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Metrological Issues in Experimental Tests of Welding Machines.
Part 2: Errors and Uncertainties in Measurements of Parameters of Selected Periodic Waveforms

Abstract: The article justifies the necessity of taking into consideration measurement accuracies in experimental tests of welding machines and presents the effect of systematic errors of measurement channels on the errors related to the determination of root-mean-square current, root-mean-square voltage and average values of momentary power. The study also presents errors and uncertainties in measurements of active, passive and apparent power in supply systems of one-phase and three-phase welding machines. For the study-related purposes it was necessary to assume the symmetry of three-phase supply voltage and the linearity of elements in load branches. The article provides primary information about digital measurements of the frequency and the angle of the phase shift of periodic wavelengths indicating sources of systematic errors. The work also presents dependences enabling the experimental determination of filling factors of rectangular wavelengths of TIG welding machines as well as describes measurement errors in three cases of rectangular wave shapes. The computational determination of errors and uncertainties was exemplified using the results of tests concerned with welding source power efficiency.

Keywords: electric arc, measurement error, measurement uncertainty

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Introduction

The operation of welding equipment is characterised by special properties, such as the following:
- non-linearity of active (arcs, semiconductors and connectors) and passive (choking coils and transformers) elements;
- inertness of active (arcs) and passive (choking coils and transformers) elements as well as of control systems;
- non-stationarity of processes in pulsed equipment (e.g. in resistance welding machines) and arc machinery (start and stop processes);
- random disturbances (fluctuations of non-stabilised arc parameters, voltage changes caused by technological or manually performed operations);
- deterministic disturbances (automatically excited changes of settings);
- asymmetry of three-phase network loads (single-phase, dual-phase and asymmetric three-phase receivers).

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The non-linearity and inertness of welding elements and systems are often related to equation tautness characterised by very large scatters of parameter values, e.g. factors of damping, impedance etc., impeding both measurements and modelling of welding machines. Due to a frequently high level and wide spectrum of generated disturbances, particular difficulties are encountered when testing electrotechnological arc welding machines in technological process conditions, which requires the use of appropriate (preferably adaptive) filters. A relatively low accuracy of measurement results would make them effects of diagnostics rather than effects of measurements.

A very wide range of accurate experimental tests involving welding machines (omitting arc stability conditions) can be obtained in arcless operation conditions. The role of static load can be played by appropriately controlled resistors and energy sources, whereas the role of dynamic load can be played by appropriate welding arc simulators [1, 2] with an inactive function of disturbance imitation. In such conditions, measurement results can be characterised by high accuracy and repeatability. This fact justifies the purposefulness of the analysis of measurement errors and uncertainties of welding machinery in design studios and diagnostic laboratories.

Errors and Uncertainties of Current, Voltage and Power Measurements in Selected Period Waveforms

The first part of article [3] presents methods used when calculating errors and uncertainties of momentary current and voltage measurements and of active power measurements using transducers provided with Hall effect sensors. Standard methods rely on multimeters with memories or measurement cards connected to transducers. In this manner, it is also possible to measure values of such quantities in DC circuits.

Table 1 presents primary formulas used when calculating average and root-mean-square values of variable waveforms. The second column contains definitions used in the operation of analogue measurement devices, whereas the third column presents definitions used in digital processing. It is assumed that waveform sampling satisfies Nyquist conditions [4]. If a signal is periodic, it can be reproduced from a finite number of N samples obtained in one period of waveform T with a frequency twice as high as harmonic frequency of the highest order in a waveform subjected to analysis. Article [5] presents three methods of determining (r.m.s.) root-mean-square values of signals.

Instead of Hall effect sensors, root-mean-square current and root-mean-square voltage can be measured using thermoelectric transducers [6, 7]. In such a case, measurements are performed regardless of waveform shapes and within a very wide range of frequency spectrum. The primary disadvantages of thermoelectric transducers include the effect of ambient

Table 1. Definitions of integral values of electric quantities of continuous and discrete period waveforms

Value of physical quantity	**Investigated continuous waveform**	**Investigated discrete waveform**
Average current I_{sr}=	$\frac{1}{T}\int_{-T/2}^{T/2} i(\tau)d\tau$	$\frac{1}{N}\sum_{k=0}^{N-1} i_k$
Average voltage U_{sr}=	$\frac{1}{T}\int_{-T/2}^{T/2} u(\tau)d\tau$	$\frac{1}{N}\sum_{k=0}^{N-1} u_k$
Root-mean-square current I_{sk}=	$\sqrt{\frac{1}{T}\int_{-T/2}^{T/2} i^2(\tau)d\tau}$	$\sqrt{\frac{1}{N}\sum_{k=0}^{N-1} i_k^2}$
Root-mean-square voltage U_{sk}=	$\sqrt{\frac{1}{T}\int_{-T/2}^{T/2} u^2(\tau)d\tau}$	$\sqrt{\frac{1}{N}\sum_{k=0}^{N-1} u_k^2}$
Average momentary power (active power) P_{sr}=	$\frac{1}{T}\int_{-T/2}^{T/2} u(\tau)i(\tau)d\tau$	$\frac{1}{N}\sum_{k=0}^{N-1} u_k i_k$
	$\frac{1}{T}\int_{-T/2}^{T/2} p(\tau)d\tau$	$\frac{1}{N}\sum_{k=0}^{N-1} p_k$

temperature and of the non-linearity of a thermoelectric element on measurements. For this reason, thermoelectric transducers are used sporadically, only when measuring strongly deformed waveforms or waveforms of very high frequency.

Due to a multiplying action, the Hall effect is also used in the manufacturing of power transducers and Hall effect analogue wattmeters, requiring appropriate temperature compensation, but enabling the obtainment of accuracies similar to those of electrodynamic meters and being capable of operation in frequency up to several hundred kilohertz. In spite of the advantages mentioned, these devices have not become popular in measurement technique.

In cases of periodic waveforms, errors of momentary value measurements can affect errors of average values of root-mean-square current, voltage and power. Table 2 presents related formulas. These deliberations assume the additive character of systematic errors of direct and indirect measurements. Therefore, such errors do not depend on a value being measured (or on

Table 2. Errors of measurements of electric quantities of continuous and discrete waveforms (Δi – systematic error of momentary current; Δu – systematic error of momentary voltage; $\delta I_{sk}=\Delta I_{sk}/I_{sk}$, $\delta U_{sk}=\Delta U_{sk}/U_{sk}$)

Error of measurement of	Error of continuous waveform quantity measurement	Error of discrete waveform quantity measurement	Error of sinusoidal waveform quantity measurement	Type B uncertainty
average current $\lvert\Delta I_{sr}\rvert=$	$\lvert\Delta i\rvert$	$\lvert\Delta i\rvert$	$\lvert\Delta i\rvert$	$u_{isr}=\frac{\lvert\Delta I_{sr}\rvert}{\sqrt{3}}$
average voltage $\lvert\Delta U_{sr}\rvert=$	$\lvert\Delta u\rvert$	$\lvert\Delta u\rvert$	$\lvert\Delta u\rvert$	$u_{usr}=\frac{\lvert\Delta U_{sr}\rvert}{\sqrt{3}}$
root-mean-square current current $\lvert\Delta I_{sk}\rvert\approx$	$\left\lvert\frac{1}{I_{sk}T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\lvert i\rvert\cdot\lvert\Delta i\rvert\cdot dt\right\rvert=\frac{\lvert\Delta i\cdot I_{sr}(\lvert i\rvert)\rvert}{I_{sk}}$	$\left\lvert\frac{1}{I_{sk}N}\sum_{k=0}^{N-1}\lvert i_k\rvert\cdot\lvert\Delta i\rvert\right\rvert=\frac{\lvert\Delta i\cdot I_{sr}(\lvert i_k\rvert)\rvert}{I_{sk}}$	$\left\lvert\frac{\Delta i}{I_{sk}}\frac{2I_m}{\pi}\right\rvert=\frac{2\sqrt{2}\lvert\Delta i\rvert}{\pi}$	$u_{isk}=\frac{\lvert\Delta I_{sk}\rvert}{\sqrt{3}}$
root-mean-square voltage $\lvert\Delta U_{sk}\rvert\approx$	$\left\lvert\frac{1}{U_{sk}T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\lvert u\rvert\cdot\lvert\Delta u\rvert\cdot dt\right\rvert=\frac{\lvert\Delta u\cdot U_{sr}(\lvert u\rvert)\rvert}{U_{sk}}$	$\left\lvert\frac{1}{U_{sk}N}\sum_{k=0}^{N-1}\lvert u_k\rvert\cdot\lvert\Delta u\rvert\right\rvert=\frac{\lvert\Delta u\cdot U_{sr}(\lvert u_k\rvert)\rvert}{U_{sk}}$	$\left\lvert\frac{\Delta u}{U_{sk}}\frac{2U_m}{\pi}\right\rvert=\frac{2\sqrt{2}\lvert\Delta u\rvert}{\pi}$	$u_{usk}=\frac{\lvert\Delta U_{sk}\rvert}{\sqrt{3}}$
average active power $\lvert\Delta P_{sr}\rvert\approx$	$\left\lvert\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\lvert u\rvert\cdot\lvert\Delta i\rvert\cdot dt\right\rvert+\left\lvert\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\lvert i\rvert\cdot\lvert\Delta u\rvert\cdot dt\right\rvert=\lvert\Delta i\cdot U_{sr}(\lvert u\rvert)\rvert+\lvert\Delta u\cdot I_{sr}(\lvert i\rvert)\rvert$	$\left\lvert\frac{1}{N}\sum_{k=0}^{N-1}\lvert u_k\rvert\lvert\Delta i\rvert\right\rvert+\left\lvert\frac{1}{N}\sum_{k=0}^{N-1}\lvert i_k\rvert\lvert\Delta u\rvert\right\rvert=\lvert\Delta i\cdot U_{sr}(\lvert u_k\rvert)\rvert+\lvert\Delta u\cdot I_{sr}(\lvert i_k\rvert)\rvert$	$\left\lvert\Delta i\frac{2U_m}{\pi}\right\rvert+\left\lvert\Delta u\frac{2I_m}{\pi}\right\rvert=\frac{2\sqrt{2}}{\pi}(\lvert\Delta iU_{sk}\rvert+\lvert\Delta uI_{sk}\rvert)=S(\lvert\delta I_{sk}\rvert+\lvert\delta U_{sk}\rvert)$	$u_{psr}=\frac{\lvert\Delta P_{sr}\rvert}{\sqrt{3}}$

its sign). Conclusions resulting from the definitions contained in Table 2 are the following:

1. In the case of sinusoidally variable current excitation operating in a circuit, $I_{sr}(i) = 0$ A and $I_{sr}(|i|) = 2I_m/\pi$, the error of root-mean-square value measurement is expressed by the formula presented in Table 2. However, for situations when DC is used, the formula is $|\Delta I_{sr}|=|\Delta I_{sk}|=|\Delta i|$.

2. If dynamic characteristics of a load are linear and current excitation is sinusoidal, $U_{sr}(u) = 0$ V and $U_{sr}(|u|)=2U_m/\pi$, the error of root-mean-square value measurement is expressed by the formula presented in Table 2. However, for situations when DC is used, the formula takes the following form $|\Delta U_{sr}|=|\Delta U_{sk}|=|\Delta u|$.

3. On the basis of previous assumptions concerning sinusoidal current excitation and the linearity of dynamic characteristics of active loads, it can be stated that an error of active power measurement is expressed by the formula presented in Table 2. However, for situations when DC is used, the error of measurement $|\Delta P_{sr}|\approx P\cdot(|\delta_u|+|\delta_i|)$, where $P=UI$, and relative errors are: $\delta_u=\Delta u/U$, $\delta_i=\Delta i/I$.

Similar to Part 1 [3], it is assumed that uncertainties resulting from limiting systematic errors of probability distribution (e.g. uniform/rectangular distribution) specified by an experimenter are classified as **type B uncertainties** in the following form:

$$u_B=\frac{\Delta_g x}{\sqrt{3}} \qquad (1)$$

The value of root-mean-square AC voltage can also be measured using measurement devices equipped with a rectifier system at the input. A similar manner of measurement can be used in computer systems. Such a solution does not affect the maximum value of a systematic error.

Powering one-phase welding machines from a power grid is usually related to quasi-sinusoidal AC. In such a case, the active power (momentary power average value) formula can be expressed in the following form:

$$P=UI\cos\varphi \qquad (2),$$

where U, I – root-mean-square voltage and current; φ – phase angle between current and voltage. The error of absolute maximum power measurement can be expressed using the following formula:

$$\begin{aligned}\Delta P &= \left|\frac{\partial P}{\partial U}\Delta U\right|+\left|\frac{\partial P}{\partial I}\Delta I\right|+\left|\frac{\partial P}{\partial \varphi}\Delta \varphi\right|= \\ &=\left|I\cos\varphi\cdot\Delta U\right|+\left|U\cos\varphi\cdot\Delta I\right|+\left|-UI\sin\varphi\cdot\Delta\varphi\right|\end{aligned} \qquad (3)$$

where $\Delta\varphi=\Delta\varphi_u+\Delta\varphi_i$ absolute errors of voltage and current angle measurements. In turn, the relative error is described by the following formula [8]:

$$\begin{aligned}\delta P &= \frac{\Delta P}{P}=\left|\frac{\Delta U}{U}\right|+\left|\frac{\Delta I}{I}\right|+\left|-tg\varphi\Delta\varphi\right|= \\ &=\left|\delta_U\right|+\left|\delta_I\right|+\left|-tg\varphi\Delta\varphi\right|\end{aligned} \qquad (4)$$

Active power measurement uncertainty can be expressed by the following formula:

$$\begin{aligned}u_{pC}(P) &= \sqrt{\left(\frac{\partial P}{\partial U}\right)^2 u_{vC}^2+\left(\frac{\partial P}{\partial I}\right)^2 u_{iC}^2+\left(\frac{\partial P}{\partial \varphi}\right)^2 u_{\varphi C}^2}= \\ &=\sqrt{(I\cos\varphi)^2 u_{vC}^2+(U\cos\varphi)^2 u_{iC}^2+(-UI\sin\varphi)^2 u_{\varphi C}^2}= \\ &=UI\cos\varphi\sqrt{\left(\frac{1}{U}\right)^2 u_{vC}^2+\left(\frac{1}{I}\right)^2 u_{iC}^2+\left(\frac{-\sin\varphi}{\cos\varphi}\right)^2 u_{\varphi C}^2} \\ &=P\sqrt{\tilde{u}_{vC}^2+\tilde{u}_{iC}^2+(-tg\varphi)^2 u_{\varphi C}^2}\end{aligned} \qquad (5)$$

where u_{vC}, u_{iC} – standard complex uncertainties of current (31) and voltage (35) [3] measurements in a system with transducers, $u_{\varphi C}$ – standard complex uncertainty of angle measurement, $\frac{\partial P}{\partial U}$, $\frac{\partial P}{\partial I}$, $\frac{\partial P}{\partial \varphi}$ – sensitivity coefficients, $\tilde{u}_{vC}$, $\tilde{u}_{iC}$ – standard relative uncertainties.

Relative complex standard uncertainty of power uncorrelated indirect measurements is the following:

$$\begin{aligned}\tilde{u}_{pC}(P) &= \frac{u_{pC}(P)}{P}=\frac{P\sqrt{\tilde{u}_{vC}^2+\tilde{u}_{iC}^2+(-tg\varphi)^2 u_{\varphi C}^2}}{P}= \\ &=\sqrt{\tilde{u}_{vC}^2+\tilde{u}_{iC}^2+(-tg\varphi)^2 u_{\varphi C}^2}\end{aligned} \qquad (6)$$

Taking into consideration expansion coefficient k_e, **absolute extended uncertainty** of uncorrelated indirect measurements is expressed by the following dependence:

$$U_{pR}(P)=k_e\cdot u_{pC}(P) \quad (7).$$

Using expression (44) [3], it is possible to determine **relative extended uncertainty** $\widetilde{U}_{pE}(P)$, in the same manner as in expressions (10) and (43) [3].

One-phase receiver reactive power can be expressed by the following formula:

$$Q=UI\sin\varphi \quad (8).$$

The absolute value of a reactive power measurement error can be expressed by the following formula:

$$\Delta Q=\left|\frac{\partial Q}{\partial U}\Delta U\right|+\left|\frac{\partial Q}{\partial I}\Delta I\right|+\left|\frac{\partial Q}{\partial \varphi}\Delta \varphi\right|= \\ =\left|I\sin\varphi\Delta U\right|+\left|U\sin\varphi\Delta I\right|+\left|UI\cos\varphi\cdot\Delta\varphi\right| \quad (9)$$

In turn, the relative error is expressed by the following formula:

$$\delta Q=\frac{\Delta Q}{Q}=\left|\frac{\Delta U}{U}\right|+\left|\frac{\Delta I}{I}\right|+\left|ctg\varphi\cdot\Delta\varphi\right|= \\ =\left|\delta_U\right|+\left|\delta_I\right|+\left|ctg\varphi\Delta\varphi\right| \quad (10)$$

Reactive power measurement uncertainty can be expressed by the following formula:

$$u_{pC}(Q)=\sqrt{\left(\frac{\partial Q}{\partial U}\right)^2u_{vC}^2+\left(\frac{\partial Q}{\partial I}\right)^2u_{iC}^2+\left(\frac{\partial Q}{\partial \varphi}\right)^2u_{\varphi C}^2}= \\ =\sqrt{(I\sin\varphi)^2u_{vC}^2+(U\sin\varphi)^2u_{iC}^2+(UI\cos\varphi)^2u_{\varphi C}^2}= \\ =UI\sin\varphi\sqrt{\left(\frac{1}{U}\right)^2u_{vC}^2+\left(\frac{1}{I}\right)^2u_{iC}^2+\left(\frac{\cos\varphi}{\sin\varphi}\right)^2u_{\varphi C}^2}= \\ =Q\sqrt{\widetilde{u}_{vC}^2+\widetilde{u}_{iC}^2+ctg\varphi u_{\varphi C}^2} \quad (11)$$

where $\frac{\partial Q}{\partial U}$, $\frac{\partial Q}{\partial U}$, $\frac{\partial Q}{\partial U}$ – sensitivity coefficients.

Relative complex standard uncertainty of uncorrelated indirect measurements is

$$\widetilde{u}_{pC}(Q)=\frac{u_{pC}(Q)}{Q}=\frac{Q\sqrt{\widetilde{u}_{vC}^2+\widetilde{u}_{iC}^2+ctg\varphi u_{\varphi C}^2}}{Q}= \\ =\sqrt{\widetilde{u}_{vC}^2+\widetilde{u}_{iC}^2+ctg\varphi u_{\varphi C}^2} \quad (12)$$

Taking into consideration expansion coefficient k_e, **absolute extended uncertainty** of uncorrelated indirect measurements is expressed by the following dependence:

$$U_{pR}(Q)=k_e\cdot u_{pC}(Q) \quad (13).$$

Using expression (44) [3], it is possible to determine **relative extended uncertainty** $\widetilde{U}_{pE}(Q)$, in the same manner as in expressions (10) and (43) [3].

One-phase receiver reactive power can be expressed by the following formula:

$$S=UI \quad (14),$$

where U, I – root-mean-square voltage and current. The value of a measurement error can be expressed by the following formula:

$$\Delta S=\left|\frac{\partial S}{\partial U}\Delta U\right|+\left|\frac{\partial S}{\partial I}\Delta I\right|= \\ =\left|I\cdot\Delta U\right|+\left|U\cdot\Delta I\right| \quad (15)$$

In turn, the relative error is described by the following formula:

$$\delta S=\frac{\Delta S}{S}=\left|\frac{1}{U}\Delta U\right|+\left|\frac{1}{\partial I}\Delta I\right|= \\ =\left|\delta_U\right|+\left|\delta_I\right| \quad (16)$$

Reactive power measurement uncertainty can be expressed by the following formula:

$$u_{pC}(S)=\sqrt{\left(\frac{\partial S}{\partial U}\right)^2u_{vC}^2+\left(\frac{\partial S}{\partial I}\right)^2u_{iC}^2}= \\ =UI\sqrt{\left(\frac{1}{U}\right)^2u_{vC}^2+\left(\frac{1}{I}\right)^2u_{iC}^2}= \\ =S\sqrt{\widetilde{u}_{vC}^2+\widetilde{u}_{iC}^2} \quad (17)$$

where u_{vC}, u_{iC} – standard complex uncertainties of current (31) and voltage (35) measurements [3] in a system with transducers, $\frac{\partial S}{\partial U}$, $\frac{\partial S}{\partial I}$ – sensitivity coefficients.

Relative complex standard uncertainty of uncorrelated indirect measurements is

$$\widetilde{u}_{pC}(S)=\frac{u_{pC}(S)}{S}=\frac{S\sqrt{\widetilde{u}_{vC}^2+\widetilde{u}_{iC}^2}}{S}=\sqrt{\widetilde{u}_{vC}^2+\widetilde{u}_{iC}^2} \quad (18)$$

Taking into consideration expansion coefficient k_e, **absolute extended uncertainty** of uncorrelated indirect measurements is expressed by the following dependence:

$$U_{pR}(S)=k_e \cdot u_{pC}(S) \quad (19).$$

Using expression (44) [3], it is possible to determine **relative extended uncertainty** $\widetilde{U}_{pE}(S)$, in the same manner as in expressions (10) and (43) [3].

Three-phase supply circuits can be provided with four-conductor (star-shaped connection of transformer windings) or three-conductor systems (triangle or star-shaped connection of transformer windings). Welding machines are usually powered by a four-conductor network (with a neutral conductor) + and an earth-wire PE.

If a welding machine is powered by a three-phase power grid, individual resultant powers are expressed by the following formulas:

$$P_{3f}=P_A+P_B+P_C \quad (20),$$

$$Q_{3f}=Q_A+Q_B+Q_C \quad (21),$$

$$S_{3f}=\sqrt{P_{3f}^2+Q_{3f}^2} \quad (22)$$

Maximum absolute resultant errors of each power measurement will be the sums of modules of absolute errors. In the case of asymmetric load, it is possible to use formulas (20)-(22), thus to use current and three voltage (phase) transducers. The remaining electric quantities can be determined using appropriate geometric sums. Applying Blondel's principle [9] to a three-conductor system leads to the formation of a simplified measurement system (so-called Aron system) requiring the use of only two current transducers and two voltage (linear) transducers:

$$P_{3f}=P_1+P_2 \quad (23).$$

In the case of symmetric load (complex impedances of a receiver satisfy the condition $\underline{Z_A}=\underline{Z_B}=\underline{Z_C}$), individual phase power components are equal ($P_1=P_A=P_B=P_C$, $Q_1=Q_A=Q_B=Q_C$), leading to the obtainment of the following:

$$P_{3f}=3P_1 \quad (24),$$

$$Q_{3f}=3Q_1 \quad (25),$$

$$S_{3f}=3S_1=3\sqrt{P_1^2+Q_1^2} \quad (26)$$

In such a case it is sufficient to use only one current transducer and one voltage transducer. Absolute systematic errors will be then triple multiples of measurement errors related to each power.

It should also be noted that the measurement errors and uncertainties determined by formulas presented before should be increased by errors of numerical operation roundings performed in accordance with specific computational algorithms applied. However, these values are negligibly low [10], e.g. 10^{-6}, in relation to amplitudes of sinusoidal and weakly deformed waveforms and as such were omitted in these deliberations.

Frequency Measurement Error

In welding equipment there are periodical waveforms of changes of physical waveforms of various frequencies. They usually include changes of the following quantities:

1. electric:
   a) waveforms of current and voltage in circuits of devices powered from power grids (AC, 50 Hz);
   b) current and voltage waveforms in electronic power-carrying circuits and rectifier control circuits (including frequency processing), generators, choppers, inductors for induction and ultrasonic welding;
   c) current and voltage waveforms in circuits with electric arcs (AC MMA, MIG/MAG (0,5-200 Hz) and TIG (0,5-300Hz) welding);
2. mechanical:
   a) vibration of inductors, concentrators and sonotrodes for ultrasonic welding (above 20 kHz);
   b) vibration of electrodes for CMT welding (up to 130 Hz)
   c) vibration of welding robot arms (several Hz).

Frequency measurements related to the vibration of mechanical systems require appropriate

sensors [7] changing them into electric signals. Due to frequent deformations of these waveforms, the range of measured frequencies can be very wide. In practice, digital measurements of frequency can be performed using two alternative methods [11]:
- direct, i.e. by counting the number of periods on a standard time section, e.g. within 1 s;
- indirect, i.e. by measuring period T_x of a measured signal and determining measured frequency as the inverse of the period:

$$f_x = \frac{1}{T_x} \tag{27}$$

In the first method, a tested waveform of frequency f_x is transformed (in a forming system) into a pulsed waveform of the same frequency. A generator of the standard time section generates an impulse of duration T_w, opening gate for a time of measurement. When the gate is open, the counter counts $N_x = T_w/T_x$ impulses. As $f_x = 1/T_x$, measured frequency is

$$f_x = N_x \frac{1}{T_w} \tag{28}$$

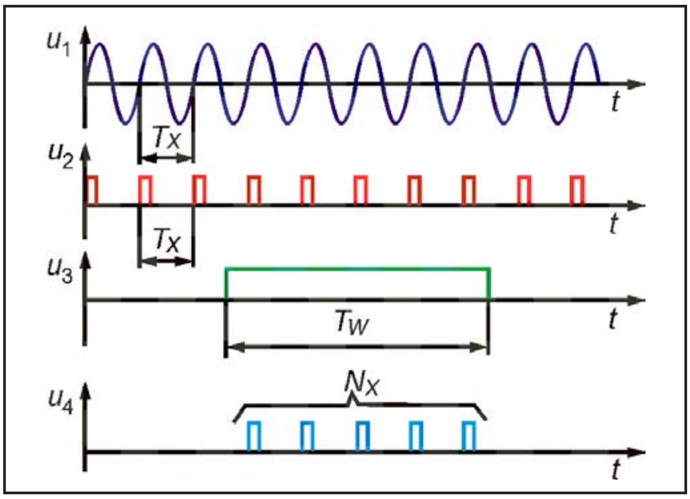

Fig. 1. Waveforms of signals in a system for frequency measurements using the direct method

In the second method, when the gate is open, a time is determined on the basis of the period of a measured signal; the counter counts impulses coming from a standard generator of frequency f_w. Therefore, the result of a period measurement is the following:

$$T_x = N_x \cdot T_w = \frac{N_x}{f_w} \tag{29}$$

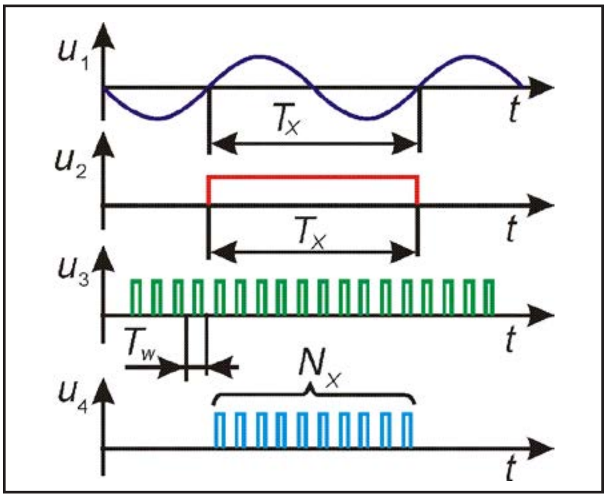

Fig. 2. Waveforms of signals in a system for frequency measurements using the indirect method

Frequency measurements are usually burdened with a certain error resulting from the following:

a) counting (quantisation) error occurring when the duration of strobing impulses is not the entire multiple of the counted periods;

b) standard generator frequency error – as this frequency may vary from nominal frequency, and, also, can change during operation;

c) strobing error, resulting from differences in delays between leading and trailing slopes of a strobing impulse; this error has a random character.

Quartz generators used presently on frequency meters and time measurers are characterised by the following parameters:
- $\delta_{Tw} \leq 2{,}5\times10^{-6}$ – ordinary quartz resonators operating at room temperature of 0-50°C,
- $\delta_{Tw} \leq 7\times10^{-9}$ – thermally compensated resonators placed in thermostats.

Usually, standard generator frequency error and strobing errors are negligibly small in comparison with the counting error [12].

The general form of a formula expressing relative errors of digital indirect measurements is the following:

$$\delta_{fx} = \frac{\Delta f_x}{f_x} = \pm\left(\left|\frac{\Delta f_w}{f_w}\right| + \left|\frac{1}{N_x}\right| + |\delta_b|\right) = \pm\left(|\delta_w| + |\delta_b| + \left|\frac{1}{T_p \cdot f_x}\right|\right) \tag{30}$$

where f_x – measured frequency, f_w – frequency of the standard (oscillator), Δf_w – absolute error of standard generator frequency, δ_w – relative error of standard generator frequency, $1/N_x = \delta_d$ – relative error of signal discretisation (quantisation), δ_b – relative strobing error, T_p – time of measurement. Resultant frequency measurement errors can also be minimised by increasing the time of measurement, assuming at the same time that the frequency of the tested time waveform does not change in time.

The manufacturer of the NI DAC PXI 6259 measurement card suggests frequency measurements based on one of three available methods [12]. Individual methods, intended for low or high frequency measurements, rely on one or two built-in counters. The selection of an appropriate measurement method depends on several factors such as expected the frequency value, expected measurement uncertainty, number of card internal counters used or measurement time. A selected method consists in the measurement of the period (one or several) duration on the basis of counter indications and a known card oscillator frequency in accordance with the following expression:

$$f_x = \frac{k \cdot f_w}{\sum_{i=1}^{k} N_{Ki}} \tag{31}$$

where k – number of periods, N_{Ki} – number of impulses counted in an i^{th} period. This method uses one counter and has been developed to measure relatively low frequencies. For instance, on the basis of a measurement of one signal period having a frequency of 50 kHz, the measurement relative error amounts to 0.06%. In turn, a measurement of 5 MHz is encumbered with an error of 6.67% [12]. In both cases, the standard frequency of a time base clock amounts to 80 MHz. Greater accuracy can be obtained by averaging results through measurements of a greater number of periods. The absolute error of the method has been expressed by the manufacturer of the card in the following formula:

$$\Delta f_m = \left(f_w \frac{f_x}{f_w - f_x} \right) - f_x \tag{32}$$

In fact, the frequency of an internal card oscillator (according to a catalogue note), is encumbered with a relative error δ_w = 50 ppm. The total relative error of a frequency measurement is the following:

$$\delta_{fc} = \delta_{fm} + \delta_w \qquad (33),$$

where δ_{fm} – relative error of the frequency measurement method.

Phase Angle Measurement Error

In experimental tests of welding equipment, phase angle measurements are used, among others, for the following:

- measurements of power and energy,
- measurements of impedance parameters (e.g. choking coils, inductors),
- identification of dynamic object;
- diagnosing of power supply systems;
- distance measurements.

Common equipment-based methods measuring phase angles of vibration are overly slow and enable the obtainment of a result not more often than two times per the period of measurement signals. However, due to the development of digital measurement technique, it is possible to measure phase angles in an algorithmic manner, enabling continuous tracking of signal phase variability.

Phase angles can be measured directly or using the comparison [11]. Results are expressed in degrees or radians. In the range of hertz fractions up to tens of megahertz, the most accurate measurement results are obtained using digital phase meters.

The operational basis of digital phase meters of all types is the principle of transformation of a measured phase angle into a proportional time interval. The duration of such an interval is determined directly, i.e. by counting discrete signals emitted by a standard generator or indirectly, i.e. by transforming it into the proportional value of voltage or current. Phase meters with the direct transformation of time interval duration into a code can be divided into the following:

1. phase meters with an input voltage change within one period;
2. phase meters with an input voltage change within several periods.

The phase metres mentioned first are used for measurements of angle momentary values, whereas the phase metres of the second group – for measurements of average values. The most popular phase metres are those belonging to the second group (of a constant measurement time), characterised by very good metrological properties.

The value of an angle measured using a phase meter of momentary values is determined by the formula following formula:

$$\varphi_x = \frac{t_x}{T_x} 360^o = \frac{n_x T_w}{N_x T_w} 360^o = \frac{n_x}{N_x} 360^o \qquad (34)$$

where t_x – time interval duration proportional to phase angle of signals; n_x – number of standard signal impulses in interval t_x; T_x – period of measured signal; N_x – number of impulses in a period of measured signal; T_w – period of standard signal.

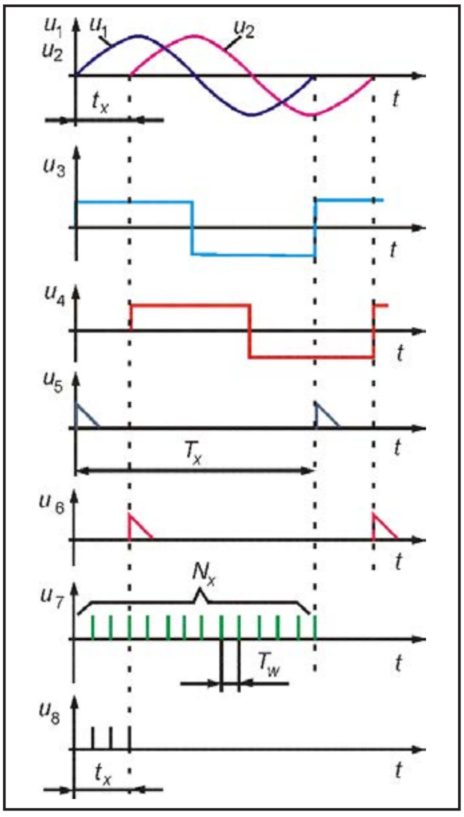

Fig. 3. Waveforms of signals in the system measuring the momentary value of a phase angle

Relative components of phase angle measurement errors are the following:

- counting error ($\delta_z = 1/n_x$),
- error resulting from standard frequency instability (δ_g),
- strobing error (δ_b),
- error resulting from different signal delay in both channels (δ_p),
- error resulting from measured signal frequency instability (δ_s),
- error determined by deformations of a tested waveform (δ_{zn}).

The above situation can be expressed by the following formula:

$$\delta\varphi_x = \delta_z + \delta_g + \delta_b + \delta_p + \delta_s + \delta_{zn} \qquad (35).$$

In a phase meter, values of average time waveforms of two signals are transformed into periodic short impulses released when they intersect with a reference level while signals increase. The phase angle of signals corresponds to the shift of signals by the time interval

$$t_x = \frac{\varphi_x}{360^o} T_x \qquad (36)$$

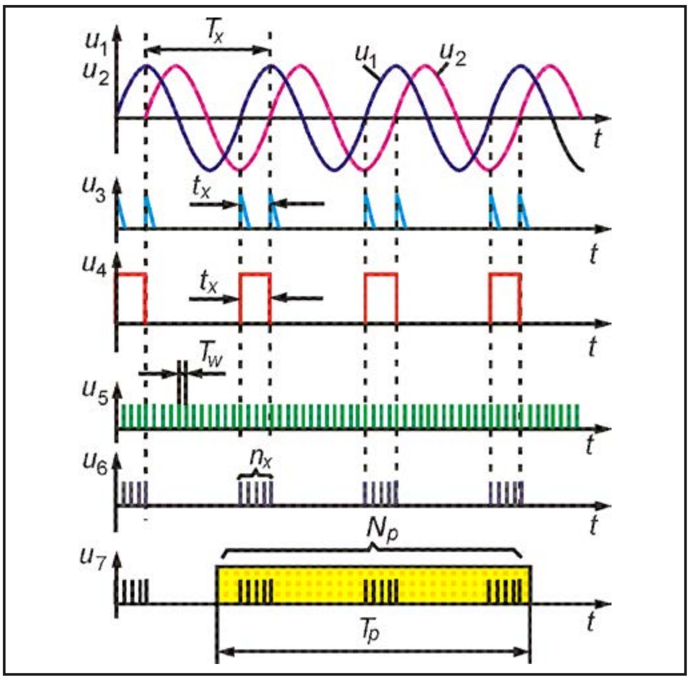

Fig. 4. Waveforms of signals in the system measuring the average value of a phase angle

where T_x – period of tested signals equal to the period of a couple of short impulses. At the time interval there is the transmission of packages composed of n_x impulses generated by a standard generator operating at high frequency $f_w = 1/T_w$. Using the condition $t_x >> T_w$, the following formula is obtained:

$$n_x = \frac{t_x}{T_w} = \frac{\varphi_x}{360^o} \frac{T_x}{T_w} \tag{37}$$

If measurement time T_p is sufficiently long ($T_p > T_g$), the number of measurement impulses from received packages is expressed by the following formula:

$$N_p = n_x \frac{T_p}{T_x} = \frac{\varphi_x}{360^o} \frac{T_p}{T_w} \tag{38}$$

where T_g – period of the lowest frequency signal tested using a phase meter. The value of an angle measured using a phase meter of average values is expressed by the following formula:

$$\varphi_x = 360^o N_p \frac{T_w}{T_p} = 360^o N_p \frac{1}{T_p f_w} \tag{39}$$

where N_p – number of standard generator impulses contained in packages transmitted in time T_p. A measurement error is the lower, the greater number of impulses n_x is contained in each package and the longer the cycle of each measurement is ($T_p >> T_g$).

A discretisation error of a phase meter of average values has two components caused by the following:

1. limited number of impulses n_x in each package;
2. limited number of impulse packages per interval T_p.

Changes of input signal frequency are accompanied by changes of discretisation errors in opposite directions. An increase in frequency is accompanied by a decrease in the number of impulses n_x in each package, and by an increase in the number of packages. Within the frequency range of 20 Hz to 1 MHz, the measurement error of a digital phase meter is usually restricted within the range of 0.1° to 0.5°.

Measurement Error of Current Rectangular Waveform Filling Coefficient

TIG welding processes often utilise rectangular-shaped current excitations [13]. Figure 5 presents examples of such waveforms.

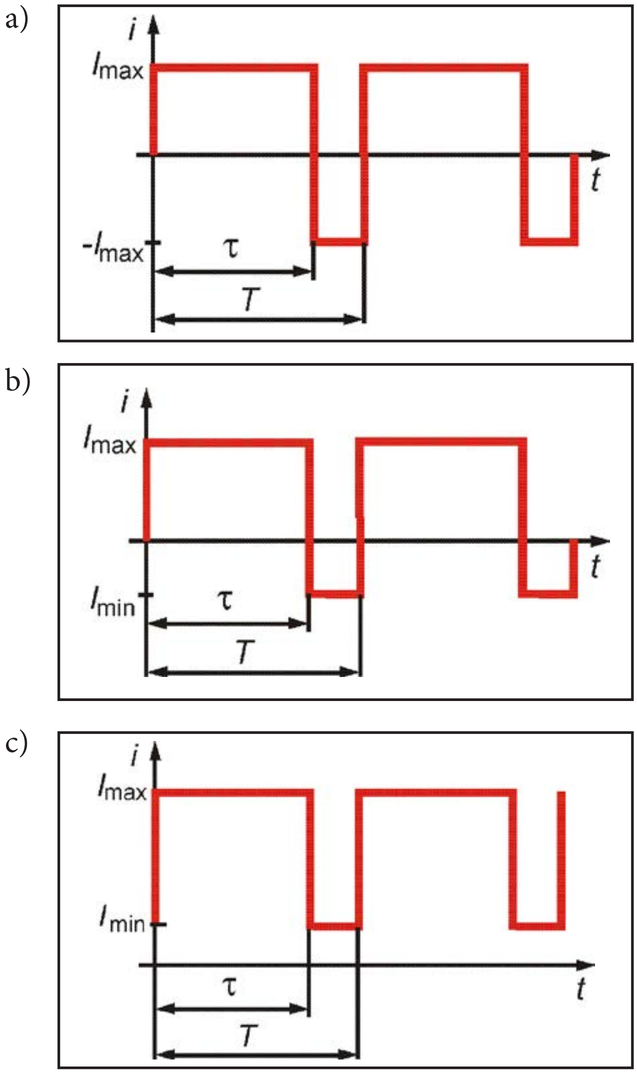

Fig. 5. Rectangular current excitation shapes: a) bipolar symmetric; b) bipolar asymmetric; c) unipolar

In welding machines, the frequency of current waveforms in a circuit with arc changes within the several hertz to approximately 500 Hz. Lower frequencies correspond to wide, whereas higher frequencies to narrow welds. The impulse filling coefficient expressed by the following formula:

$$k_w = \frac{\tau}{T} \tag{40}$$

changes in the range of 0.3 to 0.7. In this manner, it is possible to modify the average value of voltage and heat distribution between the electrode and the weld. The choice depends on the type of workpiece materials (AC – aluminium, unipolar waveform with a constant component – so-called "pulse" – stainless steel, etc.).

If the prevalence of the positive potential is on the electrode side, a tungsten rod is heated more intensively and its wear is faster, the penetration of elements is shallower and wider and oxides are removed from the surface of an aluminium element by an active cathode spot. If the prevalence of the positive potential is on the workpiece side, the workpiece is heated more intensively and deep and narrow penetration is obtained.

As the value of the filling coefficient significantly affects the quality of the technological process, its experimental determination in the analysis of welding machine control system functioning structure is important.

In the case of <u>bipolar symmetric</u> current excitation (Fig. 5a), the value of filing coefficient k_w is expressed by the following formula:

$$k_w = 0{,}5\left(\frac{I_{sr}}{I_{max}} + 1\right) \tag{41}$$

In order to verify if welding source settings are correct, it is necessary to measure the average and maximum values of current waveform. The value of the maximum absolute measurement error related to coefficient k_w can be determined using the following formula:

$$\Delta k_w = \left|\frac{\partial k_w}{\partial I_{sr}}\Delta I_{sr}\right| + \left|\frac{\partial k_w}{\partial I_{max}}\Delta I_{max}\right| = \left|\frac{0{,}5}{I_{max}}\Delta I_{sr}\right| + \left|-\frac{0{,}5 I_{sr}}{I_{max}^2}\Delta I_{max}\right| \tag{42}$$

If the additivity of errors is assumed, $\Delta I_{sr} = \Delta I_{max}$. In turn, the relative error is expressed by the following formula:

$$\delta k_w = \frac{\Delta k_w}{k_w} = \frac{I_{sr}}{I_{sr} + I_{max}}\left(|\delta I_{sr}| + |\delta I_{max}|\right) \tag{43}$$

where $\delta I_{sr} = \Delta I_{sr}/I_{sr}$, $\delta I_{max} = \Delta I_{max}/I_{max}$.

If a welding power source generates the bipolar asymmetric waveform of current (Fig. 5b), the value of filling coefficient is expressed by the following formula:

$$k_w = \frac{|I_{min}| + I_{sr}}{|I_{min}| + I_{max}} \tag{44}$$

where I_{max} – maximum value, I_{min} – minimum value, I_{sr} – average value of waveform.

In order to verify if welding source settings are correct, it is necessary to measure the average, minimum and maximum values of current waveform. The value of the absolute measurement error related to coefficient k_w can be determined using the following formula:

$$\Delta k_w = \left|\frac{\partial k_w}{\partial I_{sr}}\Delta I_{sr}\right| + \left|\frac{\partial k_w}{\partial I_{min}}\Delta I_{min}\right| + \left|\frac{\partial k_w}{\partial I_{max}}\Delta I_{max}\right| = \tag{45}$$
$$= \frac{\left|\Delta I_{max}\left(I_{sr} + |I_{min}|\right)\right| + \left|\Delta I_{min}\,\mathrm{sgn}(I_{min})\cdot(I_{sr} - I_{max})\right| + \left|I_{max} + |I_{min}|\right|\cdot|\Delta I_{sr}|}{\left(I_{max} + |I_{min}|\right)^2}$$

If the additivity of errors is assumed, $\Delta I_{sr}=\Delta I_{min}=$ $=\Delta I_{max}$. In turn, the relative error is expressed by the following formula:

$$\delta k_w = \frac{\Delta k_w}{k_w} = \left|\frac{I_{sr}}{|I_{min}| + I_{sr}}\delta I_{sr}\right| + \left|\frac{(I_{max} - I_{sr})|I_{min}|}{(|I_{min}| + I_{max})(|I_{min}| + I_{sr})}\delta I_{min}\right| +$$
$$+ \left|\frac{I_{max}}{|I_{min}| + I_{max}}\delta I_{max}\right| \tag{46}$$

where $\delta I_{sr}=\Delta I_{sr}/I_{sr}$, $\delta I_{min}=\Delta I_{min}/I_{min}$, $\delta I_{max}=\Delta I_{max}/I_{max}$.

If a welding power source generates <u>rectangular unipolar</u> waveform (Fig. 5c), it corresponds to a filling coefficient expressed by the following formula:

$$k_w = \frac{I_{sr} - I_{min}}{I_{max} - I_{min}} \tag{47}$$

In order to verify if welding source settings are correct, it is necessary to measure the average, minimum and maximum values of current waveform. The value of the absolute measurement error related to coefficient k_w can be determined using the following formula:

$$\Delta k_w = \left|\frac{\partial k_w}{\partial I_{sr}}\Delta I_{sr}\right| + \left|\frac{\partial k_w}{\partial I_{min}}\Delta I_{min}\right| + \left|\frac{\partial k_w}{\partial I_{max}}\Delta I_{max}\right| =$$
$$= \frac{\left|\Delta I_{min}(I_{sr} - I_{max})\right| + \left|\Delta I_{max}(I_{sr} - I_{min})\right| + |I_{min} - I_{max}|\cdot|\Delta I_{sr}|}{(I_{min} - I_{max})^2} \tag{48}$$

If the additivity of errors is assumed, $\Delta I_{sr}=\Delta I_{min}=$ $=\Delta I_{max}$. In turn, the relative error is expressed by the following formula:

$$\delta k_w = \frac{\Delta k_w}{k_w} = \left| \frac{I_{sr}}{I_{sr} - I_{\min}} \delta I_{sr} \right| + \left| \frac{(I_{sr} - I_{\max}) I_{\min}}{(I_{\max} - I_{\min})(I_{sr} - I_{\min})} \delta I_{\min} \right| + \left| \frac{I_{\max}}{I_{\max} - I_{\min}} \delta I_{\max} \right| \quad (49)$$

where $\delta I_{sr}=\Delta I_{sr}/I_{sr}$, $\delta I_{min}=\Delta I_{min}/I_{min}$, $\delta I_{max}=\Delta I_{max}/I_{max}$.

In formulas (41)-(49), the high steepness of leading and trailing edges of current impulses $|di/dt| \to \infty$ has been assumed. The efficient application of these formulas depends on the degree at which this condition has been satisfied. Usually, the steepness decreases along with increasing current waveform frequency and is difficult to obtain in cheap and simple welding power sources.

In addition to sinusoidal and rectangular waveforms, arc welding can also be based on current of trapezoidal [14], triangular or other waveforms. Due to comfort, it is advisable to perform quantitative assessments of variously shaped waveforms using spectrum analysers [15].

Errors and Uncertainness related to the Determination of Welding Power Source Efficiency

Among electrotechnological equipment, welding machines belong to a group of medium power machines, where the power of machines for manual welding is usually lower than that of robotic welding machines. This fact is not only related to dimensions and weight (as these are becoming increasingly smaller), but also to user safety. As regards the assessment of welding machine quality, consumed electric power and power efficiency are of secondary importance. Significantly more important are static and dynamic characteristics of welding power sources as well as shapes and frequencies of generated current (sometimes also voltage) waveforms. However, as regards the energy balance of a whole selected industrial plant, important aspects also include the time, number and power of welding machines used at the same time, as well as various detrimental effects of machines on company power networks.

Characteristics of welding machines include very wide ranges of changes in relation to settings of root-mean-square and average (frequency and shape) current values, which could be recognised as nominal. The value of voltage (arc length) limited to the safety of users is responsible for the fact that changes of root-mean-square welding current are almost proportionally accompanied by changes of power absorbed from the power network. In comparison with older, i.e. rotating machinery-based sources, today's welding power sources are characterised by high power efficiency within wide ranges of changes related to root-mean-square load current.

In turn, the evaluation of power efficiency of welding sources can be useful when performing the qualitative assessment of control systems and in the diagnostics of welding machines. A significant decrease in the expected efficiency can imply the existence of damaged internal power-carrying or electronic circuits. Relatively easy repeatability and explicitness of measurement results as well as a wide range of experimentation can be obtained in tests of arcless welding machines.

The tests were performed using a Esab Origo™ Tig 3000i ac/dc welding power source powered from a 230/400 V three-phase network burdened with a constant resistance of 0.3 Ω. The analysis presented below involved pre-set dc excitation of I_2=75A, with mma operational regime. According to the manufacturer, the efficiency with the maximum current amounts to 76%.

Table 3 presents the budget of errors of direct current and voltage measurement channels on the welding power source load side. The tests involved the use of lem transducers with Hall effect sensors, described in publication [3]. The sampling frequency setting in a computer measurement system amounted to 20 kHz, whereas

Table 3. Budget of current and voltage measurement channel errors on the load side

Indirect current measurement – on the load side – LB200-S/S4 transducer (δ_i=0.64%)

Input quantity		**Maximum limiting error Δ_g**		**Relative error δ_g**	
K_I, 1	1/1000	ΔK_I, 1 (13 [3])	$6.4 \cdot 10^{-6}$	δK_I	$6.4 \cdot 10^{-3}$
R_{MA}, Ω	27	ΔR_{MA}, Ω (16 [3])	$45.225 \cdot 10^{-3}$	δR_{MA}	$1.675 \cdot 10^{-3}$
U_{MA}, V	2.001	ΔU_{MA}, V (26 [3])	$2.220 \cdot 10^{-3}$	δU_{MA}	$1.109 \cdot 10^{-3}$
Output quantity		**Maximum indirect measurement error (45 [3])**		**Maximum indirect measurement relative error (46 [3])**	
I_2, A	74.12	ΔI_2, A	0.681	δI_1	$0.918 \cdot 10^{-2}$

Indirect voltage measurement – on the load side – LV25-P

Input quantity		**Maximum limiting error Δ_g**		**Relative error δ_g**	
K_U, 1	2500/1000	ΔK_U, 1	$6.500 \cdot 10^{-2}$	δK_U	$2.600 \cdot 10^{-2}$
R_{MV}, Ω	200	ΔR_{MV}, Ω	$3.350 \cdot 10^{-1}$	δR_{MV}	$1.675 \cdot 10^{-3}$
R_{p2}, Ω	250	ΔR_{p2}, Ω	$4.188 \cdot 10^{-1}$	δR_{p2}	
R_{v2}, Ω	6.3·103	ΔR_{v2}, Ω	10.553	δR_{v2}	
R_{vp2}, Ω	6.55·104	ΔR_{vp2}, Ω	10.971	δR_{vp2}	
U_{MV}, V	1.66	ΔU_{MV}, Ω	$2.220 \cdot 10^{-3}$	δU_{MV}	$1.338 \cdot 10^{-3}$
Output quantity		**Maximum indirect measurement error** (47 [3])		**Maximum indirect measurement relative error** (48 [3])	
U_2, V	21.743	ΔU_2, V	0.667	δU_2	$3.069 \cdot 10^{-2}$

the number of collected samples N>100 000. Such a large number of samples eliminate the random error effect. Three ferrodynamic analogue wattmeters were used on the supply side from a network having a voltage of 400 V. The absolute error maximum value is expressed by the following formula:

$$\Delta_{\max} = \frac{k_l}{100\%} Z_p \qquad (50)$$

where k_1 – class of device; Z_p – range of measurement. Table 4 presents the budget of power measurement errors on the supply and load side calculated using the total differential method.

The efficiency of a tested welding source is determined by the following formula and amounts to

$$\eta = \frac{P_2}{P_1} = \frac{1612\text{ W}}{2132\text{W}} = 0{,}756;\ (75{,}6\%) \qquad (51)$$

This value is very close to the catalogue nominal value specified for the machine (76%) during operation with maximum current of 300 A. The maximum absolute error related to efficiency determination using the total differential method can be calculated using the following formula:

$$\Delta\eta = \left|\frac{\partial\eta}{\partial P_1}\Delta P_1\right| + \left|\frac{\partial\eta}{\partial P_2}\Delta P_2\right| = \frac{|P_1 \cdot \Delta P_2| + |P_2 \cdot \Delta P_1|}{P_1^2} = 0{,}035 \qquad (52)$$

In turn, the relative error correlated with the determination of efficiency is expressed by the following formula:

$$\delta\eta = \frac{\Delta\eta}{\eta} = 0{,}046;\ (4{,}64\%) \qquad (53)$$

In accordance with existing guidelines, measurement results should be presented taking uncertainty into consideration. Table 5 presents the budget of indirect current measurement uncertainties on the load side, whereas Table 6 presents the budget of indirect voltage measurement uncertainties on the load side.

In order to simplify calculations of DC power measurement uncertainty, the deliberations

Table 4. Budget of power measurement errors on the supply and load side

Direct power measurement – on the supply side – ferrodynamic wattmeters class 0.5

Input quantity		**Maximum limiting error Δ_g**		**Relative error δ_g**	
P_{11}, W	710 (range 1000W)	ΔP_{11}, W	5	δP_{11}	$7.042\cdot10^{-3}$
P_{12}, W	672 (range 800W)	ΔP_{12}, W	4	δP_{12}	$5.952\cdot10^{-3}$
P_{13}, W	750 (range 1000W)	ΔP_{13}, W	5	δP_{13}	$6.667\cdot10^{-3}$
Output quantity $P_1=\Sigma P_i$		**Maximum direct measurement error $\Delta P_1=\Sigma\Delta P_i$**		**Maximum direct measurement relative error**	
P_1, W	2132	ΔP_1, W	14	δP_1	$6.567\cdot10^{-3}$

Indirect power measurement – on the load side – (LV25-P + LB200-S/S4 transducers)

Input quantity		**Maximum limiting error Δ_g**		**Relative error δ_g**	
I_2, A	74.12	ΔI_2, A	0.681	δI_1	$0.918\cdot10^{-2}$
U_2, V	21.743	ΔU_2, V	0.667	δU_2	$3.069\cdot10^{-2}$
Output quantity		**Maximum indirect measurement error** (49 [3])		**Maximum indirect measurement relative error** (50 [3])	
P_2, W	1612	ΔP_2, W	64.257	δP_2	0.04

Table 5. Budget of indirect current measurement uncertainties on the load side

Parameter		**Parameter value estimate**	**Unit**	**Type of uncertainty**	**Distribution**	**Value of standard uncertainty (total)**	**Sensitivity coefficient**	**Relative content of uncertainty (%)**
Input quantity	K_I	1/1000	1	B	Rectangular $\sqrt{3}$	$3.695\cdot10^{-6}$	$-7.411\cdot10^{4}$ (c_1)	0.013
	R_{MA}	27	Ω			$2.611\cdot10^{-2}$	-2.745 (c_2)	95.308
	U_{MA}	2.001	V			$1.282\cdot10^{-3}$	3.704·101 (c_3)	4.678
Current measurement complex and extended uncertainty in a system with a LB200-S/S4 transducer								
Output quantity	I_2	74.12	A	**complex uncertainty** (31 [3])		0.287	sensitivity coefficients c_1, c_2, c_3 (Table 3 [3])	
				extended uncertainty k_e=2 (p=95%) (33 [3])		0.574		

assumed the lack of correlation between current and voltage measurements. Such an approach is frequent when determining type B uncertainty on the basis of systematic error of machines.

Complex absolute standard uncertainty correlated with the value of efficiency quantity is expressed by the following formula:

$$u_C(\eta)=\sqrt{\left(\frac{\partial\eta}{\partial P_1}\right)^2\cdot u_{P1C}^2+\left(\frac{\partial\eta}{\partial P_2}\right)^2\cdot u_{P2C}^2}=\frac{1}{P_1^2}\sqrt{P_1^2\cdot u_{P2C}^2+P_2^2\cdot u_{P1C}^2}=$$
$$=\frac{1}{P_1^2}\sqrt{P_1^2P_2^2\left(\frac{u_{P2C}^2}{P_2^2}+\frac{u_{P1C}^2}{P_1^2}\right)}=\eta\sqrt{\left(\frac{u_{P2C}^2}{P_2^2}+\frac{u_{P1C}^2}{P_1^2}\right)}=\eta\sqrt{\tilde{u}_{P2C}^2+\tilde{u}_{P1C}^2} \quad (54)$$

Relative complex standard uncertainty of uncorrelated indirect measurements related to the determination efficiency is expressed by the following formula:

$$\widetilde{u}_C(\eta) = \frac{u_C(\eta)}{\eta} = \sqrt{\widetilde{u}^2_{P2C} + \widetilde{u}^2_{P1C}} \qquad (55)$$

Taking into consideration expansion coefficient k_e, the **absolute extended uncertainty** of uncorrelated indirect measurements related to the determination efficiency is expressed by the following formula:

$$U_R(\eta) = k_e \cdot u_C(\eta) \qquad (56).$$

Table 6. Budget of indirect voltage measurement uncertainties on the load side

Parameter		Parameter value estimate	Unit	Type of uncertainty	Distribution	Value of standard uncertainty (total)	Sensitivity coefficient	Relative content of uncertainty (%)
Input quantity	K_U	2500/1000	1	B	Rectangular $\sqrt{3}$	$3.753 \cdot 10^{-2}$	-8.698 (c_1)	0.571
	R_{MV}	200	Ω			$1.934 \cdot 10^{-1}$	-0.109 (c_2)	2.945
	R_{p2}	250	Ω			$2.418 \cdot 10^{-1}$	0.003 (c_3)	3.684
	R_{v2}	6.3·103	Ω			$6.093 \cdot 10^{1}$	0.003 (c_4)	92.78
	U_{MV}	1.66	V			$1.282 \cdot 10^{-3}$	13.1 (c_5)	0.02
Current measurement complex and extended uncertainty in a system with a LB200-S/S4 transducer								
Output quantity	U_2	21.743	V	**complex uncertainty** (35 [3])		0.329	sensitivity coefficients c_1, c_2, c_3, c_4, c_5 (Table 4 [3])	
				extended uncertainty k_e=2 (p=95%) (40 [3])		0.658		

Table 7. Budget of power measurement uncertainties on the supply and load side

Parameter		Parameter value estimate	Unit	Type of uncertainty	Value of standard uncertainty (total)	Sensitivity coefficient	Relative content of uncertainty (%)
Direct power measurement – on the supply side – ferrodynamic wattmeters class 0.5							
Input quantity	P_{11}	710	W	B (3 [3]), rectangular distribution $\sqrt{3}$	2.887	1 (c_{11})	35.7
	P_{12}	672			2.309	1 (c_{12})	28.6
	P_{13}	750			2.887	1 (c_{13})	35.7
Power measurement complex and extended uncertainty – on the supply side							
Output quantity	P_1	2132	W	**standard uncertainty** (5 [3])	4.690	$c_{1i} = \frac{\partial P_1}{\partial P_{1i}}$	
				extended uncertainty k_e=2 (p=95%) (44 [3])	9.380		
Indirect power measurement – on the load side – (measuring transformers LV25-P + LB200-S/S4)							
Input quantity	I_2	74.12	A	complex I_2 (tab. 5)	0.287	21.743 (c_1)	60.1
	U_2	21.743	V	complex U_2 (tab. 6)	0.329	74.12 (c_2)	39.9
Power measurement complex and extended uncertainty – on the load side							
Output quantity	P_2	1612	W	**complex uncertainty** (42 [3])	25.171	$c_1 = \frac{\partial P_2}{\partial I_2} = U_2$; $c_2 = \frac{\partial P_2}{\partial U_2} = I_2$	

In the experimental tests conducted, the absolute complex standard uncertainty amounted to $u_C(\eta)$=1,201·10^{-2}, whereas absolute extended uncertainty $U_R(\eta)$ =2,402·10^{-2} (k_e=2).

Table 7 presents the budget of uncertainties of measurements of active power on the supply and load side of a welding power source.

Conclusions

1. In research practice of welding machines it is often possible to observe variable waveforms of voltage and current and related two-half straightened waveforms; measurements of root-mean-square values and of average momentary power (active power) value lead to non-zero values of systematic errors.

2. The methods of measurements and calculations of errors and of reactive power uncertainty described above are concerned only with current and voltage sinusoidal waveforms.

3. The efficiency of the method used for measuring the coefficient of filling of current rectangular waveforms depends on the shape of tested impulses.

4. Effective measurements of frequencies of strongly deformed waveforms (in relation to the sinusoid) require the use of a set of filters or analysers of the spectrum.

5. The measurement methods described above are not immune to strong disturbances present during technological tests of arc welding equipment. For this reason, in such cases arcless tests involving connected source-loading resistors or arc simulators should be preferred.

References:

[1] Вавуло И.В.: Имитатор сварочной дуги для настройки и исследования дуговых датчиков. Сварочное производство, 1984, № 1, с 33-34.

[2] Сидорец В.Н., Пентегов И.В., Кирилюк В.В., Тупицын С.И.: Имитатор сварочной дуги. Номер патента: 1600937, Опубликовано: 23.10.1990.

[3] Sawicki A., Haltof M.: Metrological problems of experimental research welding devices. Part 1. Errors and uncertainties in measuring current, voltage and power with application of hallotron sensors). Biuletyn Instytutu Spawalnictwa, 2015, no. 4. http://dx.doi.org/10.17729/ebis.2015.4/6

[4] Szabatin J.: Podstawy teorii sygnałów. WKiŁ, Warszawa, 2007.

[5] Karolewski B., Uracz P.: Obliczanie wartości skutecznych prądu ruszającego silnika indukcyjnego. Pomiary Automatyka Robotyka, 2005, no. 12, pp. 20-23.

[6] Chwaleba A., Czajewski J.: Przetworniki pomiarowe i defektoskopowe. OWPW, Warszawa, 1998.

[7] Zakrzewski J.: Czujniki i przetworniki pomiarowe. WPŚl, Gliwice, 2004.

[8] Godec Z., Banović M., Cindrić V.: Automated measurement uncertainty estimation – a new paradigm of measuring instruments and systems. Proceedings, XVII IMEKO World Congress, June 22-27, 2003, Dubrovnik, Croatia, pp. 507-512.

[9] Bielański K., Śreniawski J.: Laboratorium miernictwa elektrycznego: skrypt dla Wydziału Elektrycznego, Część 1. Politechnika Częstochowska. Instytut Elektroenergetyki, 1972.

[10] Swerlein R.L.: A 1 Oppm accurate digital AC measurement algorithm. August 09, 1991 www.agilent.com/find/products

[11] Stabrowski M.: Cyfrowe przyrządy pomiarowe. Wydawnictwo PWN, 2002.

[12] NI. Making Accurate Frequency Measurements. Publish Date: Feb 18, 2013.

[13] Dobaj E.: Maszyny i urządzenia spawalnicze, WNT, Warszawa, 1998.

[14] Dzelnitzki D.: Progress in MIG/MAG Welding With the Help of Modern Multi-Process Welding Power Sources. 2000 EWM HIGHTEC WELDING GmbH, pp. 1-9.

[15] Rauscher Ch.: Fundamentals of Spectrum Analysis. Rohde & Schwarz GmbH & Co. KG Mühldorfstrasse 15, 81671 München, Germany.