resistance in a circuit. The accurate representation of low-current arc physical characteristics requires the use of a non-linear function (Fig. 3a). It is then comfortable to model these voltage drops using appropriately controlled voltage sources. However, in most cases related to the operation of electrotechnological devices, root-mean-square current values are sufficiently high so that these voltage drops can be treated as constants. In such situations, it is possible to use a simplified approximation (function) corresponding to uncontrolled voltage sources (Fig. 3b). Such an approach significantly facilitates the experimental determination of near-electrode voltage drops [7]. The non-linearity of near-electrode voltage drops has a negligible effect on voltage-current characteristics of long (high-voltage) arcs. In DC arcs or unipolar impulse arcs, near-electrode voltage drops constitute a mere additional constituent constant of resultant arc voltage.

In addition to non-linear phenomena, electric arcs are also characterised by parametric phenomena. Standard operational states of electrotechnological devices require the use of homogenous materials as regards electrodes and the molten charge or elements being welded. A strongly non-uniform structure is characteristic of poorly sorted steel scrap during melting in an arc furnace. Similarly, elements subjected to welding do not always have the identical chemical composition. Due to economic and thermal aspects, cathodes of plasma torches also do not have a uniform structure. In emergency states of operation, copper holders of cathodes may momentarily (with catastrophic erosion) take over the emissive role, i.e. characteristic of inserts made of refractory materials (e.g. tungsten, zirconium or hafnium).

Therefore, external factors can affect momentary values of voltage drops if dissimilar electrodes are used. Changes of gas pressure or magnetic field intensity alone do not trigger visible effects of changes of near-electrode voltage drops [7]. However, many types of electrotechnological devices utilise arcs with one or two moving arc spots. The movements of arc spots can be caused by various mechanical factors, e.g. dissimilar electrodes moving in a linear or rotating manner, or transverse gas flows. The factor responsible for excitation is usually a transverse magnetic field. The mobility of electrode spots may vary. An anodic spot is less tied to the base and is characterised by greater mobility than a cathodic spot, which is often used in stream plasma torches [8].

Wide ranges of current changes also lead to changing operational states of thermo-emissive cathodes. A decrease in current may result in a transition from a diffusive regime without a cathodic spot to a state with a cathodic spot accompanied by an increase in near-cathode voltage drop.

One of the symptoms revealing the non-linearity of arc is the asymmetry of near-electrode voltage drops. This asymmetry is caused by various emissive properties of electrode materials, yet it can be eliminated using special electronic systems.

Non-Linearity and Parametrisation of Arc Column Static Characteristics

There are many factors affecting the value of inter-electrode gap breakdown voltage, e.g. the shapes of the electrodes and the chemical composition, pressure and temperature of the gaseous environment. Almost the same factors affect the value of voltage on the column of burning arc. Figure 4 presents families of voltage-current characteristics of arcs burning in gases having various parameters. Relatively long and nearly linear sections of characteristics enable treating the arc as a linear element in DC electric circuits or circuits of pulsed current having a low frequency unipolar waveform. Sometimes the linearization of static characteristics is obtained by imposing a high-frequency waveform on low-variable current. Particularly advantageous is the synchronisation of a generator generating high-voltage and high-frequency impulses, in areas where low-variable current passes through the zero.

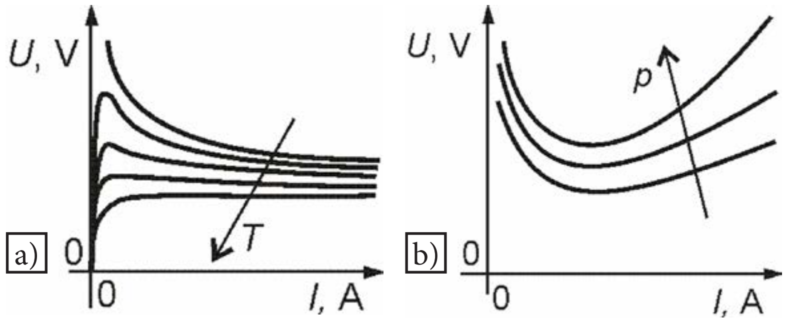

Fig. 4. Arc static voltage–current characteristics: a) in gas of various temperatures; b) in gas of various pressures

The most commonly used approximations of non-linear static voltage-current characteristics include a hyperbolic-linear approximation (horizontal) and a hyperbolic-linear approximation (increasing). The first of the above named characteristics can be described using the formula

$$U_{st}(I) = U_{AK} \cdot \operatorname{sgn}(I) + U_{col}(I) = \\ = (U_{AK} + U_{CP}) \cdot \operatorname{sgn}(I) + \frac{P_{MP}}{I} \quad (1)$$

where U_{CP}, P_{MP} – constant coefficients of approximation. In turn, the second of the above named characteristics can be described using the formula

$$U_{st}(I) = U_{AK} \cdot \operatorname{sgn}(I) + U_{col}(I) = \\ = U_{AK} \cdot \operatorname{sgn}(I) + R_0 I + \frac{P_{MR}}{I} \quad (2)$$

where R_0, P_{MR} – constant coefficients of approximation. In both cases it was assumed that the length of the arc column is constant. However, if due to various external factors, the length of the arc column is variable, the parameters referred to above are functions of time and then $R_0(L(t))$ and $P_{MR}(L(t))$. Natural disturbances of arc column length caused by the convective motion of gases are responsible for the parametric character of dependences (1) and (2) as well as of equations describing dynamic states of a circuit with arc. In electrotechnological devices, changes of pressure are usually quasi-static in nature [9] and do not cause the parametrisation of equations.

Linear and Non-Linear Mathematical Models of Electric Arc Columns

The equation of column power balance constitutes the primary input assumption when creating many one-dimensional mathematical models of arc and can be expressed in the form

$$\frac{dQ}{dt} = P_{el} - P_{dys} \tag{3}$$

where Q – enthalpy of plasma; $P_{el}=u_{col}i=i^2/g$ – supplied electric power; P_{dys} – dissipated thermal power. The adoption of various simplifying assumptions [10] enables the creation of models expressed by linear or non-linear differential equations. Simulations of processes taking place in circuits in low-current ranges are usually performed using the Mayr linear model expressed by the dependence

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M} \tag{4}$$

where g – electric conductance, θ_M – time constant of the Mayr model, P_M – power of the Mayr model. In turn, high-current ranges require the use of the Cassie model in its linear form, i.e.

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2} \tag{5}$$

where θ_C – time constant of the Cassie model, U_C – voltage of the Cassie model.

The generalisation of dependences (4) and (5) constitutes the Pentegov model, created by the relationship between the square of state current i_θ and the square of arc actual current i in the form of the following first-order linear differential equation:

$$\theta \frac{di_\theta^2}{dt} + i_\theta^2 = i^2 \tag{6}$$

where θ – time constant of the Pentegov model. Momentary voltage on the arc column can be expressed using the dependence

$$u_{col}(i) = R_{col}(i_\theta)i = \frac{U_{col}(i_\theta)}{i_\theta}i \tag{7}$$

where U_{col} – static voltage-current characteristic of the arc column, R_{col} – static resistance of the arc column.

Non-linear arc models include the Schwarz-Avdonin model, and its variants, i.e. the Mayr-Schwarz, Cassie-Schwarz [11] and Belkin [12]. In non-linear Schwarz-Avdonin models, a variation of previously constant parameters is applied. The Mayr-Schwarz model can be expressed using the equation

$$\theta_M(g)\frac{dg}{dt} + g = \frac{i^2}{P_M(g)} \tag{8}$$

whereas the Cassie-Schwarz model is defined by the formula

$$\theta_c(g)\frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2(g)} \tag{9}$$

where approximations of parameter variability using power functions $P_M(g)=P_{MS}g^\alpha$, $\theta_M(g)=\theta_{MS}g^\beta$, $U_C(g)=U_{CS}g^\chi$, $\theta_C(g)=\theta_{CS}g^\delta$ are applied. In the generalised Pentegov model, the time constant is subjected to variation [10, 13], making equation (6) non-linear.

One of the symptoms of arc non-linearity is the asymmetry of voltage drops on the column. This asymmetry is caused by external factors periodically affecting the column and causing column length changes or changes of heat exchange conditions [14]. The asymmetry can be eliminated by stabilising the position of electrodes and of the arc column using special magnetic screens.

Non-Linear Dynamic Characteristics of Electric Arcs

The research involved electric arc experimental tests performed in a chamber filled with argon under a pressure of 800 mbar, burning between graphite electrodes, each having a diameter of 12,85 mm. The primary objective was to represent dynamic characteristics of an arc discharge with a permanently changing arc length of L (0-64 mm). The power supply source was composed of two STB250 welding transformers, in which secondary windings were connected in series. In turn, the near-electrode voltage drop was determined using an Esab OrigoTM 3000i AC/DC power source. Due to the non-ideality of the welding transformer as the power source resulting in the lack of external characteristic great falling steepness, during simulation it was necessary to take into consideration time constant changes in expression (6) along with changes of root-mean-square current flowing through arc.

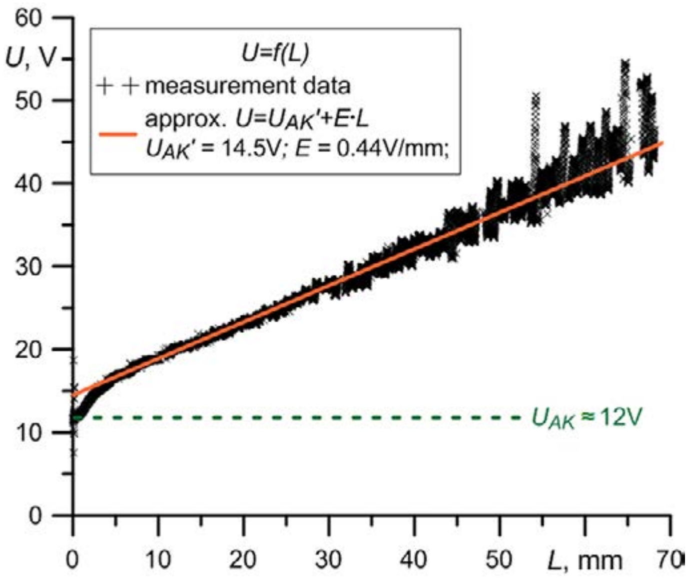

Fig. 5. Gradient characteristic of the electric arc subjected to tests

The method of linear extrapolation was used to determine a near-electrode voltage drop amounting to $U_{AK'}$=14,5 V. Figure 5 reveals that the extrapolation does not include the curved part of the characteristic, resulting from processes taking place in the conical part of the arc column. The more accurate value of the near-electrode voltage drop amounts to U_{AK}≈12 V.

During calculations, the hyperbolic-linear form of arc column static characteristic (2) was assumed. The choice was justified by arc voltage increasing along with increasing excitation current. The results of experimental tests and parameters of model (6) calculated using an integral method [15] are presented in Table 1.

The constant change of the arc length of L leads to changes of column physical parameters and changes of mathematical model parameters. In order to take into consideration changes presented in Table 1, it was necessary to use several approximations. Figure 6 presents related curves of parameters P_M and R_0.

Calculations involved the use of the following approximating functions:

$$P_M=0.0615\cdot L^2+8.828\cdot L-1.987,$$

$$R_0=0{,}04048\cdot L^{0{,}5401}.$$

In the examined case, changes of the arc model time constant can be taken into account in two ways, i.e. by being described as the function of interelectrode distance change or as the function of root-mean-square current changes (Fig. 7).

Table 1. Arc column experimental and computational parameters

	Measurements – rms values			Measurements – arc column average values		Calculations		
L, mm	I, A	U_{arc}, V	U_{col}, V	P_{av}, W	R_{av}, Ω	P_M, W	R_0, Ω	θ, 10^{-5} s
4.9	75.603	19.290	7.523	524.330	0.327	36.149	0.085	4.168
9.9	73.251	24.837	13.083	798.426	0.704	94.339	0.131	4.889
16.9	71.291	29.959	18.105	1089.662	1.073	109.694	0.193	3.905
26.8	67.653	38.091	26.546	1394.142	1.691	202.048	0.260	4.915
36.7	66.713	39.741	28.106	1484.529	1.595	233.734	0.281	6.367
46.1	65.416	42.894	31.259	1608.613	1.716	278.006	0.311	7.369
60.3	63.050	46.635	34.942	1772.879	1.855	305.465	0.369	8.243

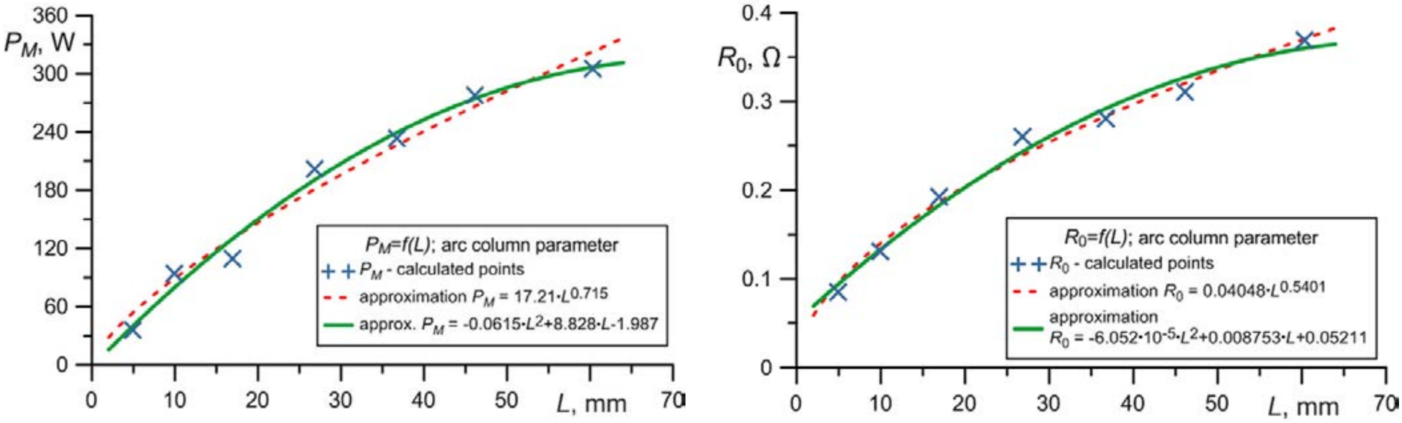

Fig. 6. Approximations of arc column parameters: a) $P_M=f(L)$; b) $R_0=f(L)$

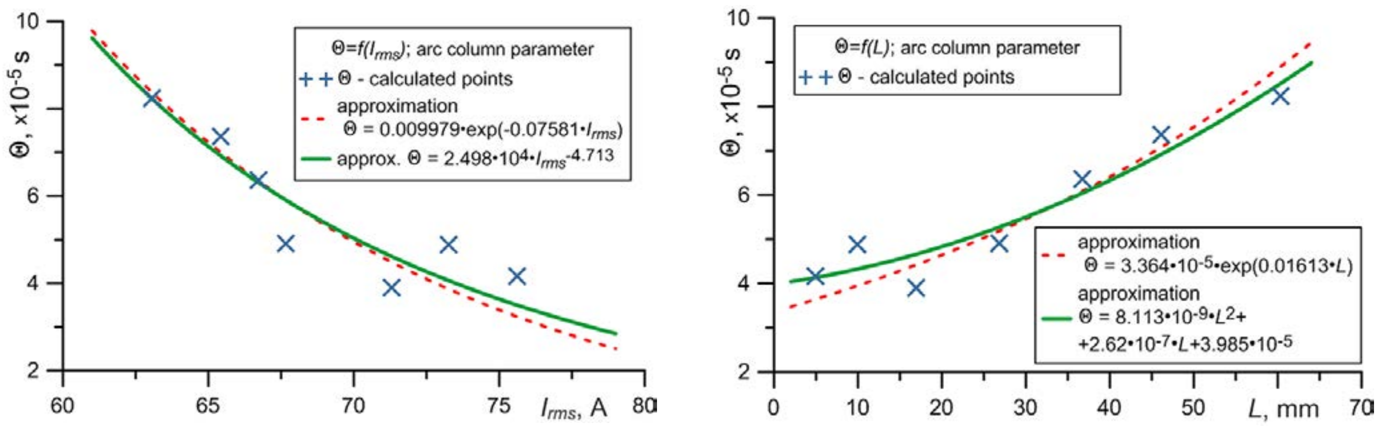

Fig. 7. Approximation of arc model time constant: a) $\theta=f(I_{rms})$; b) $\theta=f(L)$

Both of the proposals presented above are proper. The simulations took into consideration changes of the time constant of model θ from the root-mean-square current through the approximating function of $\theta(I_{rms})=2.498\cdot10^4\cdot I_{rms}^{-4,713}$. The association of the model parameters and taking into consideration the changes of root-means-square current I_{rms} along with the changes of interelectrode gap length enabled the determination of voltage on the arc extended using

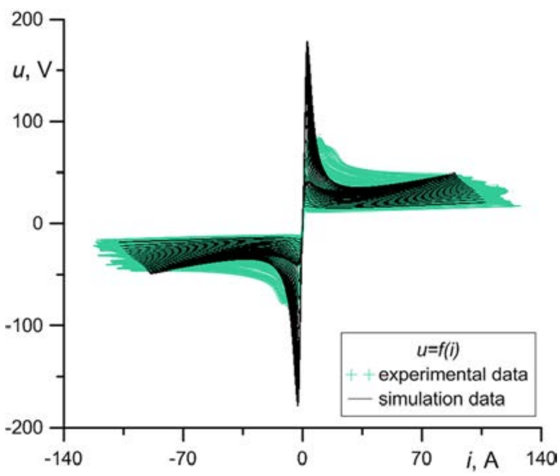

Fig. 8. Dynamic characteristics of experimentally tested arc and simulated arc during tension

expression (7) in the form

The simulation results and measurement points

$$u(i)=U_{AK}\cdot\operatorname{sgn}(i)+\frac{U_{col}(i_\theta)}{i_\theta}i \qquad (10)$$

are presented in the form of a hysteresis loop in Figure 8. Measurement points constitute the background of arc dynamic loops obtained through simulations. In spite of the free burning state, it was possible to obtain significant arc representation compatibility. The accuracy depends not only on the character of discharge, external factors affecting the arc and values of calculated parameters but also on applied approximations taking into consideration changes of column parameters in time.

Conclusions

1. The significant effect of the chemical composition of plasma-forming gases on the non-linearity of arc static characteristics is limited to the temperature range below 10,000 K, corresponding to relatively weak currents of arc discharges.

2. The effect of the chemical composition of electrode materials on the non-linearity of arc dynamic characteristics is primarily concerned with AC low-voltage discharges.

3. The effect of the chemical composition of electrode materials on the parametrisation of arc dynamic characteristics is primarily concerned with low-voltage discharges with moving arc spots.

4. The non-linearity and parametrisation of arc column static characteristics are affected by thermal factors and pressure as well as by mechanical and electromagnetic disturbances triggering changes of arc column length.

5. The adequacy of simulation results concerning processes in a circuit with an arc in relation to measurements performed on a real object depends not only on the assumed voltage-current static characteristics but also on the accurate representation of column parameter changes in time.

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