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Research

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Spectral Method for Determining the Parameters of the Mayr and Cassie Electric Arc Models

Abstract: The paper describes the effect of various external factors on the static and dynamic voltage-current characteristics of electrical arcs as well as presents the spectral method for determining parameters of the Mayr and Cassie mathematical models. This method was also used for the coarse classification of arcs. The effect of column length disturbances on the uncertainty of arc model parameter determination was investigated using the MATLAB-Simulink program.

Keywords: electric arc, Mayr model, Cassie model, spectral method

Introduction

In technical reference publications it is possible to observe mutual relationships:

- low current, steeply drooping static characteristic and high time constant;
- high current, flat static characteristic and low time constant.

In the publication [1] the authors indicated the possibility that the notions mentioned above can exist independently of each other, which can be ascribed to various electric arc burning conditions in electrotechnological devices.

The processes of electrode cooling and gas deionisation always lead to an increase in arc ignition voltage. This effect also depends on the chemical composition of the gas, the material and shape of electrodes, the time interval between successive ignitions etc. The methods and measures lowering ignition voltage include, among other things, the forced ionisation of an electrode gap, the thermal insulation of the discharge area, imposing high-frequency voltage on a low-frequency waveform forcing a discharge and a momentary electrode gap reduction. The high voltage of DC arc ignition can significantly impede the normal operation of electrotechnological arc, and in particular plasma-arc devices. In the case of alternating current it is the reason for frequent problems with the obtainment of discharge stability and electromagnetic compatibility of devices with the supply network.

The column structure and the shape of short arc static characteristics are strongly affected by electrode properties. In most electrotechnological devices intended for welding elements or melting metal charges, electrodes are the basic source of metal vapours. Particularly intense electrode evaporation and high vapour concentration are observed in arcs burning in gases of lower pressure. Metal additives to plasma-forming gases strongly affect the shape of

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conductivity physical characteristics $\sigma(T)$ and radiation power volumetric density $\sigma_\varepsilon(T)$. An increase in vapour concentration is accompanied by increases in both of the quantities mentioned above. In turn, the maximum value of dynamic viscosity decreases [2]. Metal vapours are also responsible for a decrease in arc ignition voltage and a voltage decrease on an arc column as well as for a significant increase in plasma conductivity, an increase in discharge power and an increase in thermal radiation energy dissipation. Replacing plasma torch copper electrodes with steel electrodes increases the conductivity of both air and nitrogen plasma. In turn, replacing plasma torch steel electrodes with copper electrodes decreases radiation energy by 90% [3]. Even without providing metal additives it is possible to observe an increase in radiation power in arcs burning in gases of significantly higher pressure, which in turn can lead to a voltage-current characteristic increase. In general, the shape of a dynamic characteristic is affected by a number of factors, i.e. [4]

a) column length; an increase in a column length is accompanied by an increase in arc ignition voltage and an increase in the width of hysteresis;
b) decrease in thermal conductivity, gas enthalpy and electrode material, which leads to a decrease in arc ignition voltage and a decrease in the width of hysteresis;
c) arc thermal insulation under a flux, which leads to the narrowing or the disappearance of the dynamic loop of hysteresis and a decrease in ignition voltage peaks;
d) gas pressure rise, increasing a damping factor function value; as a result, the loop of hysteresis becomes wider;
e) difference in the time constants of thermal processes in electrode materials, leading to the widening of the dynamic loop of hysteresis;
f) increase in excitation current frequency, leading to the narrowing of the dynamic loop of hysteresis.

Also the parameters of a supply circuit affect the shape of a hysteresis dynamic loop. Industrial practice uses real power supply sources, i.e. generating waveforms deformed in relation to the sinusoid and causing the following results:

– increasing the impedance of the serial branch of the supply circuit, leading to the widening of the dynamic loop of hysteresis;
– increasing the no-load state voltage of the source, leading to the narrowing of the dynamic loop of hysteresis and to the termination of arc extinctions.

In the circuits of electrotechnological (primarily welding) devices in addition to AC excitations alternating periodic pulse excitations are also used, leading to the formation of asymmetric or minor dynamic loops of hysteresis. There are numerous methods allowing the determination of AC arc dynamic parameters. These methods can be divided into the following [5]:

1. methods utilising natural periodic current and voltage waveforms;
2. methods introducing additional disturbances to periodic waveforms utilising additional current sources;
3. methods introducing disturbances of an arc column length (voltage);
4. methods introducing disturbances of conditions of energy dissipation from a column.

Introducing special disturbances requires extending measurement systems with additional electronic or electromechanical modules and is connected with changing arc burning conditions, which can cause unwanted control system reactions and be responsible for additional systematic errors. For this reason, the possibility of determining - the arc mathematical model time constants of devices during their normal operation is of particular importance, especially because in many cases of high power arc devices (e.g. AC-EAF) it is possible to perform tests and measurements only on an in-situ basis, which, in turn, impedes or even makes it impossible to determine arc voltage-current static characteristics. These difficulties and the

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necessity of meeting the demands of today's automation can be overcome by taking advantage of spectral methods developed for determining time constants and other parameters of arc column mathematical models.

Spectral Method for Determining Time Constants in Mayr and Cassie Electric Arc Models

Due to the fact that an electric arc has thermal inertia, the changes of its thermal state and geometrical dimensions are not immediate while the arc is exposed to external effects such as current rushes or column length changes but are characterised by a certain delay. If thermal processes are assumed to be linear in nature, the measure of the delay mentioned above is a time constant being a column enthalpy to a loss power ratio. This ratio is defined by the ability of an arc to dissipate heat contained in a column and in electrode spots. As the structure of an arc and the physical characteristics of gases alter in a non-linear manner along with temperature changes (current), such inertia can be described by a damping factor function. The experimental tests [6, 7] have demonstrated that this function is non-linear. In the case of a more precise representation of arc dynamic properties it is necessary to take into consideration two very different time constants, i.e. a small time constant related to processes taking place in the arc core and a great time constant concerned with processes taking place on the arc surface.

Arc cooling intensity directly affects arc inertness. When a burning arc affects a solid state metal element (during welding, scrap metal melting etc.) arc inertness is significantly smaller than during heating a metal bath. The character of the effect of arc transverse parameters on a damping factor depends on the manner of heat dissipation, with the column length having only a little effect on its inertness [7]. In arcs very intensively cooled by a high velocity gas flow, e.g. in stream plasma torch ducts, it is possible to observe the effect of the significant reduction of the time constant, shorter by 2-3 orders ($10^{-6}\div10^{-7}$ s) if compared with ordinary free arcs. The greater the velocity of a gas flow washing around a column or the velocity of an arc motion in the gas, the smaller the time constant. In the case of high gas flow rates, an arc time constant does not depend on the chemical composition of the gas or on the type of electrodes [8]. However, even during operation at a frequency of 200 kHz, the arcs of AC plasma torches have a hysteresis; this fact demonstrates that the time constant of such arcs is low. Very intensive cooling causes an increase in gas breakdown voltage. The simultaneous existence of these two phenomena favours the instability and termination of an arc.

This article presents a spectral method for determining the parameters of two simple arc models. To this end, it will be necessary to consider the Mayr model

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M} \tag{1}$$

where g – electric conductance, U_{kol} – arc column voltage; i – excitation current; P_M – Mayr model constant power value; θ_M – Mayr model time constant.

The voltage waveform being the analytical periodic solution of the Mayr model with the sinusoidal current excitation $I_m \sin\omega t$ is the function

$$u_{kol}(t) = \frac{2P_M \sin\omega t}{I_m\left[1 - \frac{1}{1+4(\omega\theta_M)^2}\left(\cos 2\omega t + 2\omega\theta_M \sin 2\omega t\right)\right]} \tag{2}$$

This solution can be presented using the Fourier infinite trigonometric series of odd harmonics (1, 3, 5, 7, ...) of decreasing amplitudes. The heights of periodic solution spectral lines constitute a geometric progression fulfilling the condition

$$\frac{U_{2k+1}}{U_{2k-1}} = \chi_M(k) = const. \tag{3}$$

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where $2k+1$, $2k-1$ – numbers of adjacent odd harmonics; U – amplitude of an appropriate voltage harmonic. On this basis it is possible to determine the time constant using the formula [9]

$$\theta_M = \frac{1}{4\omega}\left(\frac{1}{\chi_M} - \chi_M\right) \quad (4).$$

This formula does not define the numbers of specific harmonics, yet due to the spectrum of possible disturbances in the circuit with the arc, it may become necessary to select the number k.

As regards the arcs of quasi-flat static characteristics, dynamic processes are relatively well approximated by the Cassie model, describing the time changes of plasma conductance g

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2} \quad (5)$$

where U_C – Cassie model voltage constant value; θ_C – Cassie model time constant. Assuming that a circuit with an arc is subjected to an alternating sinusoidal current excitation, the periodic solution of such a model is the deformed voltage $u_{kol}=i/g$. The voltage waveform being the periodic solution of the voltage in the Cassie model is the function

$$u_{kol}(t) = \frac{\sqrt{2}U_C \sin\omega t}{\sqrt{1 - \frac{1}{1+(\omega\theta_C)^2}\left(\cos 2\omega t + \omega\theta_C \sin 2\omega t\right)}} \quad (6).$$

After squaring this waveform and applying in relation to u_{kol}^2 of spectral analysis it appears that [10] the heights of spectral lines constitute the geometric progression of even harmonics (2, 4, 6, 8, ...) of decreasing amplitudes, fulfilling the condition

$$\frac{A_{2(k+1)}}{A_{2k}} = \chi_C(k) = const. \quad (7)$$

where $2k$, $2(k+1)$ – numbers of adjacent even harmonics; A – amplitude of an appropriate voltage square harmonic. This justifies the conclusion [10] that the time constant θ_C can be determined using the formula

$$\theta_C = \frac{1}{4\omega}\left(\frac{1}{\chi_C} - \chi_C\right) \quad (8).$$

Spectral Method for Determining Static Parameters of Mayr and Cassie Electric Arc Models

The spectral analysis of the voltage waveform of an arc described by the Mayr model and burning in the steady state while being energised with sinusoidal current enable the determination of a parameter representing power [10]

$$P_M = U_{sk} I_{sk} \sqrt{\frac{1-\chi_M^2}{1+\chi_M^2}} \quad (9),$$

where U_{sk} stands for the root-mean-square voltage and I_{sk} signifies the root-mean-square arc current. Significantly simpler is the definition of the parameter representing the Cassie model voltage [10]

$$U_C = U_{sk} = \sqrt{\overline{U^2}} \quad (10),$$

i.e. the root-mean-square voltage on the arc, being the root of the voltage square mean value.

These two parameters make up the basis of two approximations of an arc static characteristic. While determining these parameters it is necessary to assume that arc burning is stationary in nature. Then, both external effects and internal disturbances should be characterised by low intensity. Decreasing the effect of noise on measurements can be obtained by using appropriate filters of voltage and current signals or by calculating appropriate power density spectral functions. The highest quality of results and the most accurate measurements are concerned with stabilised arcs applied in plasma torches and in arc lamps. The shapes of the static characteristics of real arcs significantly differ to those assumed, both hyperbolic and flat. In addition, a non-linear damping factor function affects dynamic characteristics. This effect strongly depends on the damping factor to the external excitation time ratio.

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Spectral Method for Electric Arc Classification

The knowledge of methods used for determining the time constants of the Mayr and Cassie models can be useful in the classification of arcs and of electrotechnological devices [11]. However, this article presents a significant number of factors (in addition to current) affecting the shapes of arc static and dynamic characteristics. As a result, the dependences (3) and (7) can be fulfilled only roughly, which may significantly impede the classification of arcs. For this reason, the average value estimate calculated from the dependence (3) is approximated (taking into consideration n first harmonics) using the following expression

$$\overline{\chi}_M = \frac{\sum_{k=1}^{n} \chi_M(k)}{n} \qquad (11).$$

Similarly calculated is the average value estimate of the dependence (7), approximated using the following expression

$$\overline{\chi}_C = \frac{\sum_{k=1}^{n} \chi_C(k)}{n} \qquad (12).$$

Therefore, it is suggested that the measures of the deviations of the real arc harmonic indicators χ from arc mathematical models should adopt the following forms

- measure of deviation in relation to the Mayr model

$$\delta_M = \max_{k=1}^{n}\left|\chi_M(k) - \overline{\chi}_M\right| \qquad (13)$$

- measure of deviation in relation to the Cassie model

$$\delta_C = \max_{k=1}^{n}\left|\chi_C(k) - \overline{\chi}_C\right| \qquad (14).$$

The spectral criterion for dividing arcs into low-current arcs and high-current arcs can adopt either of the following forms:

- If $\delta_M(I_m) < \delta_C(I_m)$, the Mayr model-based approximation is more accurate and an arc fulfilling this condition is a low-current arc.
- If $\delta_M(I_m) > \delta_C(I_m)$, the Cassie model-based approximation is more accurate and an arc fulfilling this condition is a high-current arc.

The same current amplitude value will usually correspond to various values of deviations $\delta_M(I_m)$ and $\delta_C(I_m)$. Due to the fact that the models assume constant damping factor function values, this division can be regarded as unequivocal.

Particularly significant difficulties accompany the determination of very low time constants of an electric arc. The idealised boundary cases correspond to the following dependences:

- arc voltage in the Mayr model

$$\lim_{\theta_M \to 0} u_{kol}(t) = \frac{P_M}{I_m \sin \omega t} = U_M \operatorname{cosec}\omega t \qquad (15)$$

- arc voltage in the Cassie model

$$\lim_{\theta_C \to 0} u_{kol}(t) = \pm U_C \qquad (16).$$

The case (15) corresponds to the voltage waveform described by the cosecant periodic function. Its infinitely high values create difficulties in the numerical integration of the Mayr model equation. In turn, the case (16) corresponds to the voltage waveform described by the bipolar rectangle function of the amplitude U_C. As the determination of a very small value $\theta_C > 0$ by means of the spectral method requires using the square of voltage $u_{kol}^2 \cong U_C^2$, the arc of symmetric characteristics corresponds to an almost constant value $U_C^2 \cong const$, which impedes the effective use of the spectral analysis.

In turn, the very high time constant corresponds to arc model voltage waveforms close to sinusoidal. In idealised cases

- arc voltage in the Mayr model

$$\lim_{\theta_M \to \infty} u_{kol}(t) = \frac{2P_M}{I_m} \sin \omega t \qquad (17)$$

- arc voltage in the Cassie model

$$\lim_{\theta_C \to \infty} u_{kol}(t) = \sqrt{2} U_C \sin \omega t \qquad (18).$$

It is then that the difference between the Mayr

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and Cassie model voltage waveforms disappears as sinusoidal alternating excitation currents correspond to sinusoidal alternating arc voltages. The shape of the dynamic voltage-current arc characteristic depends on the product of excitation current pulsation ω and on the arc damping factor θ [7]. If $\omega\theta\rightarrow\infty$, it is possible to observe the continuous deformation of the hysteresis loop through the form of an inclined and narrowed eight to an inclined straight line.

Electric arc experimental tests enable the obtainment of non-linear damping factor functions [6, 7]. Usually, the greatest values of these functions correspond to low current values. The information provided in the introduction to this article as well as that contained in the publication [12] states that the shape of a strongly non-linear voltage-current static characteristic depends on many physical factors. Also a damping factor function depends on these factors. The values of the boundary current between the Mayr and Cassie models in the hybrid TWV welding machine arc model [13] linking these two models usually amount to approximately 5 A. This provides the possibility of using the criterion mentioned above along with relatively high values of time constants $\theta_M > \theta_C >> \theta_{Cmin}$.

Usually the issue of classification concerns arcs with relatively low current values, i.e. those which correspond to relatively high values of mathematical model time constants [6, 11]. The selection of a model also depends on the shape of the dynamic voltage-current characteristic, which, in turn, depends on the shape of a static characteristic.

Verification of the Spectral Method for Determining the Parameters of the Mayr and Cassie Electric Arc Models

Due to the adoption of simplifying assumptions, the analytical dependences (4), (8), (9) and (10) ignore the effect of disturbances on the accuracy of arc model parameter measurements. These disturbances can be generated in the following subsystems of electric appliances [14]:

a) power supply circuits;
b) electric arc;
c) measurement circuits.

It is possible that during a technological process such disturbances can occur simultaneously and/or with variable intensity.

The dependence (4) underwent simulation tests using MATLAB-Simulink software. To this end, it was necessary to create a simple circuit with current excitation (of amplitude I_m=5 A and frequency f=50 Hz). The load of this circuit is the arc described by the Mayr model (P_{M0}=60 W, θ_M=1·10^{-3} s). The model assumes that power is constant (P_M =const.). However, the approximation of the static characteristic $P_M = U_{stat}I_{stat}$ reveals that arc length disturbances lead to the changes of arc power. As a result, it was necessary to assume that P_M=P_{M0}(1+ζ_M/100) where the values of the maximum disturbance amplitude ζ_M=(0; 1; 2; 5; 10)% were selected to serve as examples. The generation $\zeta_M(t)$ required the use of a random generator with a timing frequency of 300 Hz, connected in a cascade manner with the first-order inert module of a time constant T= 0.00035 s. The results of the simulation presented in Figure 1 reveal that in cases of very low disturbance values the selection of harmonic numbers very poorly affects the accuracy of determining a time constant θ_M. However, an increase in disturbance amplitude is accompanied by an increase in error $\delta_{\theta M}$, where it is possible to observe a minimum $\delta_{\theta M}$ corresponding to the quotient of harmonics 7/5.

The lack of disturbances ζ_M=0% corresponds to the error $\delta_{\theta M}$>0 resulting from numerical method calculation errors. If ζ_M>0%, the calculation errors are aggravated by additionally detrimental effects due to the deviations of parameters from assumptions enabling the obtainment of the Mayr model analytical solutions.

Similar tests of processes in an electric circuit were performed with an arc described by

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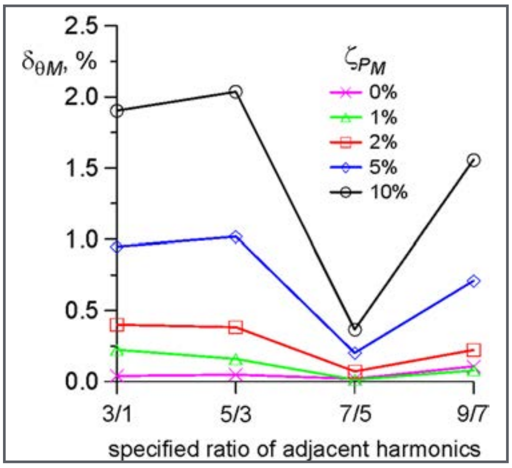

Fig. 1. Effect of various intensity disturbances on the error of determining the time constant θ_M as the function of the quotient of the numbers of adjacent odd harmonics $(2k+1)/(2k-1)$ of arc voltage

the Cassie model. The pre-set value of the sinusoidal current excitation amplitude amounted to 200 A; the frequency being f = 50 Hz. The Cassie model assumes that voltage is constant (U_C = const.). However, the approximation of the static characteristic reveals that arc length disturbances lead to changes of arc voltage $U_C = U_{C0}(1+\zeta_C/100)$, where the values of ζ_M= (0; 1; 2; 5; 10)% were selected to serve as examples. The remaining model parameters are U_{C0}=40 V and θ_C=1·10^{-3}s. Similarly, as in the previously performed generation $\zeta_M(t)$, it was necessary to use a random generator with a timing frequency of 300 Hz, connected in a cascade manner with a first-order inert module (T=0.00035 s). The simulation results are presented in Figure 2a. The results of tests repeated with a timing frequency of 150 Hz are presented in Figure 2b. The results reveal that an increase in the Cassie model disturbance intensity is accompanied by an increase in time constant determination error, yet it does not exceed 10%. A particularly small error accompanies the lack of disturbances ζ_C =0%. Also in this case it is possible to observe a minimum $\delta_{\theta C}$, corresponding to the quotient of harmonics 8/6. The presence of the minimum is not affected by a noise generator timing frequency nor is it influenced by the noise generator initial condition.

Conclusions:

1. The spectral method presented is limited to testing arcs supplied from sinusoidal variable current sources.
2. The spectral method presented is predisposed to measuring arcs of relatively high time constant values.
3. The disturbances of electric arc parameters usually increase the error of mathematical model time constant determination.
4. The selection of harmonic numbers of voltage affects the error of mathematical model time constant determination in relation to arcs subjected to random disturbance effect.
5. By appropriately selecting harmonic numbers of voltage it is possible to minimise the random disturbance effect on the error of determining mathematical model time constants of arcs.
6. The selection of approximations of arc dynamic characteristic using a (Mayr or Cassie) mathematical model does not depend on the mathematical model time constant $\theta(\chi)$, but also on the shape of a static characteristic.

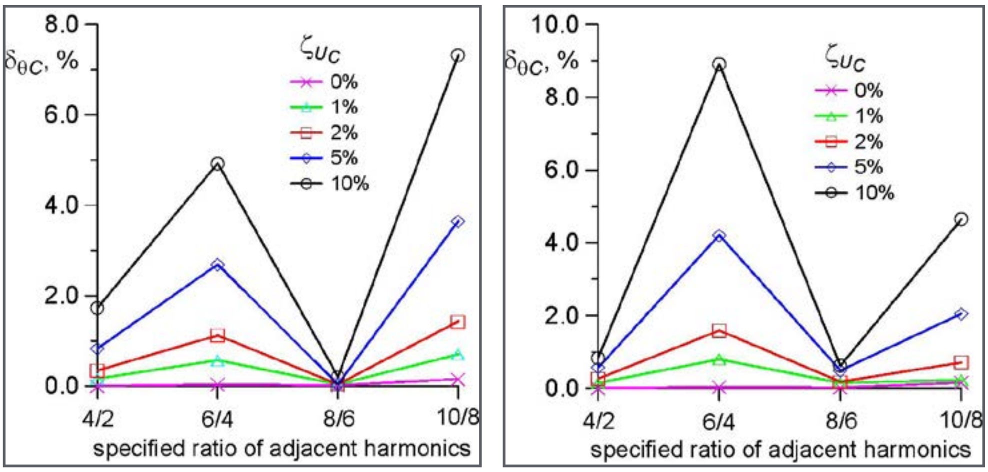

Fig. 2. Effect of various intensity disturbances ζ_{UC} on the error of determining the time constant θ_C as the function of the quotient of the numbers of adjacent even harmonics $2(k+1)/2k$ of the arc voltage square: a) disturbance timing frequency of 300 Hz; b) disturbance timing frequency of 150 Hz.

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