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Metrological Issues in Experimentation and Mathematical Modelling of Welding Machines.
Part 1. Errors and Uncertainty in Measurements of Current, Voltage and Power using Hall-effect sensors

Abstract: The article describes the methodology used in calculating errors and uncertainty of current, voltage and power measurement channels containing Hall-effect sensors and digital measurement devices. The deliberations involved Lem-manufactured current and voltage transducers whose nominal ranges enable testing electrotechnological devices intended for welding-related applications. The recommendations of the International Organisation for Standardisation (Iso) were used to present universal dependences enabling various measurement uncertainties, i.e. absolute, relative, standard, complex and extended. For some specific cases related to the selection Lem-manufactured transducers, voltmeters or measurement cards connected to a computer, relative measurement uncertainties budgets were created.

Keywords: Hall-effect sensor, current transducer, measurement uncertainty, measurement error, intermediate measurements

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Introduction

During experimental tests of electrotechnological devices, it is usually necessary to convert electric circuit quantities into signals, used, among other things, for measurements of current, voltage and power. Very high operational current (direct, sinusoidal, deformed and pulsed) as well as possible overcurrent and overvoltage requires the use of appropriate transformers capable of transferring signals within a wide range of the frequency spectrum, ensuring signal processing linearity and parameter stability as well as guaranteeing effective galvanic insulation of power-carrying and measurement circuits. One of the simplest, cheapest and reliable solutions is the use of transducers utilising the Hall effect [1]. As designers and users of electric equipment appreciate the advantages of such transducers and willingly use them, many companies produce a wide range of current and voltage transducers with Hall effect sensors intended for various ranges of electric quantities (primarily current), shapes and geometrical shapes of busses (i.e. bus-bars) and leads. The best-known producers include Lem (Switzerland), Abb (French division), Sentron, Honeywell Technologies, and Allegro MicroSystems (Usa). Less known manufacturers are 3E

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Sensor Co. Ltd, Ningbo Csr (China), Zao Npf Agrostroj, Ooo Gammamet, Tpk Elektromashina, Oao Niiem (Russian Federation) and other companies. Although the Swiss-based Lem has dominated the global market, transducers offered by other companies are sometimes characterised by slightly better parameters, e.g. the band of measurement signal transferred frequencies (Abb).

Sinusoidal current can also be measured using linear (coreless) sensors operating on the principle of the Rogowski coil, e.g. the Sidewinder Current Sensor manufactured by Pulse Electronics. In addition to linearity within a very wide range of current changes (0.1-1000 A) and very wide frequency band (up to 120 kHz), such sensors are also characterised by a very high accuracy of 0.2%. The disadvantages of these sensors include the impossibility to measure constant components of current and problems with measurements of deformed currents, requiring special integration systems.

Instead of a Lem-made voltage transducer it is possible to use integrated voltage amplifiers with optical isolation (of renowned companies, e.g. Vishay Semiconductors or Avago Technologies) providing a transfer band of up to approximately 50 kHz. A similar solution including a current shunt and an amplifier with optical isolation can be used instead of an Lem current transducer. However, systems with electronic amplifiers on the primary side are more expensive and less resistant to interference present during tests or when using Tig type arc power supplies, and, as a result, are more susceptible to damage.

Manufacturers provide users with information about basic nominal parameters and characteristics as well as about application systems of transducers with Hall-effect sensors [2-4]. Such information includes data about processing errors caused by the imperfection of products offered. As the transducers are components of measurement signal conditioning electronic systems, necessary guidelines also contain preliminary information concerning methods of calculating errors of systems with transducers caused by tolerances and temperature drifts of connected external resistors.

Scientific publications concerning the calculation of measurement uncertainty using channels with the cascade connection of Hall-effect sensors and digital devices are not particularly rich and information presented, usually when describing entire test rigs, is not complete. Due to ongoing works dedicated to the development of methods enabling the determination of characteristics of selected electrotechnological devices, the authors of this article have decided to present a methodology for the calculation of errors and uncertainty of current voltage and power measurements.

Guidelines for Calculations of Errors and Measurements of Measurement Systems

The international standard related to the assessment of measurement uncertainty, agreed in 1995 ("Guide to the expression of uncertainty in measurement" [5]) and its Polish equivalent [6] settle the dispute concerning the manners of determining measurement uncertainties. In addition to defining the basic measurement uncertainties (type A and B), the standard recommends determining the standard uncertainty u_C in the form of the statistical uncertainty transfer law and presenting subsequent results as the extended uncertainty U_E. The extended uncertainty has the form of the product of standard complex uncertainty with a properly selected extension coefficient k_e (depending on a previously assumed level of trust). However, there are also proponents of other methods [7, 8], e.g. the determination of measurement limiting errors using the total differential method (classical error propagation law). This article presents both approaches to the analysis of intermediate measurement uncertainties.

Before moving on to the complex metrological analysis of intermediate measurements, it is necessary to distribute a measurement channel into individual components in order to determine basic uncertainties obtained both by means of statistical analysis and the analysis of systematic limiting errors [9]. Random uncertainties are related to **type A uncertainty**, defined as a standard deviation from the experimental average expressed by the following formula:

$$u_A = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{1}$$

The first step to determine this uncertainty is to determine the arithmetic average of n measurements as

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2}$$

Uncertainties resulting from systematic limiting errors of probability distribution (e.g. uniform distribution) determined by the experimenter are classified as **type B uncertainties** in the following form:

$$u_B = \frac{\Delta_g x}{\sqrt{3}} \tag{3}$$

When both uncertainty types are present at the same time, it is necessary to determine the total **standard uncertainty** on the basis of the error propagation law related to direct measurements, using the following formula:

$$u_C = \sqrt{u_A^2 + u_B^2} \tag{4}$$

It may happen that there are more components of type B uncertainty. Allowing for calibration and experimenter uncertainties as well as additional uncertainties defined by the equipment manufacturer are expressed by the modified form of the following expression (4):

$$u_C = \sqrt{u_A^2 + u_{B1}^2 + u_{B2}^2 + \ldots + u_{Bn}^2} \tag{5}$$

Systems, in which quantity values searched for cannot be measured directly, require the use of intermediate measurements. A change of a measurement method entails another methodology for the determination of uncertainty. According to the Guide [5] published by the International Organisation for Standardisation (ISO), intermediate measurements are classified as **correlated** and **uncorrelated**.

The c**omplex standard uncertainty of uncorrelated measurements**, i.e. where each quantity is measured in a separate experiment or where introduced physical quantities are independent and random variables affecting these quantities are not correlated, has the following form:

$$u_{cns}(y) = \sqrt{\sum_{i=1}^{n} c_i^2 \cdot u^2(x_i)} \tag{6}$$

where c_i – sensitivity coefficients, expressed as the partial derivatives of the function y–$\partial f/\partial x_i$, $u(x_i)$ – uncertainty estimate x_i.

As opposed to the deliberations concerned with standard uncertainties, there is a notion of correlated intermediate measurements. Such a situation is present where measured quantities are interdependent or when random variables affecting these quantities are correlated, e.g. the same experimental set is used; measurements are performed at the same time and in the same conditions. The **complex standard uncertainty of correlated measurements** is defined in the following manner:

$$u_{cs}(y) = \sqrt{\sum_{i=1}^{n} c_i^2 \cdot u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} c_i c_j u(x_i) u(x_j) r(x_i, x_j)} \tag{7}$$

where $r(x_i, x_j)$ – correlation coefficient. In a special (the worst) case, where all uncertainty estimates are correlated with one another with the correlation coefficient $r(x_i, x_j) = +1$, the expression (7) adopts the following form:

$$u_{cs}(y) = \sum_{i=1}^{n} c_i u(x_i) \tag{8}$$

The complex uncertainty (8) is then expressed as the algebraic sum of the products of partial derivatives in relation to x_i with corresponding uncertainties $u(x_i)$. In spite of having a similar form, this expression cannot be identified with the classical error propagation law in the form of the total differential method

$$\Delta f = \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \Delta x_i \tag{9}$$

as limiting errors are not uncertainties. The formula (9) represents a different metrological approach, determining the maximum limiting error in intermediate measurements.

For the sake of simplification of these deliberations, measurements are treated as uncorrelated, where each measured quantity is treated as independent. Calculated uncertainties can be presented in relative non-dimensional numerical or percentage values as the following quotient:

$$\widetilde{u}(x) = \frac{u(x)}{x} \tag{10}$$

where x – specific numerical value or its estimate calculated according to (2).

Errors and Measurements of Measurement System Elements

The measurement system considered in this study is composed of current and voltage transducers, several adjusting resistances and voltmeters or a measurement card collaborating with a computer. The measurement system can include sensors with galvanic insulation using the Hall effect. The current and voltage sensors were powered using a bipolar DC source of stabilised voltage amounting to ±15V. Depending on the measurement range, the sensors may require additional external resistors. The tolerance of easily available precise resistors is restricted within the range of 0.1-1%. The tolerance of the resistors used in the study amounted to 0.1%. The temperature coefficient of resistance changes amounted to $15{\cdot}10^{-6}$ K^{-1}.

Transducer LA 200–P, presented in Table 1, can also be used for measuring current of as much as 300 A, with, obvious in such cases, deterioration of accuracy. Therefore, such measurements should be performed using sensors of greater measurement ranges. Due to the non-linearity of a transducer with a Hall-effect sensor, lower currents (of low root-mean--square value) should be measured with a transducer of a smaller measurement range, e.g. LA25-NP. Depending on the configuration of connections, the measurement ranges of LA25-NP are 5, 6, 8, 12 or 25 A. This section is focused on the 8 A range.

The signal frequency transfer band by an LV25-P voltage transducer is restricted within the range of 0-25 kHz. Although in comparison with the band of a current transducer, this voltage transducer band is not wide, yet it covers the sufficient number of voltage harmonics.

Angular errors of LEM current transducers depend on values of load current measured and are presented as maximum values in degrees. Errors of low-current transducers (up to several hundred A) are very small (0.05'), whereas errors of high-current transducers (up to several thousand A) may reach between 2.5 and 15°. As tests performed at the authors' laboratory include welding devices of power not greater than approximately 30 kW, the effect of angular errors on errors and uncertainties of active and reactive power measurements was ignored.

Initial measurements of arc root-mean-square current and voltage or of welding devices can be performed using digital voltmeters connected directly to LEM transducer conditioning systems. Both in the case of such measurements and in measurements performed using a computer, it is necessary to assess the uncertainty of such measurements. The individual componential elements of the system will be analysed on the basis of the uncertainty calculus recommended by international organisations for standardisation.

Errors and Uncertainties of a Measurement Transducer with a Hall-effect sensor

According to the manufacturer catalogue, the transmission of a transducer (conversion

Table 1. Selected parameters of current and voltage transducers and of signal conditioning systems

LEM transducer:	**LA200-P (current)**	**LA25-NP (current)**	**LV25-P (voltage)**
Nominal range of input current, I_{1max}, A	200	8	$10 \cdot 10^{-3}$ (pre-set voltage U_1=150 V)
Transfer band, kHz	0÷100	0÷150	0÷25
Output current, I_{2max} , mA	100	24	25
Sensor accuracy at a temperature of 25°C, δ_i, %	±0.40	±0.50	±0.8
Thermal drift of output current δ_{it}, %	(0-70°C), ±0.25	(25-70°C), ±1.46	(25-70°C), ±1.6
Error of transducer linearity δ_{lin}, %	<0.15	<0.20	<0.2
Maximum relative error of transducer $\boldsymbol{\delta_{imax}=\delta_i+\delta_{it}+\delta_{lin}}$, **%**	**±0.80**	**±2.16**	**±2.6**
Resistance of primary winding (at 70°C) R_{p1}, Ω	-	$2.5 \cdot 10^{-3}$	250
Resistance of additional resistor in primary circuit R_{v1}, Ω	-	-	$R_{v1}=(U_1/I_1)-R_{p1}=14\,750$
Nominal dissipated power in R_{v1}, W	-	-	$P_{v1}=I_{1\max}^2 R_{v1}=1.475$
Measurement resistance (on the secondary side), R_M, Ω	60	300	200
Nominal dissipated power in resistor R_M, $P_M=I_{2\max}^2 R_M$	0.6	0.173	0.125
Maximum output voltage of sensor, $U_{Mmax}=R_M \cdot I_{2max}$, V	6	7.2	5
Slope resistance of primary circuit R_1, Ω	-	-	$R_1=R_{v1}+R_{z1}=15 \cdot 10^3$
Resistance of secondary circuit (at 70°C), R_{z2} , Ω	76	80	110
Slope resistance of secondary circuit, $R_2=R_M+R_{z2}$, Ω	136	380	310
Transmission ***K***	1:2000	3:1000	2500:1000

coefficient) is expressed by the following dependence:

$$K=\frac{I_2}{I_1} \tag{11}$$

The manufacturer specifies transducer inaccuracies resulting, among other things, from the transducer production technological process, current temperature drift on the secondary side, or the inaccuracy resulting from transmission non-linearity (Table 1). The increment of secondary current with an unchanged primary current value leads to an appropriate transmission change:

$$K+\Delta K=\frac{I_2+\Delta I_2}{I_1} \tag{12}$$

After subtracting the undisturbed equation (11) from the disturbed equation (12), the following error value is obtained:

$$\Delta K=K\delta I_2 \tag{13}.$$

Assuming that in order to determine the systematic error of the transducer, the producer performed extensive tests involving a large group of errors; the error uncertainty can be determined using a uniform distribution:

$$u_K=\frac{\Delta K}{\sqrt{3}} \tag{14}$$

The uncertainty of a given transducer strictly depends on its type, parameters and values of relative errors ($\delta I_2=\delta_{imax}$ – Table 1) provided by the manufacturer in the catalogue card. The relative uncertainty of a transducer amounts to

$$\widetilde{u}_K = \frac{u_K}{K_I} \quad (15)$$

Errors and Uncertainties of Resistance of Additional Resistors

Outputs of current and voltage transducers have a convenient current character on the secondary side. This requires connecting a bipolar power source and resistor RM in the measurement signal conditioning system. In voltage transducers, an additional resistor is also present on the primary side. Such resistors are made with tolerance defend by a relative error provided by the manufacturer (Table 2).

Table 2. Effect of tolerance and temperature drift on resistance value error

Parameter	Relative error, %
Nominal error resulting from the selection of resistance R_M, δ_{RM}, %	±0.1
Error resulting from the temperature drift of resistance R_M δ_{RT}, %	$15 \cdot 10^{-6}$ K^{-1} (70°C-25°C)*100% = =±0.0675
Total error caused by changes of resistance $\delta_{R\max}=\delta_{RM}+\delta_{RT}$, %	±0.1675%

The systematic limiting error of resistance is defined by the following dependence:

$$\Delta_g R_i = R_i \frac{\delta_{R\max}}{100} \quad (16)$$

By assuming the rectangular distribution of probability density, it is assumed that standard uncertainty introduced by a specific resistor is dominated by type B uncertainty resulting from the systematic limiting error and amounts to

$$u_{Ri} = \frac{\Delta_g R_i}{\sqrt{3}} \quad (17)$$

The relative non-dimensional uncertainty associated with resistance is defined by the following formula:

$$\widetilde{u}_{Ri} = \frac{u_{Ri}}{R_i} \quad (18)$$

Errors and Uncertainties of a Digital Voltmeter

The absolute limiting error Δ_V of a factory digital voltmeter is defined by one of the dependences provided by the manufacturer:

$\Delta_V U = \pm a[\%] \cdot U \pm b[\%] \cdot U_z$ (19),

$\Delta_V U = \pm a[\%] \cdot U$ ±c (digits, signs, junts, dgt) (20),

$\Delta_V U = \pm a[\%] \cdot U \pm \Delta_d$ (in measured quantity units (21),

where U – voltage value measured; U_z – voltmeter measurement sub-range used; c – multiplicity of reading field resolution (if not specified, c = 1).

The relative percentage limiting error of a digital measurements amounts to

$$\delta_{V\%} = \pm \frac{\Delta_V U}{U} \cdot 100\% \quad (22)$$

After assuming that a selected digital voltmeter of DC voltage or of momentary value was characterised by a four-digit resolution and by the limiting error 2×10^{-3} of a readout +2 digits (the last digit having the value of 10^{-2} V), that the voltage measurement channel was linear, and that the distribution of probability of meter error was uniform, it was possible to calculate **type B uncertainty**:

$$u_{VB} = \frac{\Delta_v U}{\sqrt{3}} = \frac{0{,}002U + 0{,}01 \cdot 2}{\sqrt{3}}, \text{ V} \quad (23)$$

For 10 V, $u_{VB\max}$ is obtained.

In the case of multiple measurements, results obtained using a digital mustimeter are additionally burdened with **type A random uncertainty** calculated using the following expression:

$$u_{VA} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (U_i - \overline{U})^2} \quad (24)$$

where n – the number of performed measurements.

Absolute **standard uncertainty** (taking into consideration random and systematic errors) of a measurement performed using a digital voltmeter will be the following:

$$u_V = \sqrt{\left(u_{VA}(U)\right)^2 + \left(u_{VB}(U)\right)^2} \qquad (25)$$

The relative standard uncertainty is calculated analogously as presented in the publication (10).

Digital voltmeters for variable voltage measurements are equipped with additional systems transforming AC voltage into DC voltage. Most cheap digital voltmeters are adjusted for average or peak voltage measurements. Taking into consideration known waveforms of measured voltage, such devices are scaled as voltmeters of root-mean-square voltage. Such limitations are not present in expensive measurement equipment with a built-in True RMS function, e.g. collaborating with Maxim-manufactured MX536 integrated circuits. Transducers used in such devices are sources of additional additive errors (resulting from unbalance input voltage) and multiplicative errors (resulting from constant processing coefficient tolerance).

Errors and Uncertainties of a Measurement Card

The digital acquisition of data was performed using a measurement card. The limiting error of such a card is the sum of five components [10]. Within the signal range of 0-10 V, a 12-bit transducer has a resolution of 10/1098 = 2.4 mV, whereas 16-bit transducer as much as $10/2^{16}$ = 10/65536 = 0.15 mV. A 16-bit DAC PXI 6259 card was selected. As a signal introduced is bipolar, the resolution obtained is 2x lower, i.e. 0.30 mV. After amplifying the input voltage by a programmable amplifier of coefficient ku the absolute resolution improves from ΔU to $\Delta U/k_u$. However, in the case under discussion, coefficient $k_u = 1$. The maximum analogue error of the voltage signal amounts to 1920 μV, whereas the minimum error amounts to 52 μV. The total absolute error of the A/C transducer is composed of the analogue error and quantisation:

$$\Delta X_{kp}=\Delta X_{an}+\Delta X_{kw}=0{,}30\text{mV}+1{,}92\text{mV}=2{,}22\text{mV} \qquad (26)$$

The relative error amounts to

$$\delta_{kp} = \frac{\Delta X_{kp}}{\Delta U}100\% = \frac{2{,}22\cdot 10^{-3}\,\text{mV}}{10\text{V}}100\% = 2{,}22\cdot 10^{-2}\% \qquad (27)$$

On the basis of the expression (23), analogously as in the case of a digital voltmeter, it is possible to determine the **uncertainty connected with type B systematic errors** from the expression

$$u_{VB}=\frac{\Delta X_{kp}}{\sqrt{3}} = \frac{2{,}22\cdot 10^{-3}}{\sqrt{3}},\ \text{V} \qquad (28)$$

The measurement card coupled with a computer data acquisition unit performs sampling with a very high frequency yet limited by technical possibilities. As a result, it is possible to assume that the standard deviation of the average value is negligibly small, and then the s**tandard uncertainty** of the measurement card is the same in terms of value as type B uncertainty:

$$u_V \approx u_{VB} \qquad (29).$$

Uncertainties of Measurement of Electric Quantities using Standard Definitions

Current Measurement Uncertainty in a System with a Transducer

The value of current I_1 measured using an LEM transducer and voltmeter is determined from the following dependence:

$$I_1 = \frac{1}{K_I}\frac{U_{MA}}{R_{MA}} \qquad (30)$$

where K_I – transmission (transducer conversion coefficient); U_{MA} – voltage drop on the resistor RMA.

Acting in accordance with international recommendations and substituting previously determined standard uncertainties of the individual component elements of the measurement channel, the following **absolute standard complex** uncertainty of uncorrelated measurements is obtained:

$$u_{iC}(I_1)=\sqrt{\left(\frac{\partial I_1}{\partial U_{MA}}\right)^2\cdot u_V^2+\left(\frac{\partial I_1}{\partial R_{MA}}\right)^2\cdot u_{RMA}^2+\left(\frac{\partial I_1}{\partial K_I}\right)^2\cdot u_{KI}^2}=$$

$$=\sqrt{\left(\frac{1}{K_I R_{MA}}\right)^2\cdot u_V^2+\left(-\frac{U_{MA}}{K_I R_{MA}^2}\right)^2\cdot u_{RMA}^2+\left(-\frac{U_{MA}}{K_I^2 R_{MA}}\right)^2\cdot u_{KI}^2}=$$

$$=\frac{1}{K_I}\frac{U_{MA}}{R_{MA}}\sqrt{\left(\frac{1}{U_{MA}}\right)^2\cdot u_V^2+\left(\frac{1}{R_{MA}}\right)^2\cdot u_{RMA}^2+\left(\frac{1}{K_I}\right)^2\cdot u_{KI}^2}=$$

$$=\frac{1}{K_I}\frac{U_{MA}}{R_{MA}}\sqrt{\left(\frac{u_V}{U_{MA}}\right)^2+\left(\frac{u_{RMA}}{R_{MA}}\right)^2+\left(\frac{u_{KI}}{K_I}\right)^2}=I_1\sqrt{\tilde{u}_V^2+\tilde{u}_{RMA}^2+\tilde{u}_{KI}^2} \quad (31)$$

where u_V, u_{RMA}, u_{KI} – type A and type B standard uncertainties, $\frac{\partial I_1}{\partial U_{MA}}$, $\frac{\partial I_1}{\partial R_{MA}}$, $\frac{\partial I_1}{\partial K_I}$ – sensitivity coefficients. When determining the uncertainties of single measurements, a sensitivity coefficient is replaced by a specific numerical value. In multiple measurements this coefficient is constituted by the estimator of true value (2). On this basis, the **relative standard complex uncertainty** of uncorrelated intermediate measurements is determined by

$$\tilde{u}_{iC}(I_1)=\frac{u_{iC}(I_1)}{I_1}=\sqrt{\tilde{u}_V^2+\tilde{u}_{RMA}^2+\tilde{u}_{KI}^2} \quad (32)$$

the following dependence:

In practice, it is necessary to determine the measure of uncertainty covering a specific range of results around the value being measured. This is done in order to enable the comparison and verification of obtained numerical values on a previously assumed level of trust. The **absolute extended uncertainty** of intermediate measurements is the following:

$$U_{iE}(I_1)=k_e\cdot u_{iC}(I_1) \quad (33),$$

where k_e – extension coefficient, a value depending on a previously assumed level of trust (in laboratory tests usually k_e = 2 or 3).

Using the expression (33), it is possible to determine the relative extended uncertainty $\tilde{U}_{iE}(I_1)$, analogously as in the expressions (10) and (33).

Voltage Measurement Uncertainty in a System with a Transducer

The value of voltage U_1 measured using an Lem transducer and voltmeter is determined from the following dependence:

$$U_1=\frac{R_{v1}+R_{p1}}{R_{MV}}\frac{U_{MV}}{K_U} \quad (34)$$

where K_U – transmission (transducer conversion coefficient); U_{MV} – voltage drop on the resistor R_{MV}.

Applying the uncertainty propagation law, analogously as in the case of intermediate current measurement, the **standard complex absolute uncertainty** of uncorrelated intermediate measurements of voltage is expressed by the following formula:

$$u_{vC}(U_1)=\sqrt{\left(\frac{\partial U_1}{\partial U_{MV}}\right)^2\cdot u_V^2+\left(\frac{\partial U_1}{\partial R_{MV}}\right)^2\cdot u_{RMV}^2+\left(\frac{\partial U_1}{\partial R_{v1}}\right)^2\cdot u_{Rv1}^2+\left(\frac{\partial U_1}{\partial R_{p1}}\right)^2\cdot u_{Rp1}^2+\left(\frac{\partial U_1}{\partial K_U}\right)^2\cdot u_{KU}^2}=$$

$$=\sqrt{\left(\frac{R_{v1}+R_{p1}}{K_U R_{MV}}\right)^2\cdot u_V^2+\left(-\frac{U_{MV}(R_{v1}+R_{p1})}{K_U R_{MV}^2}\right)^2\cdot u_{RMV}^2+\left(\frac{U_{MV}}{K_U R_{MV}}\right)^2\cdot u_{Rv1}^2+\left(\frac{U_{MV}}{K_U R_{MV}}\right)^2\cdot u_{Rp1}^2+\left(-\frac{U_{MV}(R_{v1}+R_{p1})}{K_U^2 R_{MV}}\right)^2\cdot u_{KU}^2}=$$

$$=\frac{R_{v1}+R_{p1}}{R_{MV}}\frac{U_{MV}}{K_U}\sqrt{\left(\frac{1}{U_{MV}}\right)^2\cdot u_V^2+\left(\frac{1}{R_{MV}}\right)^2\cdot u_{RMV}^2+\left(\frac{1}{R_{v1}+R_{p1}}\right)^2\cdot u_{Rv1}^2+\left(\frac{1}{R_{v1}+R_{p1}}\right)^2\cdot u_{Rp1}^2+\left(\frac{1}{K_U}\right)^2\cdot u_{KU}^2}=$$

$$=\frac{R_{v1}+R_{p1}}{R_{MV}}\frac{U_{MV}}{K_U}\sqrt{\left(\frac{u_V}{U_{MV}}\right)^2+\left(\frac{u_{RMV}}{R_{MV}}\right)^2+\left(\frac{u_{Rv1}}{R_{v1}+R_{p1}}\right)^2+\left(\frac{u_{Rp1}}{R_{v1}+R_{p1}}\right)^2+\left(\frac{u_{KU}}{K_U}\right)^2}=$$

$$=\frac{R_{v1}+R_{p1}}{R_{MV}}\frac{U_{MV}}{K_U}\sqrt{\tilde{u}_V^2+\tilde{u}_{RMV}^2+\frac{u_{Rv1}^2+u_{Rp1}^2}{(R_{v1}+R_{p1})^2}+\tilde{u}_{KU}^2} \quad (35)$$

where u_V, u_{RMV}, u_{Rv1}, u_{Rwe}, u_{KU} – type A and type B standard uncertainties, $\frac{\partial I_1}{\partial U_{MV}}$, $\frac{\partial I_1}{\partial R_{MV}}$, $\frac{\partial I_1}{\partial R_{v1}}$, $\frac{\partial I_1}{\partial R_{we}}$, $\frac{\partial I_1}{\partial K_U}$ – sensitivity coefficients.

Using the error propagation law, it is possible to write that the total standard uncertainty of resistance present on the primary side of the transducer amounts to

$$u_{Rvp}^2 = u_{Rv1}^2 + u_{Rp1}^2 \quad (36)$$

In turn, the slope resistance of the primary circuit at the transducer input amounts to

$$R_{vp}=R_{v1}+R_{p1} \quad (37).$$

By substituting (36) and (37) to (35) and by simplifying, the final expression defining the **absolute standard complex uncertainty** of voltage measurement is obtained:

$$u_{vC}(U_1) = \frac{R_{v1}+R_{p1}}{R_{MV}} \frac{U_{MV}}{K_U} \sqrt{\widetilde{u}_V^2 + \widetilde{u}_{RMV}^2 + \widetilde{u}_{Rvp}^2 + \widetilde{u}_{KU}^2} =$$

$$= U_1 \sqrt{\widetilde{u}_V^2 + \widetilde{u}_{RMV}^2 + \widetilde{u}_{Rvp}^2 + \widetilde{u}_{KU}^2} \quad (38)$$

The **relative standard complex uncertainty** of uncorrelated intermediate measurements amounts to

$$\widetilde{u}_{vC}(U_1) = \frac{u_{vC}(U_1)}{U_1} = \sqrt{\widetilde{u}_V^2 + \widetilde{u}_{RMV}^2 + \widetilde{u}_{Rvp}^2 + \widetilde{u}_{KU}^2} \quad (39)$$

In turn, the **absolute extended uncertainty** is defined by the following dependence:

$$U_{vE}(U_1)=k_e \cdot u_{vC}(U_1) \quad (40).$$

Using the expression (40) it is possible to determine the relative extended uncertainty $\widetilde{U}_{vE}(U_1)$ analogously as in the expressions (10) and (39).

The dependences (30) and (34) define the momentary values of current and voltage measured quantities.

Power Measurement Uncertainties in a System with Current and Voltage Measurement Transducers

Due to the difficulties with the practical determination of correlation coefficients in the formula (7) related to intermediate measurements, various methods for simplifying experimentation and calculations are proposed [11]. The obtainment of specific numerical values requires many tests, due to which they are connected with the determination of type A uncertainty. Taking correlation into consideration can take place in the final phase of the extended uncertainty calculation by correcting the extension coefficient dependent, among other things, on the number of effective degrees of freedom [12].

In the case under consideration, the DC power is defined by the following formula:

$$P_1=U_1 I_1 \quad (41).$$

After using the recommended guidelines, the **absolute standard complex uncertainty**, calculated on the basis of the error propagation law, is defined by the following formula:

$$u_{pC}(P_1) = \sqrt{\left(\frac{\partial P_1}{\partial U_1}\right)^2 \cdot u_{vC}^2 + \left(\frac{\partial P_1}{\partial I_1}\right)^2 \cdot u_{iC}^2} = \sqrt{I_1^2 \cdot u_{vC}^2 + U_1^2 \cdot u_{iC}^2} =$$

$$= U_1 I_1 \sqrt{\frac{u_{vC}^2}{U_1^2} + \frac{u_{iC}^2}{I_1^2}} = P_1 \sqrt{\left(\frac{u_{vC}}{U_1}\right)^2 + \left(\frac{u_{iC}}{I_1}\right)^2} = P_1 \sqrt{\widetilde{u}_{vC}^2 + \widetilde{u}_{iC}^2} \quad (42)$$

where u_{vC}, u_{iC} – standard complex uncertainties of current (31) and voltage (35) measurements in a system with transducers, $\frac{\partial P_1}{\partial U_1}$, $\frac{\partial P_1}{\partial I_1}$ – sensitivity coefficients.

Relative standard complex uncertainty of uncorrelated intermediate measurements amounts to

$$\widetilde{u}_{pC}(P_1) = \frac{u_{pC}(P_1)}{P_1} = \frac{P_1 \sqrt{\widetilde{u}_{vC}^2 + \widetilde{u}_{iC}^2}}{P_1} = \sqrt{\widetilde{u}_{vC}^2 + \widetilde{u}_{iC}^2} \quad (43)$$

Taking into consideration the extension coefficient k_e, **the absolute extended uncertainty** of uncorrelated intermediate measurements is defined by the following dependence:

$$U_{pR}(P_1)=k_e \cdot u_{pC}(P_1) \quad (44).$$

Using the expression (44), it is possible to determine **relative extended uncertainty** $\widetilde{U}_{pE}(P_1)$, analogously as in the expressions (10) and (43).

On the basis of the function (41), the digital system of a measurement device or of a computer performs necessary calculations (with a negligibly small error of numerical roundings).

Errors of Electric Quantities Measurements using the Classical Error Propagation Law

As distinguished from the above deliberations, the maximum measurement error can be determined using the total differential method. However, this is a different metrological approach in relation to the already presented material. The classical error propagation law enables the determination of the **maximum intermediate measurement errors** and makes it possible to present them as uncertainties with the assumed level of trust $p=1$. However, this method is deterministic in nature as it does not take into consideration the randomness of signal and readout disturbances. Therefore, this method can be applied in measurements of uncorrelated quantities. In order to do so, it is necessary to analyse the total differential method used for determining the limiting errors of current, voltage and power intermediate measurements. By using special procedures, it is possible to measure power ignoring the current and voltage correlation. As previously, the deliberations are concerned with systems for conditioning signals with transducers.

The **maximum absolute error of current intermediate measurements** is the sum of the products of partial derivatives (30) and associated systematic limiting errors:

$$\begin{aligned}\Delta I_1 &= \left|\frac{\partial I_1}{\partial U_{MA}}\Delta U_{MA}\right| + \left|\frac{\partial I_1}{\partial R_{MA}}\Delta R_{MA}\right| + \left|\frac{\partial I_1}{\partial K_I}\Delta K_I\right| = \\ &= \frac{1}{K_I}\frac{U_{MA}}{R_{MA}}\left(\left|\frac{\Delta U_{MA}}{U_{MA}}\right| + \left|\frac{\Delta R_{MA}}{R_{MA}}\right| + \left|\frac{\Delta K_I}{K_I}\right|\right) = \\ &= \frac{1}{K_I}\frac{U_{MA}}{R_{MA}}\left(\left|\delta U_{MA}\right| + \left|\delta R_{MA}\right| + \left|\delta K_I\right|\right) = \\ &= I_1 \cdot \left(\left|\delta U_{MA}\right| + \left|\delta R_{MA}\right| + \left|\delta K_I\right|\right)\end{aligned} \tag{45}$$

where ΔU_{MA} – maximum systematic error of directly measured voltage U_{MA}; ΔR_{MA} – resistor tolerance provided in Table 2; ΔK_M – maximum error of transmission K_I.

The **maximum relative error of current intermediate measurements** based on the differential method is defined by the following formula:

$$\delta I_1 = \frac{\Delta I_1}{I_1} = \left|\delta U_{MA}\right| + \left|\delta R_{MA}\right| + \left|\delta K_I\right| \tag{46}$$

In turn, the **maximum absolute error of voltage intermediate measurements** is expressed by the following dependence:

$$\begin{aligned}\Delta U_1 &= \left|\frac{\partial U_1}{\partial U_{MV}}\Delta U_{MV}\right| + \left|\frac{\partial U_1}{\partial R_{MV}}\Delta R_{MV}\right| + \left|\frac{\partial U_1}{\partial R_{v1}}\Delta R_{v1}\right| + \left|\frac{\partial U_1}{\partial R_{we}}\Delta R_{p1}\right| + \left|\frac{\partial U_1}{\partial K_U}\Delta K_U\right| = \\ &= \frac{\left(R_{v1}+R_{we}\right)}{R_{MV}}\frac{U_{MV}}{K_U}\left(\left|\frac{\Delta U_{MV}}{U_{MV}}\right| + \left|\frac{\Delta R_{MV}}{R_{MV}}\right| + \frac{\left|\Delta R_{v1}\right| + \left|\Delta R_{p1}\right|}{R_{v1}+R_{we}} + \left|\frac{\Delta K_U}{K_U}\right|\right) = \\ &= \frac{\left(R_{v1}+R_{we}\right)}{R_{MV}}\frac{U_{MV}}{K_U}\left(\left|\delta U_{MV}\right| + \left|\delta R_{MV}\right| + \left|\delta R_{vp}\right| + \left|\delta K_U\right|\right) = \\ &= U_1 \cdot \left(\left|\delta U_{MV}\right| + \left|\delta R_{MV}\right| + \left|\delta R_{vp}\right| + \left|\delta K_U\right|\right)\end{aligned} \tag{47}$$

where $\left|\delta R_{vp}\right| = \dfrac{\left|\Delta R_{v1}\right| + \left|\Delta R_{p1}\right|}{R_{v1}+R_{p1}}$.

The **maximum relative error of voltage intermediate measurements** is obtained from the following formula:

$$\delta U_1 = \left|\delta U_{MV}\right| + \left|\delta R_{MV}\right| + \left|\delta R_{vp}\right| + \left|\delta K_U\right| \tag{48}$$

Analogously to (45) and (47), the **maximum absolute error of DC power intermediate measurements** amounts to

$$\begin{aligned}\Delta P_1 &= \left|\frac{\partial P_1}{\partial U_1}\Delta U_1\right| + \left|\frac{\partial P_1}{\partial I_1}\Delta I_1\right| = \left|I_1\Delta U_1\right| + \left|U_1\Delta I_1\right| = \\ &= U_1 I_1 \cdot \left(\left|\frac{\Delta U_1}{U_1}\right| + \left|\frac{\Delta I_1}{I_1}\right|\right) = P_1 \cdot \left(\left|\delta U_1\right| + \left|\delta I_1\right|\right)\end{aligned} \tag{49}$$

The **maximum relative error of power intermediate measurements** is expressed by the following dependence:

$$\delta P_1 = \frac{\Delta P_1}{P_1} = \frac{P_1 \cdot \left(\left|\delta U_1\right| + \left|\delta I_1\right|\right)}{P_1} = \left|\delta U_1\right| + \left|\delta I_1\right| \tag{50}$$

Some scientific publications present a method for the geometrical summation of total differential components in order to obtain the values of the absolute resultant error [8]. The error value obtained in this manner will be lower than the values expressed by the formulas (45)-(49).

Budges of Uncertainties of Current, Voltage and Power Measurements using Hall-effect sensors

The summing up of analyses of measurement errors and uncertainties is performed using tabular budgets of uncertainties (Table 3-5). This article is limited only to type B uncertainties. Such a case corresponds to the number of the degrees of freedom ∞ and to extension coefficient 2. In addition, it was assumed that the distribution of the function of the density of error probability in the range limited by limiting error values would be uniform (rectangular). The tables contain estimates of input and output quantities. The results in relation to uncertainty calculations shown in the tables were presented in relation to measurements performed using a digital mustimeter (M) and measurements performed using a measurement card and a computer (K). Current measurement-related data also include cases when a selected Lem transducer was used.

In addition to budgets of uncertainties, Tables 6-8 contain data related to budgets of maximum absolute errors of intermediate measurements. As expected, the values of these errors are greater than related uncertainties.

Conclusions:

1. Present Polish and international standards impose the duty of providing measurement results with calculated uncertainty. This requires developing new analytical methods taking into consideration at least systematic errors of elements in applied measurement channels.

2. The two uncertainty estimation versions presented in the article can satisfy the needs

Table 3. Budget of uncertainties of current intermediate measurement in a system with transducer

Parameter		Parameter value estimate	Unit	Type of uncertainty	Distribution	Value of standard uncertainty (total)	Sensitivity coefficient	Relative content of uncertainty (%) M	Relative content of uncertainty (%) K
Input quantity	K_I	1/2000 (I)	1	B	Rectangular $\sqrt{3}$	$2.309 \cdot 10^{-6}$	$-4 \cdot 10^{5}$ (c_1)	0.003	0.004
		3/1000 (II)				$3.741 \cdot 10^{-5}$	$-2.667 \cdot 10^{3}$ (c_1)	0.012	0.013
	R_{MA}	60 (I)	Ω	B	Rectangular $\sqrt{3}$	$5.802 \cdot 10^{-2}$	-3.333 (c_2)	75.847	97.835
		300 (II)				$2.901 \cdot 10^{-1}$	$-2.667 \cdot 10^{-2}$ (c_2)	93.582	99.547
	U_{MA}	6 (I)	V	B	Rectangular $\sqrt{3}$	$1.848 \cdot 10^{-2}$ (M)	$3.333 \cdot 10^{1}$ (c_3)	24.150	-
						$1.282 \cdot 10^{-3}$ (K)		-	2.151
		7.2 (II)				$1.986 \cdot 10^{-2}$ (M)	1.111 (c_3)	6.406	-
						$1.282 \cdot 10^{-3}$ (K)		-	0.440
Current measurement complex and extended uncertainty									
Output quantity	I_1	200 (I)	A	**complex uncertainty** (31)		1.127 (M)	$c_1 = \frac{\partial I_1}{\partial K_I} = -\frac{U_{MA}}{K_I^2 R_{MA}}$		
						0.945 (K)			
		8 (II)				0.102 (M)	$c_3 = \frac{\partial I_1}{\partial U_{MA}} = \frac{1}{K_I R_{MA}}$		
						0.100 (K)			
		200 (I)	A	**extended uncertainty** k_e=2 (p=95%) (33)		2.254 (M)	$c_2 = \frac{\partial I_1}{\partial R_{MA}} = -\frac{U_{MA}}{K_I R_{MA}^2}$		
						1.890 (K)			
		8 (II)				0.205 (M)			
						0.200 (K)			

*(M) – measurement performed using a digital multimeter;
*(K) – use of a measurement card coupled with a computer data acquisition unit;
*(I) – measurement transducer LEM LA 200-P; *(II) - measurement transducer LEM LA 25-NP.

Table 4. Budget of uncertainties of voltage intermediate measurement in a system with transducer

Parameter		**Parameter value estimate**	**Unit**	**Type of uncertainty**	**Distribution**	**Value of standard uncertainty (total)**	**Sensitivity coefficient**	**Relative content of uncertainty (%)**	
								M	**K**
Input quantity	$\boldsymbol{K_U}$	2500/1000	1	B	Rectangular $\sqrt{3}$	$3.753 \cdot 10^{-2}$	-60 (c_1)	0.254	0.255
	$\boldsymbol{R_{MV}}$	200	Ω	B	Rectangular $\sqrt{3}$	$1.934 \cdot 10^{-1}$	-0.75 (c_2)	1.311	1.312
	$\boldsymbol{R_{p1}}$	250	Ω	B	Rectangular $\sqrt{3}$	$2.418 \cdot 10^{-1}$	0.01 (c_3)	1.639	1.640
	$\boldsymbol{R_{v1}}$	$1.475 \cdot 10^{4}$	Ω	B	Rectangular $\sqrt{3}$	$1.426 \cdot 10^{1}$	0.01 (c_4)	96.679	96.784
	$\boldsymbol{U_{MV}}$	5	V	B	Rectangular $\sqrt{3}$	$1.732 \cdot 10^{-2}$ (M)	30 (c_5)	0.117	-
						$1.282 \cdot 10^{-3}$ (K)		-	0.009
Voltage measurement complex and extended uncertainty									
Output quantity	$\boldsymbol{U_1}$	150	V	complex uncertainty (35)		2.315 (M)	$c_1 = \frac{\partial U_1}{\partial K_U} = -\frac{U_{MV}(R_{v1}+R_{p1})}{K_U^{2} R_{MV}}$		
						2.257 (K)	$c_2 = \frac{\partial U_1}{\partial R_{MV}} = -\frac{U_{MV}(R_{v1}+R_{p1})}{K_U R_{MV}^{2}}$		
				extended uncertainty k_e=2 (p=95%) (40)		4.631 (M)	$c_3 = \frac{\partial U_1}{\partial R_{p1}} = \frac{U_{MV}}{K_U R_{MV}}$; $c_4 = \frac{\partial U_1}{\partial R_{v1}} = \frac{U_{MV}}{K_U R_{MV}}$		
						4.513 (K)	$c_5 = \frac{\partial U_1}{\partial U_{MV}} = \frac{R_{v1}+R_{p1}}{K_U R_{MV}}$		

Table 5. Budget of uncertainties of power intermediate measurement in a system with transducer

Parameter		**Parameter value estimate**	**Unit**	**Type of uncertainty**	**Distribution**	**Value of standard uncertainty (total)**	**Sensitivity coefficient**	
							M	**K**
Input quantity	$\boldsymbol{I_1}$	200 (I)	A	complex I_1 (31)	1.127 (M)	150 (c1)	32.743	29.513
					0.945 (K)			
	$\boldsymbol{I_1}$	150	V	complex U_1 (35)	2.315 (M)	200 (c2)	29.513	70.487
					2.257 (K)			
Power measurement complex and extended uncertainty								
Output quantity	$\boldsymbol{P_1}$	$3 \cdot 10^{4}$	W	**complex uncertainty** (42)		$4.929 \cdot 10^{2}$ (M)	$c_1 = \frac{\partial P_1}{\partial I_1} = U_1$	
						$4.731 \cdot 10^{2}$ (K)		
				extended uncertainty k_e=2 (p=95%) (44)		$9.858 \cdot 10^{2}$ (M)	$c_2 = \frac{\partial P_1}{\partial U_1} = I_1$	
						$9.463 \cdot 10^{2}$ (K)		

Table 6. Budget of errors of current measurement channel determined using the classical error propagation method

Input quantity		Maximum limiting error Δ_g		Relative error δ_g	
K_I, 1	1/2000 (I)	ΔK_I	$4.000 \cdot 10^{-6}$	δK_I	$8.000 \cdot 10^{-3}$
	3/1000 (II)		$6.480 \cdot 10^{-5}$		$2.160 \cdot 10^{-2}$
R_{MA}, Ω	60 (I)	ΔR_{MA}	$1.005 \cdot 10^{-1}$	δR_{MA}	$1.675 \cdot 10^{-3}$
	300 (II)		$5.025 \cdot 10^{-1}$		
U_{MA}, V	6 (I)	ΔU_{MA}	$3.200 \cdot 10^{-2}$ (M)	δU_{MA}	$5.333 \cdot 10^{-3}$ (M)
			$2.220 \cdot 10^{-3}$ (K)		$3.700 \cdot 10^{-4}$ (K)
	7.2 (II)		$3.440 \cdot 10^{-2}$ (M)		$4.778 \cdot 10^{-3}$ (M)
			$2.220 \cdot 10^{-3}$ (K)		$3.083 \cdot 10^{-4}$ (K)
Output quantity		**Maximum intermediate measurement error (45)**		**Maximum intermediate measurement relative error (46)**	
I_1, A	200 (I)	ΔI_1	3,002 (M)	δI_1	$1{,}501 \cdot 10^{-2}$ (M)
			2,009 (K)		$1{,}005 \cdot 10^{-2}$ (K)
	8 (II)		$2{,}244 \cdot 10^{-1}$ (M)		$2{,}805 \cdot 10^{-2}$ (M)
			$1{,}887 \cdot 10^{-1}$ (K)		$2{,}358 \cdot 10^{-2}$ (K)

Table 7. Budget of errors of voltage measurement channel determined using the classical error propagation method

Input quantity		Maximum limiting error Δ_g		Relative error δ_g	
K_I, 1	2500/1000	ΔK_I	$6.500 \cdot 10^{-2}$	δK_I	$2.600 \cdot 10^{-2}$
R_{MA}, Ω	200	ΔR_{MA}	$3.350 \cdot 10^{-1}$	δR_{MA}	$1.675 \cdot 10^{-3}$
R_{p1}, Ω	250	ΔR_{p1}	$4.188 \cdot 10^{-1}$	δR_{p1}	
R_{v1}, Ω	$1.475 \cdot 10^{4}$	ΔR_{v1}	$2.471 \cdot 10^{1}$	δR_{v1}	
R_{vp}, Ω	$1.5 \cdot 10^{4}$	ΔR_{vp}	$2.513 \cdot 10^{1}$	δR_{vp}	
U_{MA}, V	5	ΔU_{MA}	$3.000 \cdot 10^{-2}$ (M)	δU_{MA}	$6.000 \cdot 10^{-3}$ (M)
			$2.220 \cdot 10^{-3}$ (K)		$4.440 \cdot 10^{-4}$ (K)
Output quantity		**Maximum intermediate measurement error (47)**		**Maximum intermediate measurement relative error (48)**	
U_1, V	150	ΔU_1	5.303 (M)	δU_1	$3.535 \cdot 10^{-2}$ (M)
			4.469 (K)		$2.979 \cdot 10^{-2}$ (K)

Table 8. Budget of errors of current and voltage measurement channels for the determination of determined using the classical error propagation method method

Input quantity		Maximum limiting error Δ_g		Relative error δ_g	
I_1, A	200	ΔI_1	3.002 (M)	δI_1	$1.501 \cdot 10^{-2}$ (M)
			2.009 (K)		$1.005 \cdot 10^{-2}$ (K)
U_1, V	150	ΔU_1	5.303 (M)	δU_1	$3.535 \cdot 10^{-2}$ (M)
			4.469 (K)		$2.979 \cdot 10^{-2}$ (K)
Output quantity		**Maximum intermediate measurement error (49)**		**Maximum intermediate measurement relative error (50)**	
P_1, W	$3 \cdot 10^{4}$	ΔP_1	$1.511 \cdot 10^{3}$ (M)	δP_1	$5.036 \cdot 10^{-2}$ (M)
			$1.195 \cdot 10^{3}$ (K)		$3.984 \cdot 10^{-2}$ (K)

of testers of electric welding devices who wish to use both normalised and traditional metrological methods.

3. The described method for calculating uncertainties of intermediate measurements of electric quantities
- is versatile as it can be used for measurement channels with transducers (Hall-effect sensors) of various parameters and from various producers;
- can be used for evaluating determined parameters and characteristic of welding power sources (transformers, rectifiers and generators) and welding arc characteristics;
- is preferred in the case of modern diagnostic systems provided with digital devices or measurement card collaborating with computers;
- can be used for evaluating determined parameters and characteristic of various electric machines and equipment.

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