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Static Characteristics of Defined Ignition Voltage Used in the Modelling of Arc within a Wide Range of Current Excitation

Abstract: The article presents relatively extensive sets of selected functions useful for the approximation of voltage-current characteristics of static arc of defined and undefined ignition voltage values. The article contains graphs of the above-named functions presenting their effectiveness in the mathematical representation of arc within wide ranges of current excitation. Selected static characteristics were applied to the Pentegov mathematical model. The research also involved simulations of processes in circuits with models of arc having defined and undefined ignition voltage.

Keywords: electric arc, static characteristics, Pentegov model

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Introduction

Simulations of operational states of electrotechnological equipment and electric devices utilise relatively simple mathematical models of electric arc [1, 2]. Because of required thermal effects, particular attention is paid to characteristics of arc within the high-current range. The low-current range is important as regards the testing of the arc burning stability and effect on power sources. In the above-named models, the voltage of arc ignition is usually undefined, which can pose problems in tests of devices with extended arc or with arc subjected to various external effects [3, 4]. Another consequence is the deteriorated stability of numerical integration algorithms during the analysis of dynamic states in circuits with AC arc.

The value of arc ignition voltage depends on numerous factors including:

- distance between electrodes and electrode shapes;
- chemical composition of gas (concentration of metal vapours);
- gas ionisation degree (affected by temperature, pressure, activity of external sources of ionising radiation, time interval between arc termination and re-ignition);
- frequency of voltage applied to electrodes.

Publications [5, 6] investigate the properties of the modified Mayr model of low-current arc with defined ignition voltage, constituting a special case of the Pentegov mathematical model [7, 8], characterised not only by simplicity but also by high versatility. As a result, it is possible to simulate arc of nearly any static characteristics and, at the same time, satisfy the power balance equation.

This article presents modified mathematical models having specific arc ignition voltage

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and various shapes of characteristics within low-current and high-current ranges. The aforesaid models are characterised by high versatility and can be useful when representing dynamic properties of arc in welding equipment. The proper representation of physical arc characteristics can be applied in relation to the design and operation of arc simulators used in the diagnostics of welding power sources [9-11].

Quasi-static characteristics of arc with defined and undefined arc ignition voltage values

The modelling of low-current arc often involves the use of the static voltage-current characteristic proposed by Ayrton [12]. If the length of the arc is constant and if there is excitation triggered by direct current I, corresponding voltage

$$U_a = U_{AK} + U_{col} = U_{AK} + \frac{P_M}{I} \quad (1)$$

can be expressed in the form of the static conductance-current characteristic of the arc column

$$G_{col} = \frac{I}{U_{col}} = \frac{I^2}{P_M} \quad (2)$$

where U_{AK} – sum of near-electrode voltage drops, U_{col} – voltage on the arc column; P_M – constant value of dissipated power; G_{col} – arc column conductance. The shapes of the static characteristics of the above-named arc are presented in Figure 1.

As can be seen in the Figure 1, the passing of current through the value of zero corresponds to the undefined (theoretically infinite) value of voltage. However, in terms of AC arc, the value of the above-named voltage decreases because of the presence of the area of ionised plasma and the non-zero value of its residual conductance. The modified static characteristic of arc having the defined value of arc ignition voltage can be expressed by the following function

$$U_{col} = \frac{P_M I}{I^2 + P_M G_M} = \frac{P_M I}{I^2 + I_M^2} \quad (3)$$

also expressible in the following form

$$G_{col} = \frac{I}{U_{col}} = \frac{I^2 + I_M^2}{P_M} = \frac{I^2}{P_M} + G_M \quad (4)$$

where $G_M = I_M^2 / P_M$ – arc residual conductance. The coordinates of extreme $S(I, U)$ of characteristic (3) are the following:

$$I = I_M \qquad U = \frac{P_M}{2I_M} \quad (5)$$

The graphs of function (3) and (4) are presented in Figure 2.

Various variants of static characteristics with defined and undefined values of arc ignition voltage are presented in Table 1. Figures 1, 2 and 3a present static characteristics of arc (described by function A1 and A2). The aforesaid characteristics are useful for the modelling of electric

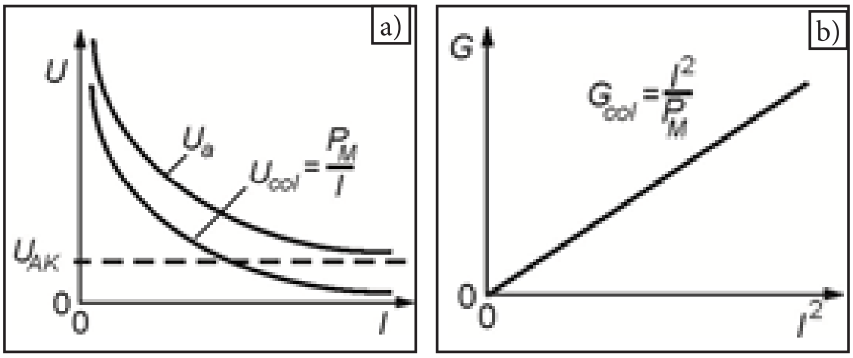

Fig. 1. Static characteristics of arc with the undefined value of arc ignition voltage: a) voltage-current characteristics $U_a(I)$ and $U_{col}(I)$ and b) conductance-current characteristic $G_{col}(I^2)$

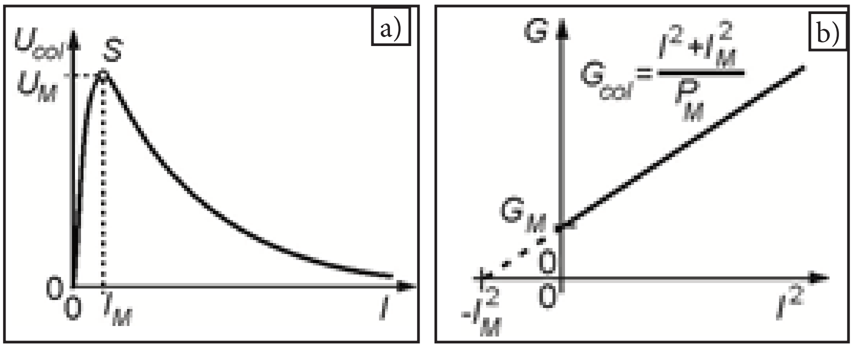

Fig. 2. Static characteristics of the arc column with the defined value of arc ignition voltage: a) static voltage-current characteristic $U_{col}(I)$; b) static conductance-current characteristic $G_{col}(I^2)$

Table. 1. Modified Ayrton functions used for the approximation of graphs presenting voltage changes in the arc column

Designation	Static characteristics with undefined arc ignition voltage $U_{col} =$	Designation	Static characteristics with defined arc ignition voltage $U_{col} =$
A1	$\frac{P_M}{I}$	A2	$\frac{P_M I}{I^2 + I_M^2}$
A3	$\frac{P_M}{I} + R_p I$	A4	$\frac{P_M I}{I^2 + I_M^2} + R_p I$
A5	$\frac{P_M}{I} + U_C$	A6	$\frac{P_M I}{I^2 + I_M^2} + U_C$
A7	$\frac{P_M}{I} + U_C + R_p I$	A8	$\frac{P_M I}{I^2 + I_M^2} + U_C + R_p I$

discharges within the low-current range. Functions designated as A3-A8 can be used for the modelling of arc in high-current ranges. The shapes of the above-named characteristics are presented in Figure 3b and 4.

In cases of functions designated as A1, A3, A5, A6, A7 and A8, the value of the residual conductance of arc amounts to zero. In turn, as regards the remaining functions, the value of conductance is higher than 0 S. Function A2 corresponds to conductance $G_{col}(I = 0\text{A}) = I_M^2 / P_M$, whereas function A4 corresponds to conductance $G_{col}(I = 0\text{A}) = I_M^2 / (P_M + R_p I_M^2)$.

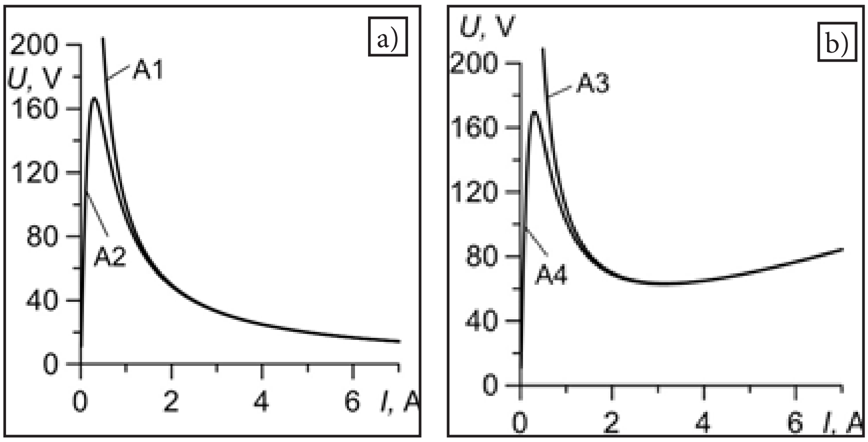

Fig. 3. Static voltage-current characteristics described using the modified Ayrton functions for the modelling of (P_M = 100 W, I_M = 0.3 A): a) low-current hyperbolic arc; b) hyperbolic-linear arc (R_p = 10 Ω)

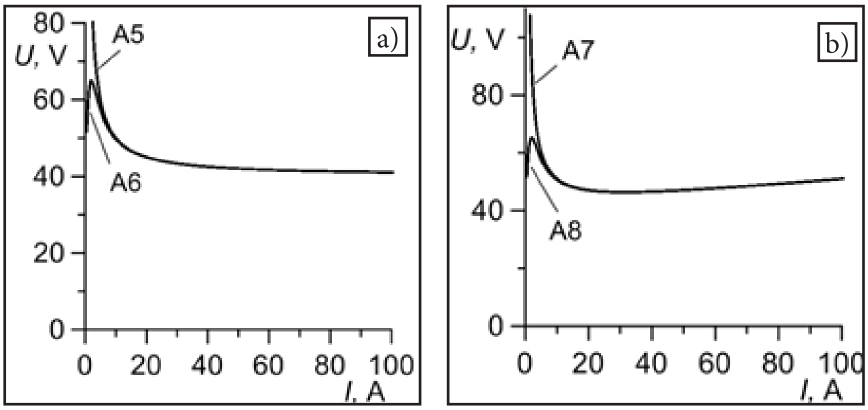

Fig. 4. Static voltage-current characteristics described using the modified Ayrton functions for the modelling of high-current (P_M = 100 W, U_C = 40 V, I_M = 2 A): a) hyperbolic-flat arc; b) hyperbolic-flat-linear arc (R_p = 0.1 Ω)

The assumption about the hyperbolic fall of the static characteristic of arc is considered to be rough. For this reason, occasionally the approximation proposed by Nottingham is applied [12]. If the length of arc is constant and if there is quasi-direct current-triggered excitation, changes in voltage can be described using the following formula

$$U_a = U_{AK} + U_{col} = U_{AK} + U_0 \cdot \left(\frac{I_0}{I}\right)^n \qquad (6)$$

where U_0, I_0 – approximation coefficients; n – power exponent of approximation ($n > 0$). In a special case, i.e. when $n = 1$, the above-named characteristic becomes identical as the Ayrton formula (1) and then $P_M = U_0 I_0$.

The electric properties of arc can also be described in the form of a static conductance-current characteristic

$$G_{col} = \frac{I}{U_{col}} = \frac{I^{n+1}}{U_0 I_0^n} \tag{7}$$

Various variants of static characteristics with defined and undefined values of arc ignition voltage are presented in Table 2. Figure 5a presents the static characteristics of arc described using functions N1 and N2, useful when modelling discharges within the low-current range. Functions designated as N3-N8 can be used for the modelling of arc within high-current ranges. The shapes of the above-named characteristics are presented in Figures 5 and 6. The modification of the Nottingham formula enables the description of the static characteristic using a formula with the defined voltage of arc ignition

$$U_{col} = U_0\left(\frac{I_0 I}{I^2 + I_M^2}\right)^n \tag{8}$$

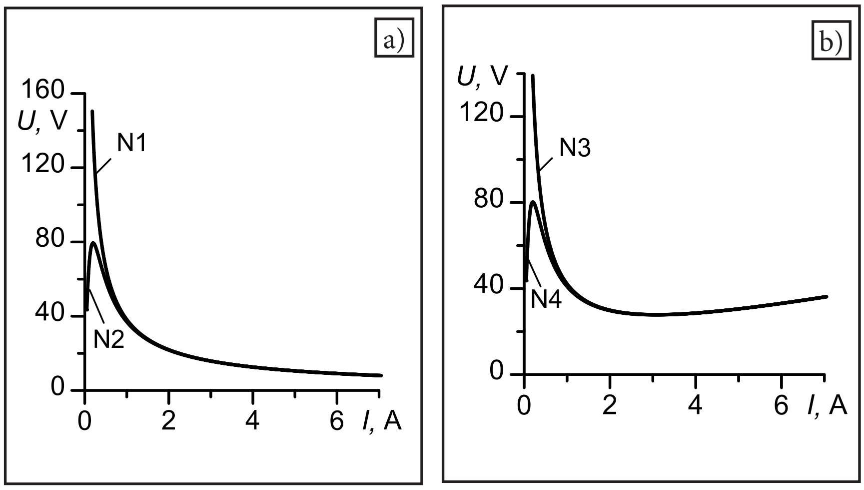

Fig. 5. Static voltage-current characteristics described using the modified Nottingham functions for the modelling of (U_0 = 100 V, I_0 = 0.3 A, n = 0.8, I_M = 0.2 A): a) low-current quasi-hyperbolic arc; b) quasi-hyperbolic-linear arc (R_p = 4 Ω)

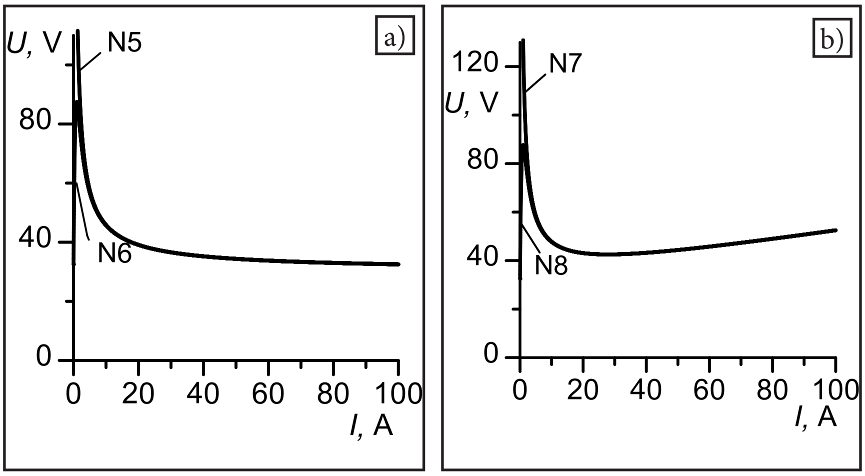

Fig. 6. Static voltage-current characteristics described using the modified Nottingham functions for the modelling of high-current (U_0 = 100 V, I_0 = 1 A, n = 0,8, I_M = 1 A): a) quasi-hyperbolic-flat arc; b) quasi-hyperbolic-flat-linear arc (R_p = 0.2 Ω)

Table. 2. Modified *Nottingham* functions used for the approximation of graphs presenting voltage changes in the arc column

Designation	**Static characteristics with undefined arc ignition voltage** $U_{col}=$	**Designation**	**Static characteristics with defined arc ignition voltage** $U_{col}=$
N1	$U_0\left(\frac{I_0}{I}\right)^n$	N2	$U_0\left(\frac{I_0 I}{I^2+I_M^2}\right)^n$
N3	$U_0\left(\frac{I_0}{I}\right)^n + R_p I$	N4	$U_0\left(\frac{I_0 I}{I^2+I_M^2}\right)^n + R_p I$
N5	$U_0\left(\frac{I_0}{I}\right)^n + U_C$	N6	$U_0\left(\frac{I_0 I}{I^2+I_M^2}\right)^n + U_C$
N7	$U_0\left(\frac{I_0}{I}\right)^n + U_C + R_p I$	N8	$U_0\left(\frac{I_0 I}{I^2+I_M^2}\right)^n + U_C + R_p I$

which can also be expressed as the static conductance-current characteristic

$$G_{col}=\frac{I}{U_{col}}=\frac{\left(I^2+I_M^2\right)^n}{U_0 I_0^n I^{n-1}} \tag{9}$$

The coordinates of extreme $S(I, U)$ of characteristic (8) are the following:

$$I=I_M \quad U=\frac{U_0 I_0^n}{\left(2I_M\right)^n} \tag{10}$$

The graphs of functions presented in Table 2 are shown in Figures 5 and 6.

The value of arc residual conductance, obtained using formulas N1, N3, N5, N6, N7 and N8, amounts to $G_{col}(I{=}0\text{A}) = 0$ S and does not depend on the value of exponent $n > 0$. Arc of characteristics designated as N2 and N4 and values of exponent $n > 1$ are characterised by residual conductance $G_{col}(I{=}0\text{A}) > 0$ S. If $0 < n < 1$, it is possible to modify power exponent n in order to obtain non-zero residual conductance in cases of N2 and N4. Then, it is possible to use the following approximation

$$n(I)=1-A_0\left[1-\exp\left(-\frac{I}{I_{01}}\right)\right] \tag{11}$$

where A_0, I_{01} – approximation coefficients ($0 < A_0 < 1$). The value of coefficient I_{01} should be very low. Then, a decrease in the value of current is accompanied by a fast change in the value of n. In turn, if $I = 0$ A, the value of power exponent $n = 1$. When considering characteristic N2, on the basis of formula (7), residual conductance amounts to

$$G_{col}(I=0\text{A})=\frac{I_M^2}{U_0 I_0}$$

This result is consistent with formula (4) as in the aforesaid case $U_0 I_0 = P_M$.

The universal, yet relatively more complex static characteristic described in publications [3, 4] has the defined value of arc ignition. The modified form of the aforesaid characteristic is the following [10]

$$U(I)=U_p\cdot\left(\frac{I}{I_p}\right)^{k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{k_1}+1\right)+ \\ +\left(U_0+R_0 I^{k_0}\right)\cdot\zeta(I)\cdot\left[1-\left(\frac{I}{I_p}\right)^{k_1}\cdot\exp\left(-\left(\frac{I}{I_p}\right)^{k_1}+1\right)\right] \tag{12}$$

where k_0, k_1 – approximation coefficients, $0 < k_1 < 1$; (I_p, U_p) – coordinates of a local maximum; U_0 – voltage and R_0 – arc resistance within the high-current range; ζ - tapering function, which may take various forms.

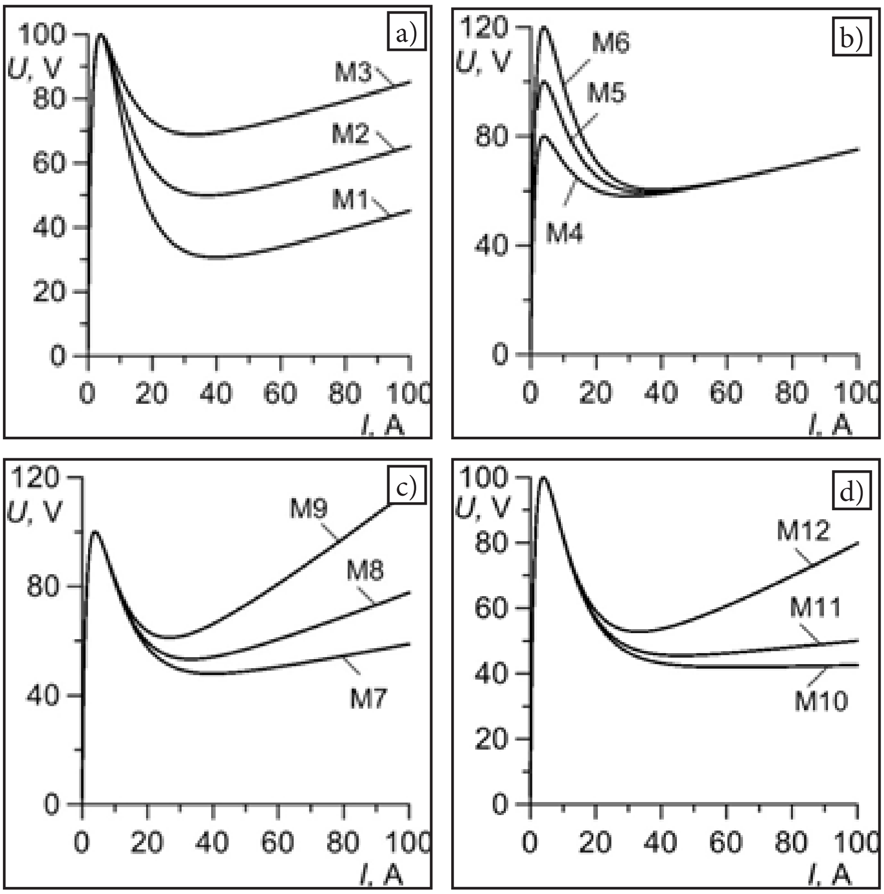

Fig. 7. Static voltage-current characteristics of arc described using formula (12) (I_p=4 A, k_1=0.8,): a) in relation to a change in voltage U_0 (R_0=0.1Ω, k_0=1.2, U_p=100 V, M1 - U_0=20 V, M2 - U_0=40 V, M3 - U_0=60V);
b) in relation to a change in voltage U_p (R_0=0.1 Ω, k_0=1.2, U_0=50V, M4 - U_p=80V, M5 - U_p=100V, M6 - U_p=120V);
c) in relation to a change in resistance R_0 (U_p=100V, U_0=40V, k_0=1.2, M7 - R_0=0.075Ω, M8 - R_0=0.15Ω, M9 - R_0=0.3Ω);
d) in relation to a change in exponent k_0 (U_p=100V, U_0=40V, R_0=0.1Ω, M10 - k_0=0.7, M11 - k_0=1, M12 - k_0=1.3)

Function $\zeta(I) = 1 - \exp(-I/I_p)$ was used in publication [12]. Coefficients U_p, U_0 and R_0 depend on the length of arc.

Figure 7 presents families of static voltage-current characteristics of arc described using formula (12). The families include changes in parameters within the high-current range. In actual arc, external effects trigger simultaneous changes in the shape of characteristics within entire ranges of current changes.

Dynamic characteristics of arc with a defined value of arc ignition voltage

The basis of initial assumptions of the Pentegov mathematical model [7, 8] is the equation of arc column energy balance. Actual electric arc is characterised by certain inertia related to thermal processes. To include thermal inertia in the Pentegov mathematic model of electric arc it is necessary to introduce hypothetical electric current $i_\theta(t)$ representing the thermal sate of arc and changing in time in relation to the square of momentary current i of arc in accordance with the first-order equation

$$\theta \frac{di_\theta^2}{dt} + i_\theta^2 = i^2 \tag{13}$$

The above-named model is electrically inertialess as, according to its assumptions, there is the following correlation

$$\frac{i}{u_{col}} = \frac{i_\theta}{U_{col}(i_\theta)} \tag{14}$$

where u_{col} represents momentary voltage on the arc column and $U_{col}(i_\theta)$ represents the static voltage-current characteristic of the arc column. All isoenergetic states are characterised by one variable, i.e. arc state current $i_\theta(t)$.

The Pentegov model represents the non-linear circuit two-terminal network, which is balanced in terms of energy; 1st degree thermally inert, linear, stationary and electrically inertialess. The advantage of the model is the possibility of using any approximation of static voltage-current characteristic $U_{col}(i_\theta)$ in equation (14). Hence

$$u_{col} = \frac{U_{col}(i_\theta)}{i_\theta} i = \frac{i}{G_{col}(i_\theta)} \tag{15}$$

and the assignment of the constant value of the function of damping factor θ. In comparison with other models, the above-presented model can use (in a physically justified manner) more accurate approximations of static arc characteristics observed during welding performed using both inert and active gases.

In certain cases of the application of approximations of static voltage-current characteristics with the zero value of residual conductance it is possible to implement the modification of formula (15) in the following form

$$u_{col} = \frac{i}{G_{col}(i_\theta) + G_{\min}} = \frac{i}{\frac{i_\theta}{U_{col}(i_\theta)} + G_{\min}} \tag{16}$$

where $G_{min} > 0$ S.

The generalised Pentegov model was used to simulate processes in a circuit with arc powered by the source of sinusoidal alternating current having frequency $f = 50$ Hz. The calculations included the assumed sum of near-electrode voltage drops $U_{AK} = 18$ V. Figures 8 and 9 present the dynamic characteristics of electric arc described using formulas (13) and (15) with static characteristics specified in Table 1 and with parameter values as presented in Figure 3 and 4. It can be seen that the use of the static characteristic with defined ignition voltage decreases voltage spikes in the area where current passes through zero. The above-named effect also depends on time constant θ.

It is possible to obtain extensive possibilities of the approximation of dynamic characteristics of arc using static characteristic (12). The figure presents examples obtained during the simulation. However, more complex correlations impede the development of simple and effective methods enabling the experimental determination of parameters of mathematical models of arc.

Conclusions

1. The above-presented formulas defining static voltage-current characteristics are characterised by formal simplicity as well as by the extensive range of possible changes in current excitation and results triggered by possible various external influences affecting the arc column.

2. The use of static characteristics of defined ignition voltage enables the more precise representation of dynamic characteristics of arc (particularly with a low value of the damping function) and facilitates performing numerical simulations of processes in electric circuits.

3. Relatively simple forms of approximating functions facilitate the development of methods enabling the experimental determination of parameters of mathematical models of arc with preset current excitation (usually rectangular or sinusoidal).

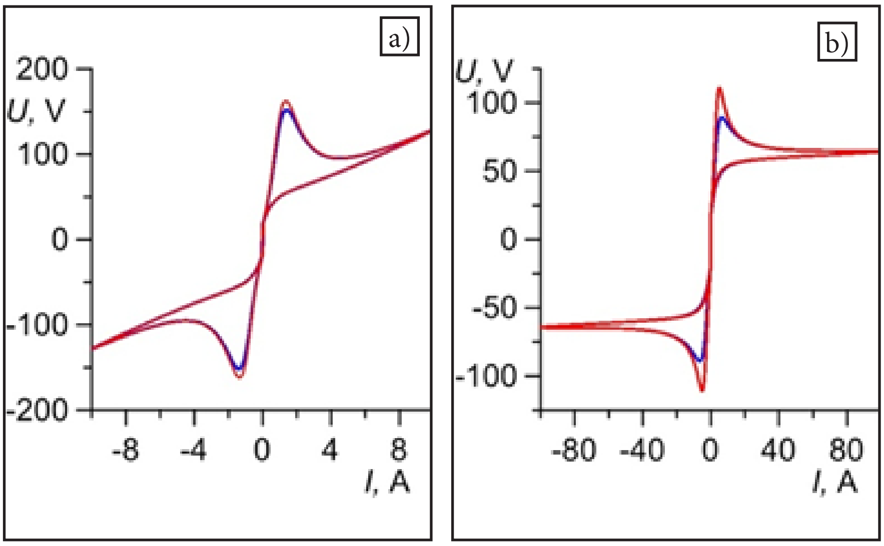

Fig. 8. Dynamic voltage-current characteristics of arc of a static characteristic described using modified Ayrton functions: a) (θ=3·10^{-4}s) red line – A3, blue line – A4; b) (θ=1·10^{-4}s) red line – A7, blue line – A8

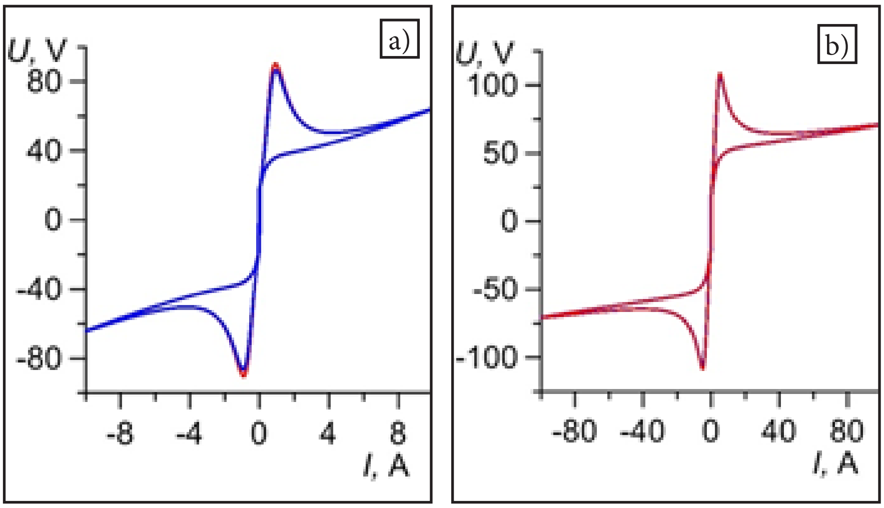

Fig. 9. Dynamic voltage-current characteristics of arc of a static characteristic described using modified Nottingham functions: a) (θ=2·10^{-4}s) red line – N3, blue line – N4; b) (θ=1·10^{-4}s) red line – N7, blue line – N8

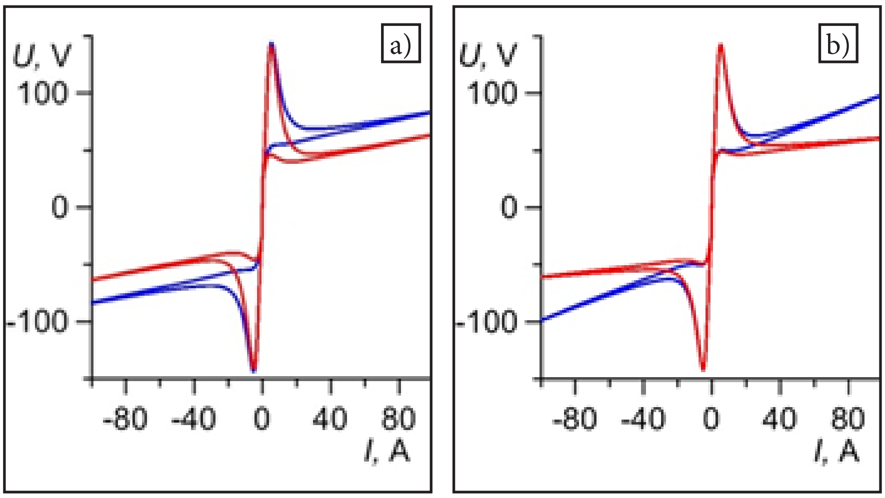

Fig. 10. Dynamic voltage-current characteristics of arc of a dynamic characteristic (12) (U_p=100 V, I_p=2 A, k_1=0.8, k_0=1.2, θ=1·10^{-4}s): a) in relation to a change in voltage U_0 (R_0=0.1Ω, red line – U_0=20 V, blue line – U_0=40 V) b) in relation to a change in resistance R_0 (U_0=30 V, red line – R_0=0.05Ω, blue line – R_0=0.2Ω)

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