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Representation of the Effect of Plasma Column Disturbances on the Static and Dynamic Characteristics of Arcs Described by the Pentegov Model.

Part 1. Modelling the Effect of One-Parameter Disturbances on Electric Arc Characteristics

Abstract: The article discusses various types of disturbances affecting an arc column such as changes in length, changes in the intensity of convective heat dissipation using a stream of gas washing around a column, changes in gas atmosphere pressure, and changes in the intensity of laser radiation penetrating the arc. The above named disturbances were included in expressions approximating static current-voltage arc characteristics. The characteristics mentioned above were used to create a number of macromodels based on the Pentegov model of disturbed arcs. The results of processes simulated in an electric circuit were used to demonstrate the usefulness of the models developed and the considerable comfort of such an approach when testing systems with electric arcs.

Keywords: Electric arc characteristics, plasma column disturbances, Pentegov model

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Introduction

Usually applied mathematical models of AC arcs do not allow for the possibility of independent discharge ignition voltage control using various sources of constant or momentary (periodical) ionisation. In addition, the models mentioned above do not allow for controlling the shape and positions of arc characteristics in high-current ranges. However, by changing pressure, the chemical composition of gas, the distance between electrodes etc., it is possible to obtain various shifts or inclinations of function diagrams in relation to the axis of coordinate system *u-i*.

Therefore, as regards low-current arcs, the most important is not ignition voltage but discharge power. In turn, in terms of high-current arcs, the most important is not power but voltage corresponding to maximum current. The problem related to the determination of ignition voltage is solved with various accuracy when representing the entire dynamic loop using a time constant, usually along with the satisfaction of the energy balance condition. To some extent, such an approach may be responsible for modelling ambiguity as various arc static voltage-current characteristics correspond to almost the same dynamic models. Effects of representation inadequacy can be then easily observed as various contents of harmonics of arc voltage waveforms. Arc burning in

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special conditions (e.g. in some gases or during intense gas flows) can be characterised by low damping function values. Narrow dynamic loops of hysteresis corresponding to such arc are not able to ignore positions of points corresponding to various values of discharge re-ignition voltage.

As, depending on technological needs, it is possible to control various fragments of arc dynamic and static characteristics [1-4], in such situations the most comfortable representation of arc properties is offered by the model proposed by Pentegov and developed in conjunction with Sidoryetz [5, 6]. This model requires the setting of the family of static voltage-current characteristics. Similar possibilities are offered by the frequently used Novikov-Shellhase model [7]. However, the latter model only represents arc in an approximate manner as this model does not satisfy the condition of energy balance.

Due to the heterogeneous arc structure, experimentally determined static voltage-current characteristics include areas of near-electrode voltage drops and column voltage drops. Values of electric arc near-electrode voltage drops very poorly depend on current and are usually adopted as constant quantities. In cases of high supply voltage (long arcs), the above named near-electrode voltage drops are negligible because of their relatively low values. However, in short welding arc burning states (low voltage); such drops should be taken into account as the value of arc column voltage U_{col} is comparable with these drops. Because of the fact that, as a rule, external factors do not or very poorly affect values of near-electrode voltage drops, the primary issue is the representation of external disturbances affecting processes taking place in the arc column. Such an approach determines manners in which static characteristics are approximated, without the separate inclusion or with separate inclusion of the sum of near-electrode voltage drops U_{AK}.

Representation of the Effect of Variable External Disturbances on Electric Arc Static Characteristics

Representation of the Effect of Column Length Changes on Electric Arc Static Characteristics

The value of power dissipated by the arc column (primarily radiation) in welding and electrothermal equipment is usually controlled by performing forced arc column length changes. Such changes are also present during the activation and termination of equipment operation, but can also result from various disturbances (magnetic fields, gas flows etc.), in turn resulting in various values of arc ignition voltage.

In low-current ranges, exemplary components of static characteristics can adopt standard forms in relation to peak (I_p, U_p) using an exponential function and be expressed by the following dependence [7]:

$$U(I, L_a) = U_p(L_a) \cdot \left(\frac{I}{I_p}\right) \cdot \exp\left(-\left(\frac{I}{I_p}\right) + 1\right) \quad (1)$$

where ignition voltage is expressed by the following formula

$$U_p(L_a) = E_{p0} \cdot L_a^{k_2} \quad (2)$$

where k_2, E_{p0} – approximation coefficients depending on the temperature and material of electrodes and on the chemical composition of gas ($0 < k_2 < 1$). Within the range of short arc lengths L_a of electrotechnological devices, the above named dependence can be adopted as linear $U_p(L_a)=E_{p0} \times L_a$. Quantity U_p also proportionally depends on the time passing after current reaches the value of zero. Arc length changes (La > 0) also affect voltage in high-current ranges

$$U_0(L_a)=E_0 \times L_a \quad (3).$$

Longer arcs are also characterised by greater resistance

$$R_0(L_a)=r_0 \times L_a \quad (4).$$

The association of these three dependences including the selected function

$$\zeta(I)=1\text{-}\exp|-(I/I_p)^{k2}| \quad (5)$$

leads to the obtainment of modified static voltage-current characteristic

$$U_{stat}(I,L_a)=\left[U_0(L_a)+R_0(L_a)I^{k_0}\right]\cdot\zeta(I)+$$
$$+\left\{U_p(L_a)-\left[U_0(L_a)+R_0(L_a)I^{k_0}\right]\cdot\zeta(I)\right\}\times$$
$$\times\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right)+U_{AK}= \quad (6)$$
$$=U_{col}(I,L_a)+U_{AK}$$

where $k_1 > 1$, $I > 0$ A provides the possibility of adjusting the steepness of characteristic fall in the range of negative differential resistance. If $k_1 = 1$, the formula provided by L. Marciniak [7] is obtained.

Figure 1 presents the family of characteristics $U_{stat}(I, L_a)$ with various arc length values. As can be seen, formula (6) can be used to approximate nearly any shape of extended arc static characteristics within the current range of I_p up to maximum values of supply current.

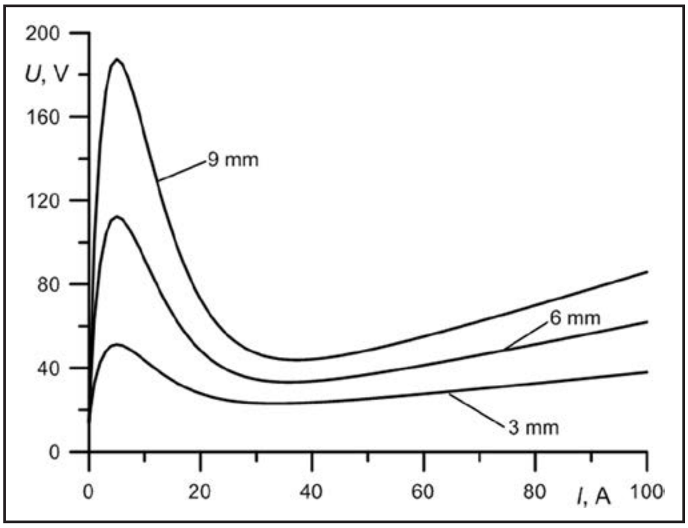

Fig. 1. Family of static voltage-current characteristics of variable length arc (U_{AK}=14 V, k_0=1.2; k_1=1.1; k_2=1.4; k_3=1.4; E_0=0.45 V/mm; r_0=0.03 Ω/mm; Ep_0=8 V/mm; I_p=5 A)

Representation of the Effect of Gas Stream Changes on Electric Arc Static Characteristics in a Stream Plasma Torch

Plasma torches with internal arc usually operate having a fixed arc length as well as changeable mass stream and chemical composition of gas washing around the column. The effect of changes in the stream of gas mass $\dot{m}_{gaz}$ longitudinally or transversely washing around arc on voltage U_{stat} is very similar to the effect of changes in column length L_a (Fig. 1).

Arc re-ignition voltage is the function of many variables $U_p(I, L_a, p, \vartheta, f)$ as it is affected by root-mean-square current I, currentless pause duration, inter-electrode distance L_a, pressure p, absolute temperature ϑ, current frequency f and gas chemical composition. Changes in the value and direction of input gas stream trigger changes in discharge area temperature [8] $\vartheta \propto \dot{m}_{gaz}^{-k_4}$, which due to $U_p \propto \vartheta^{-1}$ leads to the following dependence

$$U_p(\dot{m}_{gaz})=U_{p0}\cdot\left(1+u_{p4}\dot{m}_{gaz}^{k_4}\right) \quad (7)$$

where U_{p0}, u_{p4}, k_4 – constant parameters of approximation ($0 < k_4 < 1$). In turn, changes in ignition current are significantly smaller and reveal the inverse tendency $I_p \propto \vartheta \propto \dot{m}_{gaz}^{-1}$. For this reason, it is possible to roughly adopt the constant value of current I_p.

Similar to the case presented above (1), the exponential component (standardised in relation to peak (I_p, U_p)) of the static characteristic can be expressed by the following dependence:

$$U(I,\dot{m}_{gaz})=U_p(\dot{m}_{gaz})\cdot\left(\frac{I}{I_p}\right)\cdot\exp\left(-\left(\frac{I}{I_p}\right)+1\right) \quad (8)$$

The effect of gas parameters on voltage U_p can be reduced and voltage can be controlled independently using an additional stabiliser of discharge. An increase in the gas stream also corresponds to an increase in plasma torch working voltage manifested by a rise in static characteristics [4]. The application of formula (8) along with the similarity of arc characteristics to those described in formula (3) and (4) lead to the obtainment of new modified parameters:

$$U_0\left(\dot{m}_{gaz}\right)=U_{m0\min}+u_0 m_{gaz} \tag{9}$$

$$R_0\left(\dot{m}_{gaz}\right)=R_{m0\min}+r_0\dot{m}_{gaz} \tag{10}$$

which can be used to create the modified approximation of voltage-current characteristic

$$\begin{aligned} U_{stat}\left(I,\dot{m}_{gaz}\right)&=\left[U_0\left(\dot{m}_{gaz}\right)+R_0\left(\dot{m}_{gaz}\right)I^{k_0}\right]\cdot\zeta(I)+\\ &+\left\{U_p\left(\dot{m}_{gaz}\right)-\left[U_0\left(\dot{m}_{gaz}\right)+R_0\left(\dot{m}_{gaz}\right)I^{k_0}\right]\cdot\zeta(I)\right\}\times\\ &\times\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right)+U_{AK}=\\ &=U_{col}\left(I,\dot{m}_{gaz}\right)+U_{AK} \end{aligned} \tag{11}$$

If $\dot{m}_{gaz}=0$, formula (11) transforms into a formula describing characteristics of free arc having a constant length.

Figure 2 presents the family of characteristics $U_{stat}(I,\dot{m}_{gaz})$ with various gas mass stream values. As can be seen, formula (11) can be used to approximate nearly any shape of arc static characteristics of arc washed around by gas flow within the current range of I_p up to maximum values of supply current.

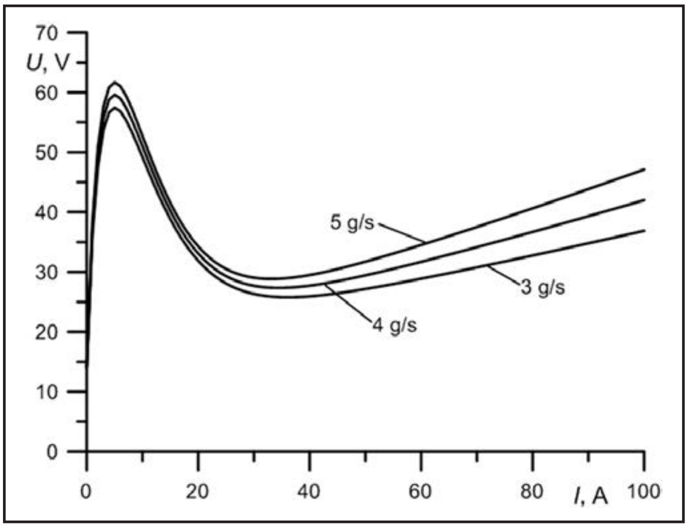

Fig. 2. Family of static voltage-current characteristic of arc in the stream plasma torch (U_{AK}=14 V, k_0=1.2; k_1=1.1; k_3=1.4; k_4=0.8; r_0=0.02 Ω/g; U_{p0}=35 V; u_{p4}=0,1 $(s/g)^{k4}$; u_0=0.01 Vs/g; $U_{0_{min}}$=5 V; $R_{0_{min}}$=0.01 Ω; I_p=5 A)

As regards gas mixtures, it is comfortable to use the parameter of gas volume stream $\dot{v}_{gaz}$ in l/min (generally $\dot{v}_{gaz}=\dot{m}_{gaz}/\rho_{gaz}$). If a mixture consists of two components *a* and *b*, formula (7) can be written in the following form:

$$U_p\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)=U_{p0}\cdot\left[1+u_{p4}\cdot\left(\rho_a v_a+\rho_b v_b\right)^{k_4}\right] \tag{12}$$

where U_{p0}, u_{p4}, k_4 – constant parameters of approximation ($0<k_4<1$). Similar to the above, it is possible to roughly adopt the constant value of current I_p.

The application of formula (8) along with the similarity of physical conditions to those described in formula (9) and (10) lead to the obtainment of new modified parameters:

$$U_0\left(\dot{v}_a+\dot{v}_b\right)=U_{0\min}+u_0\cdot\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right) \tag{13}$$

$$R_0\left(\dot{v}_a+\dot{v}_b\right)=R_{0\min}+r_0\cdot\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right) \tag{14}$$

which can be used to create the modified approximation of the static voltage-current characteristic:

$$\begin{aligned} U\left(I,\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)&=\left[U_0\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)+\right.\\ &\left.+R_0\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)I^{k_0}\right]\cdot\zeta(I)+\\ &+\left\{U_p\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)-\left[U_0\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)+\right.\right.\\ &\left.\left.+R_0\left(\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)I^{k_0}\right]\cdot\zeta(I)\right\}\times\\ &\times\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right)+U_{AK}=\\ &=U_{col}\left(I,\rho_a\dot{v}_a+\rho_b\dot{v}_b\right)+U_{AK} \end{aligned} \tag{15}$$

Representation of the Effect of Changes in Gas Pressure on Static Characteristics of Electric Arcs

The effect of an increase in gas atmosphere pressure is similar to an increase in arc length (Fig. 1). Therefore, within the range of low current, the component of static characteristic can have a standardised form in relation to peak (I_p, U_p) and can be expressed using the following dependence:

$$U(I,p)=U_p(p)\cdot\left(\frac{I}{I_p}\right)\cdot\exp\left(-\left(\frac{I}{I_p}\right)+1\right) \tag{16}$$

whereas changes in ignition voltage can be expressed using the following approximated formula:

$$U_p(p) = U_{p0} + k_p p \quad (17)$$

where U_{p0}, k_p – approximation coefficients dependent on the shape, temperature and material of electrodes as well as on the chemical composition of gas. Changes in gas atmosphere pressure also affect voltage in the high-current range [4, 9]:

$$U_0(p) = U_{p0\min} + u_{p0} \cdot p \quad (18)$$

In a chamber under greater pressure, arc has a smaller diameter and thus higher resistance:

$$R_0(p) = R_{p0\min} + r_{p0} \cdot p \quad (19)$$

The association of these three dependences including function (5) leads to the obtainment of a modified voltage-current characteristic:

$$\begin{aligned} U(I,p) &= \left[U_0(p) + R_0(p) I^{k_0}\right] \cdot \zeta(I) + \\ &+ \left\{U_p(p) - \left[U_0(p) + R_0(p) I^{k_0}\right] \cdot \zeta(I)\right\} \times \\ &\times \left(\frac{I}{I_p}\right)^{1/k_1} \cdot \exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1} + 1\right) + U_{AK} = \\ &= U_{col}(I,p) + U_{AK} \end{aligned} \quad (20)$$

Figure 3 presents the family of characteristics $U_{stat}(I, p)$ with various gas pressure values. As can be seen, formula (20) can be used to approximate nearly any shape of arc static characteristics in gas atmospheres of various pressure within the current range of I_p up to maximum values of supply current.

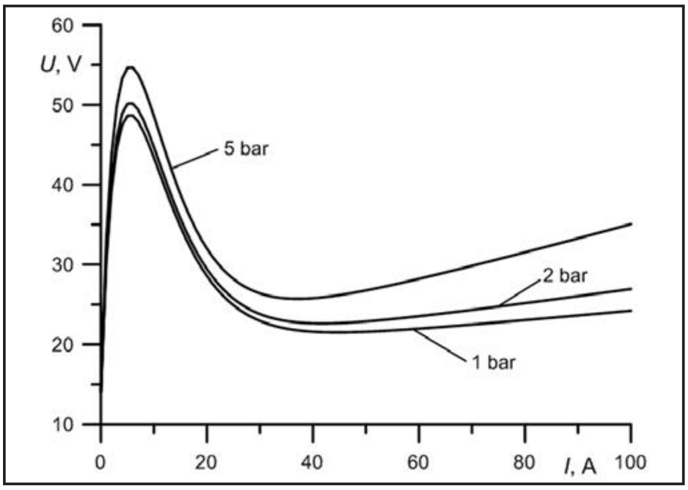

Fig. 3. Families of static voltage-current characteristic of arc in gas under variable pressure (U_{AK}=14 V, k_0=1.2; k_1=1.1; k_p=1.6; r_{p0}=0.01 Ω/bar; U_{p0}=35 V; u_{p0}=0,2 V/bar; U_{0min}=5 V; R_{0min}=0.01 Ω; I_p=5 A)

Representation of the Effect of Changes in Laser Radiation Power on Electric Arc Static Characteristics

Laser radiation can be used in arc welding by means of three methods. In the first method, laser radiation is directed at an element from the side opposite to the torch and electric arc and additionally heats the material. In this way, the direct effect of laser radiation on arc may be inexistent or minimum. However, because of a local increase in the element temperature and intensified material evaporation, an increase in laser radiation power is accompanied by a decrease in arc voltage of 1-2 V within a wide range of current changes [10].

In the second case, laser radiation is directed at a certain angle in relation to the torch and arc. The laser radiation strikes the weld pool facilitating the fixing (stabilisation) of an arc spot on the weld pool surface and supplying additional energy to the metal. In this way, the direct effect of the laser radiation on arc is minimum [11].

In the third case, laser radiation passing through the cavity of the torch cathode penetrates the plasma column and, similar to the second case described above, strikes the weld pool, also stabilising the discharge and supplying energy to metal [12, 13]. An increase in laser radiation power decreases the voltage of low-current arc (by about 1-2 V). In high-current arcs, the effect of laser radiation is minimum. The longer the arc, the greater the effect. Practically, only in the third case, it is possible to speak of the actual effect of laser radiation on arc. Similar effects can be achieved in situations when electric arc is affected by microwave radiation.

Similar to the case presented above, the exemplary component of static characteristic can adopt the standardised form in relation to peak (I_p, U_p) and be expressed by the following dependence:

$$U(I,P_L)=U_{pL}(P_L)\cdot\left(\frac{I}{I_p}\right)\cdot\exp\left(-\left(\frac{I}{I_p}\right)+1\right) \tag{21}$$

where ignition voltage changes are described by the following dependence:

$$U_{pL}(P_L)=U_{P0}-F_{L0}\cdot P_L^{k_2} \tag{22}$$

where k_2, F_{L0} – approximation coefficients; P_L – radiation power. Changes in laser radiation power poorly affect voltage in the high-current range, which can be expressed using the following approximated formula:

$$U_0(P_L)=U_{0\min}-u_{PL}\frac{P_L}{I+I_0}>0 \tag{23}$$

Greater laser radiation power supports the ionisation of plasma and reduces the resistance of the column:

$$R_0(P_L)=R_{0\min}-r_{PL}\frac{P_L}{I+I_0}>0 \tag{24}$$

where low quantity $I_0 > 0$ A. The association of these three dependences including function (5) leads to the obtainment of the following modified static voltage-current characteristic:

$$\begin{aligned}U_{stat}(I,P_L)&=\left[U_0(P_L)+R_0(P_L)I^{k_0}\right]\cdot\zeta(I)+\\&+\left\{U_p(P_L)-\left[U_0(P_L)+R_0(P_L)I^{k_0}\right]\cdot\zeta(I)\right\}\times\\&\times\left(\frac{I}{I_p}\right)^{1/k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{1/k_1}+1\right)+U_{AK}=\\&=U_{col}(I,P_L)+U_{AK}\end{aligned} \tag{25}$$

Figure 4 presents the family of characteristics $U_{stat}(I, P_L)$ with various values of laser power. As can be seen, formula (25) can be used to approximate nearly any shape of arc static characteristics with plasma additionally ionised by laser radiation within the current range of Ip up to maximum values of supply current.

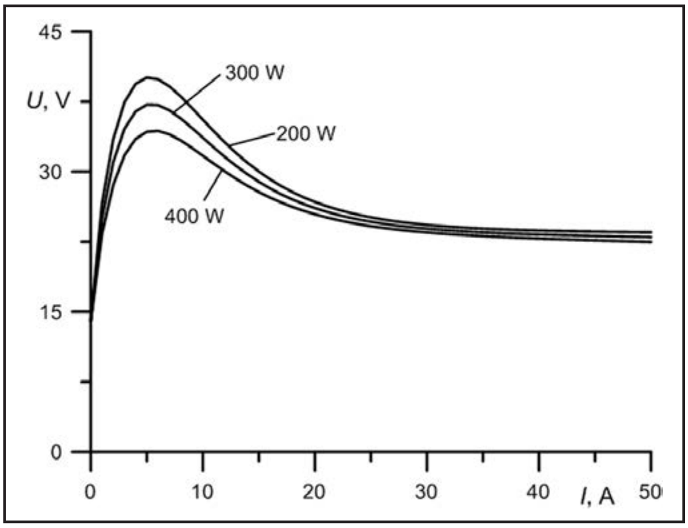

Fig. 4. Families of static voltage-current characteristics of arc with plasma additionally ionised by laser radiation of controller power (U_{AK}=14 V, k_0=1.2; k_1=1.1; I_0=1 A; r_{PL}=0.0001A^{-1}; U_{P0}=35 V; u_{PL}=0.2; U_{0min}=10 V; R_{0min}=0.01 Ω; I_p = 4 A)

Representation of External Effects on Electric Arc using the Pentegov Model

In actual inert arc, a step change of current $i(t)$ causes a voltage impulse and quasi-exponential change in arc resistance. Actual arc is replaced by hypothetical arc. The primary input assumption of the Pentegov model is the equation of energy balance [5, 6]. At the same time, in contrast to actual arc, this model is electrically inertialess. For this reason, the conductance of arc column is determined using not only actual current i, but also virtual current $i_\theta(t)$, delayed in relation to actual current, changing after step current disturbance i with time constant θ. This is a certain representation of arc thermal state. All isoenergetic states are characterised by one variable, i.e. arc state current $i_\theta(t)$.

The relationship between the square of state current i_θ and the square of arc actual current i is described by the following first-order differential linear equation [5]:

$$\theta\frac{di_\theta^2}{dt}+i_\theta^2=i^2 \tag{26}$$

The Pentegov model represents a non-linear circuit two-terminal network, which is balanced in terms of energy, first-order thermally inert, linear, stationary and electrically inertialess.

The advantage of this model is the possibility of using nearly any approximation of a static voltage-current characteristic and at the same time constant damping function value (the so-called time-constant). Simple linear models of electric arc (Mayr, Cassie, Zarudi) can be treated as specific cases of the more general Pentegov model. In the above named models, approximations of static voltage-current characteristic are usually performed using power functions [6]. However, these approximations are not sufficiently versatile to enable allowing for differences in various plasma torch designs and differences in external effects on arcs. However, approximations utilising standardised parameter-dependent functions make it possible to allow for the differences mentioned above.

State current can be used for determining arc model dynamic characteristics depending on external parameters (Table 1). In the table presented below, plasma enthalpy depends on various factors:

$$Q(T, p_{ar}) = \begin{cases} h(T_a)\cdot L_a(p_{ar})\pi r_c^2(i,T_a)\cdot \rho(T_a), & \text{if } L_a = p_{ar} \\ h(T_a, p_{ar})\cdot L_a \pi r_c^2(i,T_a,p_{ar})\cdot \rho(T_a,p_{ar}), & \text{if } L_a = const \end{cases} \quad (27)$$

where T_a - average plasma temperature, h – plasma specific enthalpy, ρ - gas density.

Using the definitions formulated by Pentegov [5], it is possible to calculate root-mean-square voltage and root-mean-square current as well as average values of arc power and resistance (Table 2). In spectral methods, the values mentioned above are used for determining parameters of mathematical models on the basis of experimental test results related to arc.

In the conductance form, the Pentegov model can be expressed using the following formula [6]:

$$\frac{2\theta}{1 - g\dfrac{dU_{col}}{di_\theta}}\frac{1}{g}\frac{dg}{dt} + 1 = \frac{i^2}{g^2 U_{col}^2} \quad (28)$$

The specific cases of this model are obtained by adopting an appropriate form of the function approximating the static characteristic. For instance, in the case of a power function with exponent μ, it is possible to obtain the following

Table 1. Definitions of physical quantities of the column of actual arc characterised by variable parameters and of the Pentegov model representing the properties of the arc

Physical quantity	**Actual arc**	**Pentegov model**
Electric current	$i\,(t, p_{ar})$	$i\,(t, p_{ar})$; $i_\theta\,(t, p_{ar}) \geq 0$
Dynamic voltage $u_{col}(t) = u(t) - U_{AK}\cdot \mathrm{sgn}(i) =$	$u_{col}(i, p_{ar})$	$\dfrac{U_{col}(i_\theta, p_{ar})}{i_\theta} i$
Supplied electric energy, $P_{el}(t)=$	$P_{el}(i, p_{ar}) = u_{col}(i, p_{ar})\times i$	$\dfrac{U_{col}(i_\theta, p_{ar})}{i_\theta} i^2$
Dissipated electric energy, $P_{dys}(t) = P_{el}(t) - \dfrac{dQ(t)}{dt} =$	$P_{dys}(i, p_{ar}) = = P_{el}(i, p_{ar}) - \dfrac{dQ(T_a, p_{ar})}{dT_a}\dfrac{dT_a}{dt}$	$U_{col}(i_\theta, p_{ar})\cdot i_\theta$
Conductance $g(t)$	$g(T_a, p_{ar}) = \dfrac{i}{u_{col}(i, p_{ar})}$	$\dfrac{i_\theta}{U_{col}(i_\theta, p_{ar})}$
Arc plasma enthalpy, $Q(t)$	$Q(T_a, p_{ar})$	$2\theta\int_0^{i_\theta} U_{col}(i_\theta, p_{ar})di_\theta$

static characteristic:

$$g\frac{dU_{col}}{di_\theta}=\frac{i_\theta}{U_{col}}\frac{dU_{col}}{di_\theta}=\mu=const. \quad (29)$$

Physical characteristics of gases depend on temperature and pressure, whereas geometrical dimensions of the column clearly depend on current. The damping function related to transient processes in electric arcs also depends on the factors mentioned above [14]. This case can be expressed using the following approximate dependence:

Table 2. Definitions of measurement quantities of the actual arc column and of the Pentegov model

Physical quantity	Actual arc	Pentegov Model
Square of root-mean-square current, $I^2_{rms}=$	$\frac{1}{T}\int_0^T i^2dt$	$\frac{1}{T}\int_0^T i_\theta^2dt$
Square of root-mean-square voltage, $U^2_{rms}=$	$\frac{1}{T}\int_0^T u^2_{col}(t,p_{ar})dt$	$\frac{1}{T}\int_0^T U^2_{col}(i_\theta,p_{ar})dt$
Average momentary power, P_{col}	$\frac{1}{T}\int_0^T u_{col}(t,p_{ar})idt$	$\frac{1}{T}\int_0^T U_{col}(i_\theta,p_{ar})\cdot i_\theta dt$
Average resistance, R_{col}	$\frac{1}{T}\int_0^T \frac{u_{col}(t,p_{ar})}{i}dt$	$\frac{1}{T}\int_0^T \frac{U_{col}(i_\theta,p_{ar})}{i_\theta}dt$

$$\theta(T_a(I),I)\propto\frac{\rho(T_a(I))\cdot c_p(T_a(I))}{\lambda(T_a(I))}r_c^2(I,T_a(I))\propto\frac{1}{T_a}\propto\frac{1}{I} \quad (30)$$

where non-linear functions: ρ – gas density ($\rho\propto 1/T_a$); c_p – specific heat of gas under pressure p (inert gases $c_p\approx$const, polyatomic gases $c_p\propto T_a$); λ – gas heat flow coefficient (non-monotonically variable $\lambda\propto T_a$); r_c – arc column radius ($r_c\propto T_a$); I – DC.

Expression (26) can be expressed in a simplified manner as non-linear

$$\theta(i_\theta)\frac{di_\theta^2}{dt}+i_\theta^2=i^2 \quad (31)$$

Theoretically, arc temperature affects this current as root-mean-square values of momentary quantities $i(t)$, $i_\theta(t)$ correspond to condition $I_{rms}=(i_\theta)_{rms}$ (Tab. 2). On this basis, $T_a(I_{rms})=T_a(i_\theta)$ is obtained. Equation (31) can be developed in relation to conductance g of the arc column, receiving

$$\frac{2\theta(i_\theta(g))}{1-\frac{g}{G_d(g)}}\frac{1}{g}\frac{dg}{dt}+1=\frac{i^2}{g^2U^2_{col}(g)} \quad (32)$$

where quantity

$$\theta(g)=\frac{2\theta(i_\theta(g))}{1-\frac{g}{G_d(g)}}=\frac{2\theta(i_\theta(g))}{U_{col}\frac{dg}{dt}} \quad (33)$$

constitutes a damping function. Dynamic conductance Gd(g) can be determined using the following formula:

$$G_d(g)=\frac{di_\theta}{dU_{col}(i_\theta)} \quad (34)$$

By introducing various additional simplifications to equation (32), it is possible to obtain its other known models being its specific cases [5]. In this manner, among other things, it is possible to obtain the Belkin model, in which the static characteristic of the arc column $U_{col}(I)$ adopts the form of power function $U_{col}/U_0=(i_\theta/I_0)^\mu$, where μ – power coefficient of approximation. Publication [15] describes a methodology used when experimentally determining parameters of this model.

The function related to the damping of waveforms of processes in low-current arcs is affected by many factors including pressure, gas mass stream, laser or microwave radiation power etc. In such a situation, the equation of state current adopts the following form:

$$\theta(i_\theta,p_{ar})\frac{di_\theta^2}{dt}+i_\theta^2=i^2 \quad (35)$$

However, in most cases of high-current arcs of electrotechnological devices, the effect of external factors on the damping function is not significant and neither are ranges of changes of these factors. For this reason, the constant value of damping function in the form of a time constant is usually adopted.

Simulations of Processes in Circuits with Arcs of Disturbed Static Characteristic

In order to test the effectiveness of the proposed methods dedicated to approximations of static characteristics, it was necessary to perform simulations of processes in a simple electric circuit with arc described by the Pentegov model. The excitation used in the simulations was performed by a source of sinusoidal current having amplitude I_m and frequency f = 50 Hz. Equations of static characteristics utilised the substitution $I=|i|$ obtaining $u=U_{stat}(|i|)\times\mathrm{sgn}(i)$. The sum of near-electrode voltage drops amounted to U_{AK}=14 V.

Figure 5 presents dynamic voltage-current characteristics of extended arc. As can be seen, the use of the static characteristic described by formula (6) enables the relatively easy representation of arc length disturbances in the created Pentegov model.

Figure 6 presents characteristics of arc in a plasma torch with an increasing gas flow. As can be seen, the use of the static characteristic described by formula (11) enables the relatively easy representation of disturbances of convective arc heat exchange in the created Pentegov model.

Figure 7 presents similar characteristics of arc burning in a chamber with compressed gas. As can be seen, the use of the static characteristic described by formula (20) enables the relatively easy representation of disturbances of gas atmosphere pressure in the chamber with arc in the created Pentegov model.

Figure 8 presents dynamic voltage-current characteristics of arc burning in the plasma torch channel with additional ionisation induced by laser radiation. As can be seen, the use of the static characteristic described by

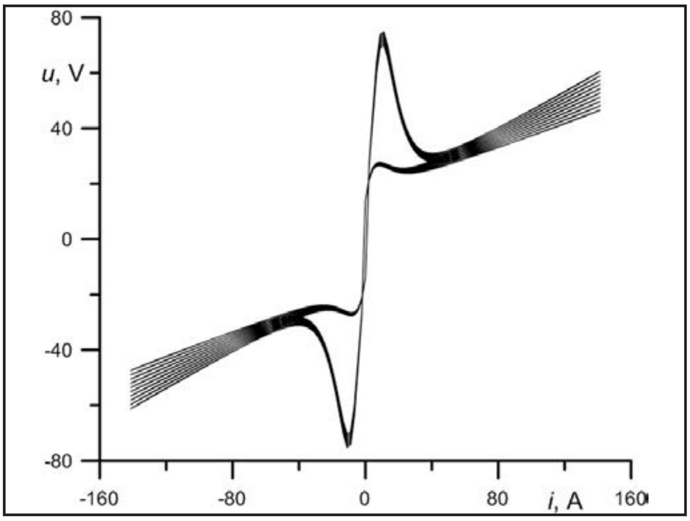

Fig. 5. Dynamic voltage-current characteristics of extended arc (θ=1,5·10^{-4} s, I_{rms}= 100 A; the remaining parameters in accordance with Fig. 1)

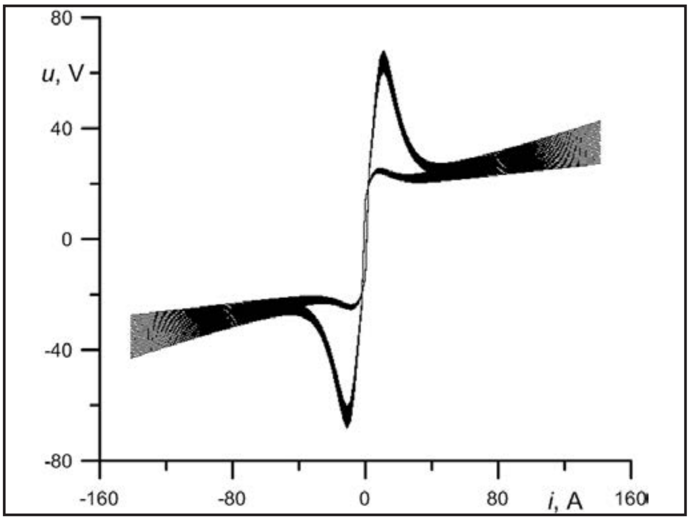

Fig. 6. Dynamic voltage-current characteristics of arc in the plasma torch with increasing intensity of convective heat dissipation (θ=1,5·10^{-4} s, I_{rms}= 100 A; the remaining parameters in accordance with Fig. 2)

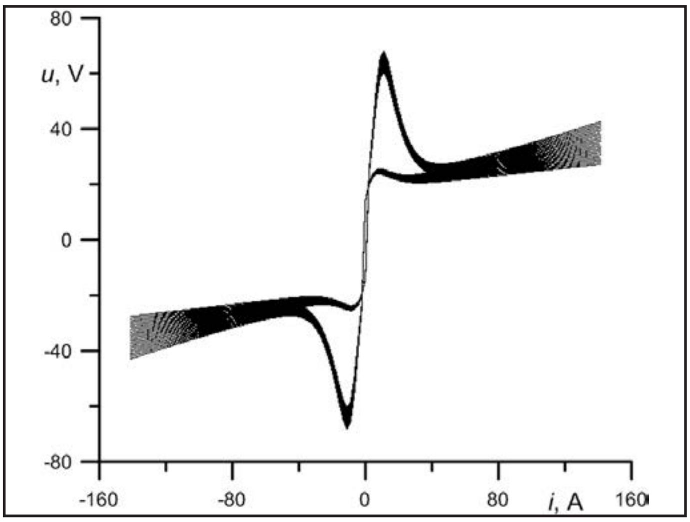

Fig. 7. Dynamic voltage-current characteristics of arc in in the chamber with the increasing pressure of gas atmosphere (θ=1,5·10^{-4} s, I_{rms}= 100 A; the remaining parameters in accordance with Fig. 3)

formula (25), enables the relatively easy representation of plasma ionisation disturbances in the created Pentegov model.

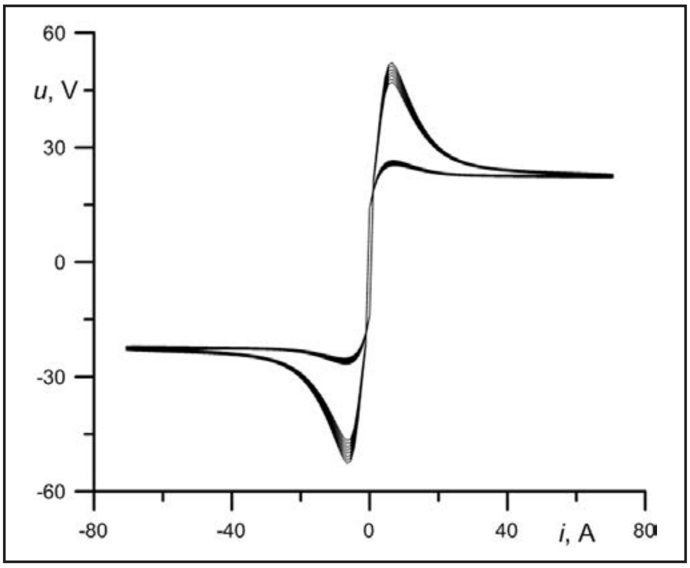

Fig. 8. Dynamic voltage-current characteristics of arc in the plasma torch with increasing intensity of laser radiation (θ=1,5·10^{-4} s, I_{rms}= 100 A; the remaining parameters in accordance with Fig. 4)

Slightly greater accuracy of the representation of dynamic characteristics can be obtained using the generalised Pentegov model (32). Such an approach does not increase the difficulty of simulations, yet may significantly complicate the determination of model parameters on the basis of data obtained in experimental tests.

Conclusions:

1. The approximations of the family of electric arc static current-voltage characteristics using the sum of standardised simple analytical functions make it possible to easily take into consideration the effect of various external factors on plasma columns.

2. The use of the proposed approximations of the family of electric arc static voltage-current characteristics when simulating processes in electric circuits with the Pentegov mathematical model demonstrates high efficiency in representing the effect of various external factors on plasma columns.

3. The Pentegov mathematical model and the set of functions approximating families of static voltage-current characteristics enable the representation of processes in disturbed arcs of many electrotechnological devices.

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