

Antoni Sawicki, Maciej Haltof

Representation of the Effect of Plasma Column Disturbances on the Static and Dynamic Characteristics of Arcs Described by the Modified Pentegov Model.

Part 2. Modelling the Effect of Rapid Disturbances on Electric Arc Characteristics

Abstract: The article presents universal functions approximating quasi-static characteristics of electric arc with undefined and defined ignition voltage. The above named approximations take into consideration the effect of disturbances on an arc column. The approximations were used to create a modified mathematical Pentegov model of arc characterised by high and low rate of disturbance changes. The simulation of processes in a circuit with electric arc involved the use of the modified Pentegov model with one and two-parameter rapid disturbances. The authors demonstrated the efficiency of the developed modifications of the Pentegov model.

Keywords: electric arc, quasi-static characteristic, modified Pentegov model, arc disturbances

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Introduction

Part 1 of the article [1] discusses the issues related to the representation of relatively slow changes in disturbances of column parameters in the modified mathematical Pentegov model of electric arc [2] using the generalised approximation of a static voltage-current characteristic [3-5]. However, sometimes the operation of electrotechnological devices is characterised by rapid changes in plasma column disturbances caused by movements of electrodes, effect of variable magnetic fields, gas pressure pulsation etc. In order to take these disturbances into consideration, it is necessary to use simplified forms of functions approximating static

voltage-current characteristics, enabling the analytical expression of integrals making up the expression of plasma enthalpy in the Pentegov model (developed with Sidoretz) of electric arc. As a result, it is possible to significantly facilitate the modelling of systems and the simulation of processes in circuits with electric arc.

The original Pentegov model [6], using the equation of power balance, involves a constant damping function, i.e. so-called time-constant. However, the changeability of operating conditions (also including arc burning) of many electrotechnological devices necessitates the variation of initially constant parameters of arc mathematical models. These modifications

dr hab inż. Antoni Sawicki (PhD (DSc) hab. Eng.), Professor at Częstochowa University of Technology;

dr inż. Maciej Haltof (PhD (DSc) Eng.) – Częstochowa University of Technology, Faculty of Electric Engineering

result in the parametrisation and delinearisation of differential equations [7]. However, such changes could lead to the infringement of preliminary simplifying assumptions, adopted when creating models. In cases of the Mayr and Cassie models, the Schwarz-Avdonin modifications lead to, among other things, non-linear damping functions, depending on arc column conductance [8]. A similar effect of the modification of the Pentegov model was described in publications [9]. The effect of changes in column length and in the rate of gas washing around arc in the plasma torch constrictor on its dynamic characteristics is discussed in publications [2, 10]. To this end, it was necessary to use data approximations obtained in experimental tests. This publication presents simplified modifications of the mathematical model of arc making it possible to take into consideration nearly any plasma column disturbances. Depending on the time of their occurrence, they could affect the value of re-ignition voltage, power dissipated in the high-current range etc.

Approximations of Static Voltage-Current Characteristics of Undisturbed Arc

These deliberations involve two forms of functions approximating static characteristics. The first case involves the use of the relatively simple and frequently used form of the approximating function [11] having the undefined value of arc ignition voltage

$$U_{stat}(I) = U_{AK} + U_C + \frac{P_M}{I^m} + R_a I^k = U_{AK} + U_{col}(I) \quad (1)$$

where U_{AK} – sum of near-electrode voltage drops; U_{col} – arc column voltage; U_C – voltage constant component; P_M – dissipated power, predominantly in the range of low-current I , R_a – arc resistance component, most strongly affecting arc voltage and arc power in the high-current range; m, k – approximation parameters.

The second case involved determining the value of ignition voltage, which is important in disturbances or special impulse excitations. The relatively general form of the approximating function can be described by the following formula:

$$U_{stat}(I) = U_{AK} + [U_0 + R_0 I] \cdot \zeta(I) + U_p \cdot \left(\frac{I}{I_p}\right) \cdot \exp\left[-\left(\frac{I}{I_p}\right) + 1\right] \cdot [1 - \zeta(I)] = U_{AK} + U_{col}(I) \quad (2)$$

where U_p – discharge re-ignition voltage; I_p – current corresponding to ignition voltage. Points of characteristic close to data having coordinates (I_p, U_p) correspond to discharge ignition voltage. Tapering function ζ depends on the required accuracy of approximation [12] and can be adopted in the following form

$$\zeta(I) = 1 - \exp\left[-\left(I / I_p\right)\right] \quad (3)$$

Function (2), in comparison with the function used in publications [1, 4, 5], poses difficulties in terms of the accurate representation of the predefined ignition point (I_p, U_p) .

Approximations of Quasi-Static Voltage-Current Characteristics of Disturbed Arc

For the purpose of simplification, it was assumed that variable p_a represented the vector of parameters related to parameters in various physical natures (length, gas mass stream, pressure etc.). In addition, it was assumed that the effects of disturbance on the quasi-static characteristic did not depend on current. Then, formula (1), having a quasi-static characteristic, depending on disturbance parameters, can be expressed in the following form:

$$U_{stat}(I, p_a) = U_{AK} + U_C(p_a) + \frac{P_M(p_a)}{I^m} + R_a(p_a) I^k = U_{AK} + U_{col}(I, p_a) \quad (4)$$

As regards characteristic (2), taking into consideration disturbance parameters leads to the following expression

$$\begin{aligned}
 U_{stat}(I, p_a) &= U_{AK} + [U_0(p_a) + R_0(p_a)I] \cdot \zeta(I) + \\
 &+ U_p(p_a) \cdot \left(\frac{I}{I_p}\right) \cdot \exp\left(-\left(\frac{I}{I_p}\right) + 1\right) \cdot [1 - \zeta(I)] = \quad (5) \\
 &= U_{AK} + U_{col}(I, p_a)
 \end{aligned}$$

If the values of disturbances are constant ($p_{ai}(t) = \text{const.}$), quasi-static voltage-current characteristics of arc become appropriately transformed static characteristics.

Representation of Rapid Multi-Parameter Disturbances of Arc Column in the Modified Pentegov Model

Entering parameter changes only into a static characteristic ignores the effect of disturbances on plasma enthalpy. However, such disturbances could lead to changes in geometrical dimension of a column, inflow or outflow of additional energy etc. Publication [10] presents special cases of the effect of length disturbances or disturbances of a washing gas stream on the equation of arc state resulting from energy balance. An appropriate system of equations can be written in the following form:

$$\frac{dQ}{dt} = P_{el} - P_{dys} = \frac{U_{col}(i_\theta, p_a)}{i_\theta} i_\theta^2 - U_{col}(i_\theta, p_a) i_\theta \quad (6)$$

$$Q = 2\theta \int_0^{i_\theta} U_{col}(i_\theta, p_a) di_\theta \quad (7)$$

where Q – plasma enthalpy; P_{el} – electric power supplied to an arc column; P_{dys} – thermal power dissipated from a column; i_θ – virtual current representing current I present in static characteristics (1)-(5) [12]. Changes in parameters p_a affect the value of column voltage. This results in the simultaneous changes in the value of supplied power, dissipated power and plasma enthalpy. The derivative of column enthalpy has the following form:

$$\begin{aligned}
 \frac{dQ}{dt} &= \frac{\partial Q}{\partial i_\theta} \frac{di_\theta}{dt} + \sum_{i=1}^n \frac{\partial Q}{\partial p_{ai}} \frac{dp_{ai}}{dt} = \quad (8) \\
 &= 2\theta U_{col}(i_\theta, p_a) \frac{di_\theta}{dt} + 2\theta \sum_{i=1}^n \left(\int_0^{i_\theta} \frac{\partial}{\partial p_{ai}} U_{col}(i_\theta, p_a) di_\theta \right) \frac{dp_{ai}}{dt}
 \end{aligned}$$

where n – dimension of the vector of parameters. If it is assumed that $U_{col}(i_\theta, p_a)$ is a constant function having constant derivatives in relation to parameters p_{ai} , after substituting (8) to (6) the dependence obtained has the following form:

$$\begin{aligned}
 2\theta i_\theta \frac{di_\theta}{dt} + \frac{2\theta i_\theta}{U_{col}(i_\theta, p_a)} \sum_{i=1}^n \int_0^{i_\theta} \left(\frac{\partial}{\partial p_{ai}} U_{col}(i_\theta, p_a) di_\theta \right) \frac{dp_{ai}}{dt} = \\
 = i_\theta^2 - i_\theta^2 \quad (9)
 \end{aligned}$$

or another form

$$\begin{aligned}
 \theta \frac{di_\theta^2}{dt} + \frac{2\theta i_\theta}{U_{col}(i_\theta, p_a)} \sum_{i=1}^n \int_0^{i_\theta} \left(\frac{\partial}{\partial p_{ai}} U_{col}(i_\theta, p_a) di_\theta \right) \frac{dp_{ai}}{dt} = \\
 = i_\theta^2 - i_\theta^2 \quad (10)
 \end{aligned}$$

If the rate of changes in parameters is relatively low ($|dp_{ai}/dt| \ll |di_\theta/dt|$), the equations of state (10) can be expressed in a significantly simpler form, i.e.

$$\theta \frac{di_\theta^2}{dt} = i_\theta^2 - i_\theta^2 \quad (11)$$

In the modified Pentegov model (6), arc column voltage is expressed by the following formula

$$u_{col}(i, p_a) = \frac{U_{col}(i_\theta, p_a)}{i_\theta} i \quad (12)$$

which should be considered in simulation calculations where $i_\theta > 0$ A.

Representation of Rapid Multi-Parameter Disturbances of Arc Column in the Pentegov Model with the Pre-Defined Quasi-Static Characteristic

This study involved such a selection of functions approximating quasi-static characteristics of arc, so that they could be characterised by the significant generalisation of experimental data and, at the same time, enable relatively easy analytical calculations of integral in expression (9).

The derivative of voltage (4) in relation to a selected parameter can be expressed in the following form:

$$\begin{aligned} \frac{\partial}{\partial p_{ai}} U_{col}(i_{\theta}, p_a) &= \quad (13) \\ &= \frac{\partial}{\partial p_{ai}} U_C(p_a) + \frac{1}{i_{\theta}^m} \frac{\partial}{\partial p_{ai}} P_M(p_a) + i_{\theta}^k \frac{\partial}{\partial p_{ai}} R_a(p_a) \end{aligned}$$

The integral determined analytically from (13), indispensable in expression (9), has the following form:

$$\begin{aligned} \int_0^{i_{\theta}} \frac{\partial}{\partial p_{ai}} U_{col}(i_{\theta}, p_a) di_{\theta} &= \quad (14) \\ &= \frac{d}{dp_{ai}} U_C(p_a) \cdot i_{\theta} + \frac{d}{dp_{ai}} P_M(p_a) \cdot \frac{i_{\theta}^{1-m}}{1-m} + \frac{d}{dp_{ai}} R_a(p_a) \cdot \frac{i_{\theta}^{k+1}}{k+1} \end{aligned}$$

Likewise, the derivative of voltage (5) in relation to a select parameter can be expressed in the following form:

$$\begin{aligned} \frac{\partial}{\partial p_{ai}} U_{col}(i_{\theta}, p_a) &= \left[\frac{\partial}{\partial p_{ai}} U_0(p_a) + \frac{\partial}{\partial p_a} R_0(p_a) i_{\theta} \right] \cdot \zeta(i_{\theta}) + \quad (15) \\ &+ \frac{\partial}{\partial p_{ai}} U_p(p_a) \cdot \left(\frac{i_{\theta}}{I_p} \right) \cdot \exp\left(- \left(\frac{i_{\theta}}{I_p} \right) + 1 \right) \cdot [1 - \zeta(i_{\theta})] \end{aligned}$$

The integral determined analytically from (15), in expression (9) has the following form:

$$\begin{aligned} \int_0^{i_{\theta}} \frac{\partial}{\partial p_{ai}} U_{col}(i_{\theta}, p_a) di_{\theta} &= \quad (16) \\ &= \frac{d}{dp_{ai}} U_0(p_a) \cdot \left\{ i_{\theta} + I_p \left[\exp\left(- \frac{i_{\theta}}{I_p} \right) - 1 \right] \right\} + \\ &+ \frac{d}{dp_{ai}} R_0(p_a) \cdot \left[\frac{1}{2} i_{\theta}^2 + I_p (i_{\theta} + I_p) \exp\left(- \frac{i_{\theta}}{I_p} \right) - I_p^2 \right] + \\ &+ \frac{1}{4} \frac{d}{dp_{ai}} U_p(p_a) \cdot \left[- (2i_{\theta} + I_p) \exp\left(1 - 2 \frac{i_{\theta}}{I_p} \right) + e I_p \right] \end{aligned}$$

where e – Neper number.

Simulations of Transient Processes in a Circuit with the Disturbed Model of Electric Arc

The circuit with electric arc was powered by the source of sinusoidal current having an amplitude $I_m = 100\sqrt{2}$ A and frequency of 50 Hz. The disturbance of column length was represented by $p_a = p_{av}$, changing in a quasi-step manner in accordance with dependence

$p_a = l_a = l_{a1} + l_{a2} [1 - \exp(-(t - t_1)/T_I)] H(t - t_1)$, where H – Heaviside step function. In turn, changes in the mass stream of gas washing around arc are linear in nature $p_{a2} = \dot{m} = \dot{m}_1 + \mu_2 t$. The simplification of the tests required the adoption of constant parameters of disturbances: $l_{a1} = 2$ mm; $l_{a2} = 4$ mm; $T_I = 0.04$ s; $t_1 = 0.025$ s; $\dot{m}_1 = 0$ g/s; $\mu_2 = 100$ g. The range of simulated time was 0-0.5 s.

The simulation tests related to the circuit with disturbed electric arc involved four cases. The first case was connected with only one disturbing factor, leading to changes in the coefficients of function (4). The individual components of the function depend on the type of disturbance and, in some conditions, can be approximated using the following linear formulas:

$$U_C(p_a) = U_{C0} + U_{C1} p_a \quad (17)$$

$$P_M(p_a) = P_{M0} + P_{M1} p_a \quad (18)$$

$$R_a(p_a) = R_{a0} + R_{a1} p_a \quad (19)$$

The derivatives of these functions in relation to parameters are constant quantities:

$$\frac{d}{dp_a} U_C(p_a) = U_{C1} \quad (20)$$

$$\frac{d}{dp_a} P_M(p_a) = P_{M1} \quad (21)$$

$$\frac{d}{dp_a} R_a(p_a) = R_{a1} \quad (22)$$

In the simplest case, the entire form of the differential equation of state (9) is the following:

$$\theta \frac{di_{\theta}^2}{dt} + 2\theta i_{\theta} \times \quad (23)$$

$$\times \frac{U_{C1}i_{\theta} + P_{M1} \cdot \frac{i_{\theta}^{1-m}}{1-m} + R_{a1} \cdot \frac{i_{\theta}^{k+1}}{k+1}}{U_{C0} + U_{C1}p_a + \frac{P_{M0} + P_{M1}p_a}{i_{\theta}^m} + (R_{a0} + R_{a1}p_a)i_{\theta}^k} \times$$

$$\times \frac{dp_a}{dt} = i^2 - i_{\theta}^2$$

where $m < 1$. Figure 1 presents the results of the simulations of processes in the circuit with arc and with one parameter, i.e. column length, disturbed in a quasi-step manner. To this end it was necessary to build the macromodel of arc using equations (4), (9), (12)-(14) and (17)-(23). In this case it was assumed that parameter changes concerned only the length of arc. Then, the value of voltage depends on the current of electric field 1-20 V/mm affected by the type of gas, concentration of metal vapours, temperature etc.

The second case was connected with two disturbing factors, leading to changes in the coefficients of function (4). In certain conditions they can be approximated using simple polynomials (bass 1, $p_{a1}, p_{a2}, p_{a1}p_{a2}$)

$$U_C(p_{a1}, p_{a2}) = U_{C0} + U_{C1}p_{a1} + U_{C2}p_{a2} + U_{C12}p_{a1}p_{a2} \quad (24)$$

$$P_M(p_{a1}, p_{a2}) = P_{M0} + P_{M1}p_{a1} + P_{M2}p_{a2} + P_{M12}p_{a1}p_{a2} \quad (25)$$

$$R_a(p_{a1}, p_{a2}) = R_{a0} + R_{a1}p_{a1} + R_{a2}p_{a2} + R_{a12}p_{a1}p_{a2} \quad (26)$$

The derivatives of these functions in relation to parameters are the following:

$$\frac{\partial}{\partial p_{a1}} U_C(p_{a1}, p_{a2}) = U_{C1} + U_{C12}p_{a2} \quad (27)$$

$$\frac{\partial}{\partial p_{a2}} U_C(p_{a1}, p_{a2}) = U_{C2} + U_{C12}p_{a1} \quad (28)$$

$$\frac{\partial}{\partial p_{a1}} P_M(p_{a1}, p_{a2}) = P_{M1} + P_{M12}p_{a2} \quad (29)$$

$$\frac{\partial}{\partial p_{a2}} P_M(p_{a1}, p_{a2}) = P_{M2} + P_{M12}p_{a1} \quad (30)$$

$$\frac{\partial}{\partial p_{a1}} R_a(p_{a1}, p_{a2}) = R_{a1} + R_{a12}p_{a2} \quad (31)$$

$$\frac{\partial}{\partial p_{a2}} R_a(p_{a1}, p_{a2}) = R_{a2} + R_{a12}p_{a1} \quad (32)$$

Figure 2 presents the results of the simulations of processes in the circuit with arc and

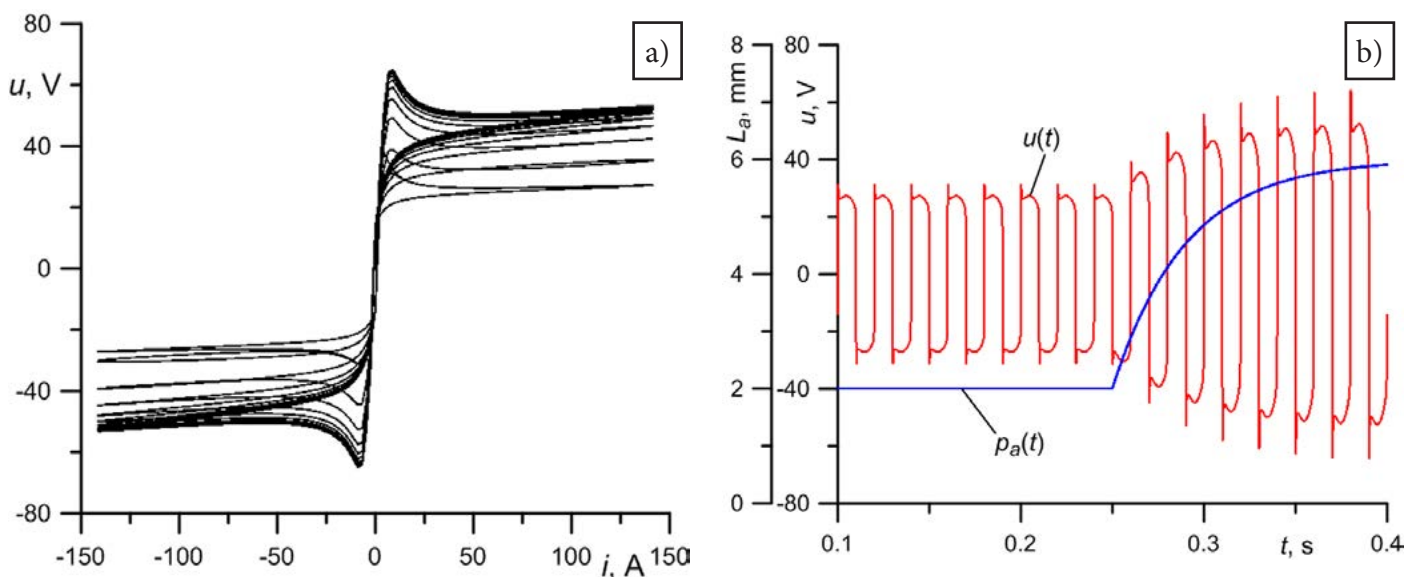


Fig. 1. Characteristics and transient processes in the circuit with arc described by the modified Pentegov model with quasi-static characteristic (4), with one parameter disturbed: a) dynamic voltage-current characteristic; b) waveforms of changes in disturbances and waveforms of arc voltage ($U_{AK} = 14$ V; $\theta = 1 \cdot 10^{-4}$ s; $U_{C0} = 0$ V; $U_{C1} = 5$ V/mm; $P_{M0} = 0$ W; $P_{M1} = 5$ W/mm; $R_{a0} = 0$ Ω ; $R_{a1} = 0.01$ Ω /mm; $m = 0.9$; $k = 1$)

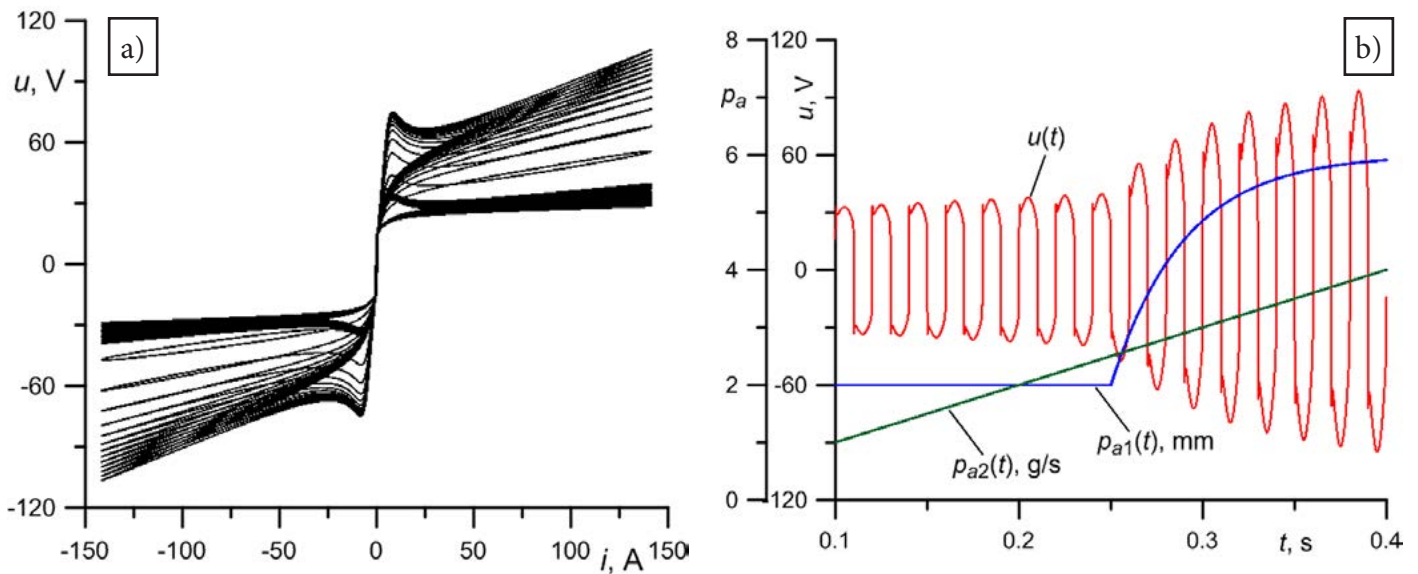


Fig. 2. Characteristics and transient processes in the circuit with arc described by the modified Pentegov model with quasi-static characteristic (4), with two parameters disturbed: a) dynamic voltage-current characteristic; b) waveforms of changes in disturbances and waveforms of arc voltage ($U_{AK} = 14V$; $\theta = 1 \cdot 10^{-4} s$; $U_{C0} = 0.2 V$; $U_{C1} = 5 V/mm$; $U_{C2} = 0.5 Vs/g$; $U_{C12} = 0.05 Vs/(g \cdot mm)$; $P_{M0} = 0 W$; $P_{M1} = 5 W/mm$; $P_{M2} = 0.5 Wg/s$; $P_{M12} = 0.05 Wg/(s \cdot mm)$; $R_{a0} = 0 \Omega$; $R_{a1} = 0.01 \Omega/mm$; $R_{a2} = 0.01 \Omega s/g$; $R_{a12} = 0.01 \Omega s/(g \cdot mm)$; $m = 0.9$; $k = 1$)

with two parameters disturbed, i.e. column length disturbed in a quasi-step manner and a gas mass stream disturbed in a linear manner. To this end it was necessary to build the macro-model of arc using equations (4), (10), (12)-(14) and (24)-(32). In this case it was assumed that parameter changes were not synchronised and included an increase in arc length and an increase in the stream of the mass of gas washing around a column, thus leading to an increase in the amount of dissipated heat. In the second case, the entire form of the differential equation of state was more difficult to present and, because of that, was omitted.

In the third case, it was assumed again that only one disturbing factor was present, leading to changes in the coefficients of function (5). The individual components of the function are described by the following formulas:

$$U_0(p_a) = U_{00} + U_1 p_a \tag{33}$$

$$R_0(p_a) = R_{00} + R_1 p_a \tag{34}$$

$$U_p(p_a) = U_{p0} + U_{p1} p_a \tag{35}$$

The derivatives of these functions in relation to parameter p_a are the following:

$$\frac{\partial}{\partial p_a} U_0(p_a) = U_1 \tag{36}$$

$$\frac{\partial}{\partial p_a} R_0(p_a) = R_1 \tag{37}$$

$$\frac{\partial}{\partial p_a} U_p(p_a) = U_{p1} \tag{38}$$

It is assumed that parameter changes are concerned only with the length of arc. The entire form of the differential equation of state is the following:

$$\begin{aligned} & U_1 \cdot \left\{ i_\theta + I_p \left[\exp\left(-\frac{i_\theta}{I_p}\right) - 1 \right] \right\} + \\ & \theta \frac{di_\theta^2}{dt} + 2\theta i_\theta \frac{\rightarrow}{\left[U_{00} + U_1 p_a + (R_{00} + R_1 p_a) i_\theta \right] \cdot \zeta(i_\theta) +} \rightarrow \\ & + R_1 \cdot \left[\frac{1}{2} i_\theta^2 + I_p (i_\theta + I_p) \exp\left(-\frac{i_\theta}{I_p}\right) - I_p^2 \right] + \frac{1}{4} U_{p1} \left[-(2i_\theta + I_p) \exp\left(1 - 2\frac{i_\theta}{I_p}\right) + e I_p \right] \quad (39) \\ & \rightarrow \frac{\rightarrow}{+ (U_{p0} + U_{p1} p_a) \left(\frac{i_\theta}{I_p} \right) \exp\left(-\left(\frac{i_\theta}{I_p}\right) + 1\right) [1 - \zeta(i_\theta)]} \times \\ & \times \frac{dp_a}{dt} = i^2 - i_\theta^2 \end{aligned}$$

Figure 3 presents the results of the simulations of processes in the circuit with arc and with one parameter, i.e. column length, disturbed in a quasi-step manner. To this end it was necessary to build the macromodel of arc using equations (5), (9), (12), (15), (16) and (33)-(39).

In the fourth case it was assumed that there were two disturbing factors, leading to changes in the coefficients of function (5). In certain conditions they can be approximated using simple polynomials. In certain conditions they can be approximated using simple polynomials (base 1, p_{a1} , p_{a2} , $p_{a1}p_{a2}$)

$$U_0(p_{a1}, p_{a2}) = U_{00} + U_1 p_{a1} + U_2 p_{a2} + U_{12} p_{a1} p_{a2} \quad (40)$$

$$R_0(p_{a1}, p_{a2}) = R_{00} + R_1 p_{a1} + R_2 p_{a2} + R_{12} p_{a1} p_{a2} \quad (41)$$

$$U_p(p_{a1}, p_{a2}) = U_{p0} + U_{p1} p_{a1} + U_{p2} p_{a2} + U_{p12} p_{a1} p_{a2} \quad (42)$$

The derivatives of these functions in relation to parameters are the following

$$\frac{\partial}{\partial p_{a1}} U_0(p_{a1}, p_{a2}) = U_1 + U_{12} p_{a2} \quad (43)$$

$$\frac{\partial}{\partial p_{a2}} U_0(p_{a1}, p_{a2}) = U_2 + U_{12} p_{a1} \quad (44)$$

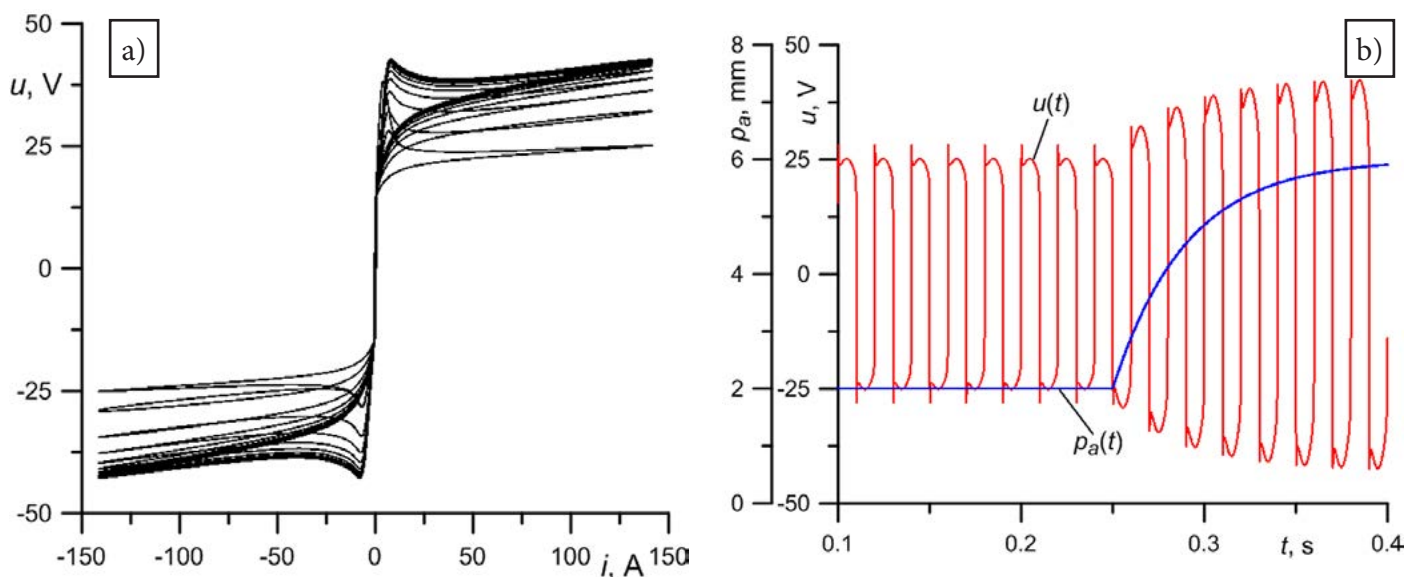


Fig. 3. Characteristics and transient processes in the circuit with arc described by the modified Pentegov model with quasi-static characteristic (5), with one parameter disturbed: a) dynamic voltage-current characteristic; b) waveforms of changes in disturbances and waveforms of arc voltage ($U_{AK} = 14V$; $\theta = 1 \cdot 10^{-4} s$; $U_{00} = 2 V$; $U_1 = 3 V/mm$; $R_{00} = 0 \Omega$; $R_1 = 0.01 \Omega/mm$; $U_{p0} = 22 V$; $U_{p1} = 1 V/mm$; $I_p = 3 A$)

$$\frac{\partial}{\partial p_{a1}} R_0(p_{a1}, p_{a2}) = R_1 + R_{12} p_{a2} \quad (45)$$

$$\frac{\partial}{\partial p_{a2}} R_0(p_{a1}, p_{a2}) = R_2 + R_{12} p_{a1} \quad (46)$$

$$\frac{\partial}{\partial p_{a1}} U_p(p_{a1}, p_{a2}) = U_{p1} + U_{p12} p_{a2} \quad (47)$$

$$\frac{\partial}{\partial p_{a2}} U_p(p_{a1}, p_{a2}) = U_{p2} + U_{p12} p_{a1} \quad (48)$$

Figure 4 presents the results of the simulations of processes in the circuit with arc and with two parameters disturbed, i.e. column length disturbed in a quasi-step manner and a gas mass stream disturbed in a linear manner. To this end it was necessary to build the macro-model of arc using equations (5), (9), (12), (15), (16) and (40)-(48). In this case it was assumed that parameter changes were not synchronised and included an increase in arc length and an increase in the stream of the mass of gas washing around a column. In the fourth case the entire form of the differential equation of state was also more difficult to present and, because of that, was omitted.

Conclusions:

1. The approximations of quasi-static voltage-current characteristics of arc with disturbed parameters presented in the article are characterised by significant generality enabling their use in the modelling and simulating of the operating conditions of various electrotechnical devices.

2. In spite of the significant generalisation of quasi-static voltage-current characteristics, the obtained modified mathematical Pentegov model of arc has a convenient form enabling the representation of one and two-parameter column disturbances.

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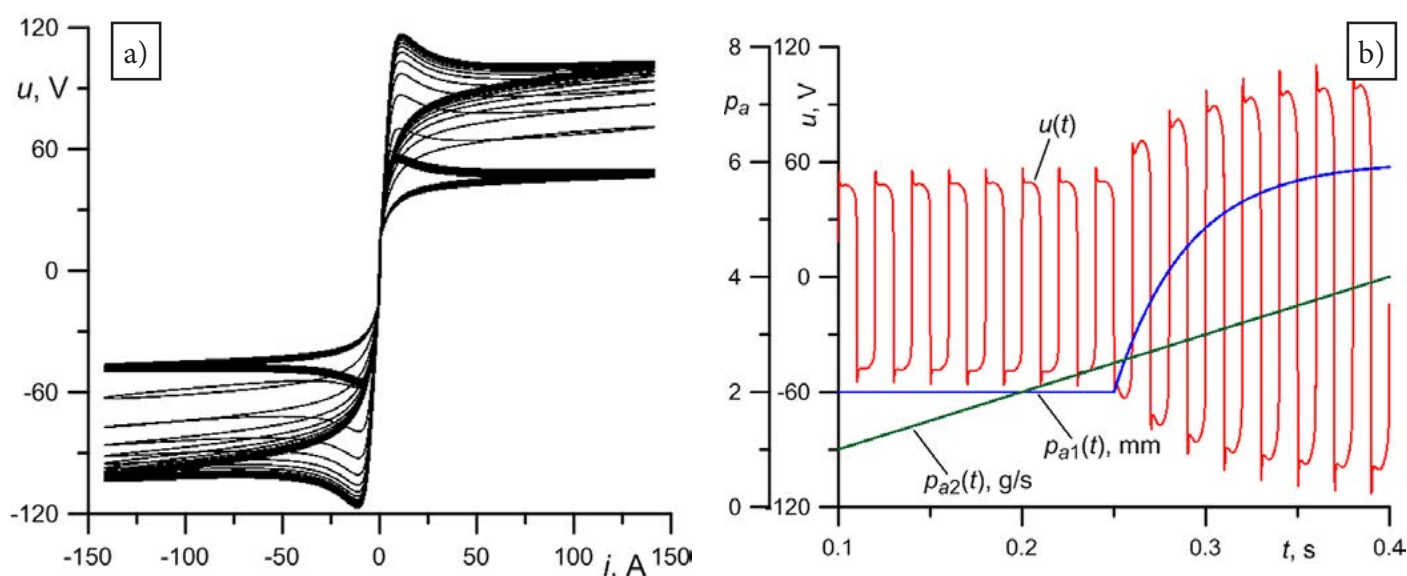


Fig. 4. Characteristics and transient processes in the circuit with arc described by the modified Pentegov model with quasi-static characteristic (5), with two parameters disturbed: a) dynamic voltage-current characteristics; b) waveforms of changes in disturbances and waveforms of arc voltage ($U_{AK} = 14V$; $\theta = 1 \cdot 10^{-4} s$; $U_{00} = 9 V$; $U_1 = 10 V/mm$; $U_2 = 1 Vs/g$; $U_{12} = 0.15 Vs/(g \cdot mm)$; $R_{00} = 0 \Omega$; $R_1 = 0.01 \Omega/mm$; $R_2 = 1 \cdot 10^{-4} \Omega/s/g$; $R_{12} = 5 \cdot 10^{-4} \Omega/s/(g \cdot mm)$; $U_{p0} = 22 V$; $U_{p1} = 1 V/mm$; $U_{p2} = 0.05 Vs/g$; $U_{p12} = 0.01 Vs/(g \cdot mm)$; $I_p = 3 A$)

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