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Modelling the Effect of External Disturbances in Static Characteristics of Unspecified and Determined Ignition Voltages on Dynamic Characteristics of Arc in a Circuit with the Current Source.
Part. 1. Primary Analytical Correlations

Abstract: The article presents a set of functions useful when approximating non-linear current-voltage characteristics of static arc disturbed by external factors. To this end, modified Ayrton and Nottingham functions were used. The study involved characteristics with undefined and determined arc ignition voltages. The mapping of the above-named arc and the simulation of processes in the electric circuit with the current source were performed using the generalized Pentegov model, involving the use of appropriately transformed components of functions approximating static current-voltage characteristics of arc (taking into consideration the effect of single-parameter disturbances of the column length, mass stream or gas pressure.

Keywords: electric arc, Pentegov model, ignition voltage

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Introduction

Electric arc is rated among strongly non-linear and poorly inertial elements of electric circuits. The selection of functions approximating static arc characteristics and mathematical models approximating dynamic arc characteristics depends on many factors [1-4] including, among other things, ranges of electric excitation (amplitudes, frequencies), ranges of changes in internal parameters triggered by control actions and the intensity of various external disruptions. Even if ranges of changes in excitations or parameters are small, their linearization in analytical tests is often very difficult.

Various physical (e.g. changes in the column length, stream of gas washing around the column, pressure and temperature of gas) and chemical (chemical composition of gas and electrode material) factors trigger changes in the position and shape of static and dynamic characteristics of arc [5-7]. The manner in which such factors are taken into consideration in numerical and analytical calculations depends on required accuracy, permissible computational complexity and the intended use of mathematical models.

Publication [8] presents the mathematical Pentegov model [9, 10] utilising static

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current-voltage characteristics of arcs with unspecified and determined arc ignition values. This article presents tests extended by the creation of mathematical models variants taking into consideration changes of single parameters, e.g.: length, gas mass stream or gas pressure.

Non-linear static characteristics of arc disturbed by external factors

Because of the heterogeneous structure of electric arc and the varied behaviour of individual areas of plasma affected by slow changes in current excitation, voltage between electrodes is described by the following expression

$$U_a(I) = U_{AK} + U_{col}(I) \tag{1}$$

where U_{AK} – sum of near-electrode voltage drops in areas with non-equilibrium plasma; U_{col} – voltage on the arc column with equilibrium plasma.

Usually, it is roughly assumed that near-electrode voltage drops have constant values independent of current. As a matter of fact, these drops depend on types of electrode materials and electrode (primarily) cathode shapes, usually not changing during technological operations. In addition, to some extent, the aforesaid drops depend on the type and pressure of plasma-forming gas as well as on the length of arc. Voltage drops in the arc column can be approximated using various functions, the choice of which depends on the distribution of measurement data obtained during experiments involving various (e.g. constant, sinusoidal or rectangular) current excitations [11, 12]. The use of variable excitations indicates the possibility of lowering ignition voltage (see Table 1). Functions presented in Table 1 are modifications of Ayrton formula [1]; they contain segments with the hyperbolic falling of the voltage characteristic within the low-current range. In addition, the above-named modifications take into consideration various distributions of measurement data within the high-current range.

External disruption of arc may affect near-electrode areas or the plasma column. Changes in electrode (primarily cathode) properties can be relatively easily taken into consideration by entering values of near-electrode voltage drops. They can be triggered mechanically by a mowing (e.g. rotating) electrode having a non-uniform material structure as well as gasodynamically or magnetically, by deflecting arc along with the displacement of the arc spot area. Obviously, changes in physical properties of spots occur with thermal inertia depending on the type of electrode material as well as on the shape and dimensions of the electrode. Arc column disruption may temporarily affect column conductance and energy dissipation intensity. Because of thermal and gasodynamic processes, the reaction of column is characterised by certain inertia. For this reason, the use of the ordinary modification of static characteristic (2) may prove insufficient

$$U_a(I, p_a) = U_{AK} + U_{col}(I, p_a) \tag{2}$$

Table 1 presents dependences expressing arc static characteristics, the components of which depend on changes in parameters affected by external disruptions. It was assumed that the effect of external disruptions on the value of current I_M was negligible. In addition, the following designations were adopted: P_M – arc power within the low-current range; R_p – arc resistance within the high-current range; U_C – arc voltage within the high-current range.

Changes in parameter p_a affect the value of residual conductance (in relation to A2, being $G_M = I_M^2 / P_M(p_a)$) and the value of ignition voltage. The coordinates of extreme point $S(I, U)$ on characteristic (A2) are the following:

$$I = I_M, \quad U = \frac{P_M(p_a)}{2I_M} \tag{3}$$

Some experimental tests performed within the low-current range pointed to the shape of the falling of the static current-voltage characteristic described by a power function having

Table 1. Modified Ayrton functions of disturbed components of voltage on the arc column, useful when approximating static characteristics

Designation	Static characteristic with unspecified arc ignition voltage $U_{col}(I, p_a)=$	Designation	Static characteristic with determined arc ignition voltage $U_{col}(I, p_a)=$
A1	$\frac{P_M(p_a)}{I}$	A2	$\frac{P_M(p_a)\cdot I}{I^2+I_M^2}$
A3	$\frac{P_M(p_a)}{I}+R_p(p_a)\cdot I$	A4	$\frac{P_M(p_a)\cdot I}{I^2+I_M^2}+R_p(p_a)\cdot I$
A5	$\frac{P_M(p_a)}{I}+U_C(p_a)$	A6	$\frac{P_M(p_a)\cdot I}{I^2+I_M^2}+U_C(p_a)$
A7	$\frac{P_M(p_a)}{I}+U_C(p_a)+R_p(p_a)\cdot I$	A8	$\frac{P_M(p_a)\cdot I}{I^2+I_M^2}+U_C(p_a)+R_p(p_a)\cdot I$

exponent n, i.e. to the possibility of obtaining better approximation using the formula proposed by Nottingham [13]. Table 2 contains various functions constituting modifications of the above-presented formula. The functions also take into consideration various distributions of measurement data within the high-current range. It was assumed that the effect of external disruptions on values of current I_s and I_M was negligible.

Changes in parameter p_a affect the value of ignition voltage. The coordinates of extreme point $S(I, U)$ on characteristic (N2) are the following:

$$I = I_M, \quad U = \frac{U_s(p_a)I_s^n}{(2I_M)^n} \tag{4}$$

The modelling of arc having unspecified and determined ignition voltage and static characteristics depending on external disruption

Entering parameter changes only into the static characteristic ignores the effect of disruptions on plasma enthalpy. Disruptions may change the geometrical dimensions of the column as well as the inflow and outflow of additional energy. Publications [9, 10, 14-16] present the modelling

Table 2. Modified Nottingham functions of disturbed components of voltage on the arc column, useful when approximating static characteristics

Designation	Static characteristic with unspecified arc ignition voltage $U_{col}(I, p_a)=$	Designation	Static characteristic with determined arc ignition voltage $U_{col}(I, p_a)=$
N1	$U_s(p_a)\cdot\left(\frac{I_s}{I}\right)^n$	N2	$U_s(p_a)\cdot\left(\frac{I_sI}{I^2+I_M^2}\right)^n$
N3	$U_s(p_a)\cdot\left(\frac{I_s}{I}\right)^n+R_p(p_a)\cdot I$	N4	$U_s(p_a)\cdot\left(\frac{I_sI}{I^2+I_M^2}\right)^n+R_p(p_a)\cdot I$
N5	$U_s(p_a)\cdot\left(\frac{I_s}{I}\right)^n+U_C(p_a)$	N6	$U_s(p_a)\cdot\left(\frac{I_sI}{I^2+I_M^2}\right)^n+U_C(p_a)$
N7	$U_s(p_a)\cdot\left(\frac{I_s}{I}\right)^n+U_C(p_a)+R_p(p_a)\cdot I$	N8	$U_s(p_a)\cdot\left(\frac{I_sI}{I^2+I_M^2}\right)^n+U_C(p_a)+R_p(p_a)\cdot I$

of specific cases concerning how disruptions of length or the stream of gas washing around the column affected the equation of arc state resulting from energy balance. An appropriate system of equations can be written in the following form

$$\frac{dQ}{dt} = P_{el} - P_{dis} = \frac{U_{st}(i_\theta, p_a)}{i_\theta} i^2 - U_{st}(i_\theta, p_a) \cdot i_\theta \tag{5}$$

$$Q = 2\theta \int_0^{i_\theta} U_{st}(i_\theta, p_a) di_\theta \tag{6}$$

where Q – plasma enthalpy; P_{el} – supplied electric power; P_{dis} – dissipated thermal power; i_θ – virtual state current; θ – time constant [10].

The derivative of enthalpy in relation to time t has the following form

$$\frac{dQ}{dt} = 2\theta U_{st}(i_\theta, p_a)\frac{di_\theta}{dt} + 2\theta \int_0^{i_\theta} \left(\frac{\partial}{\partial p_a} U_{st}(i_\theta, p_a) di_\theta \right) \frac{dp_a}{dt} \tag{7}$$

It is assumed that $U_{st}(i_\theta, p_a)$ is a continuous function having a continuous derivative in relation to the parameter. After substituting equation (7) to equation (5), the following differential equation is obtained

$$2\theta i_\theta \frac{di_\theta}{dt} + 2\theta i_\theta \frac{\int_0^{i_\theta} \frac{\partial}{\partial p_a} U_{st}(i_\theta, p_a) di_\theta}{U_{st}(i_\theta, p_a)} \frac{dp_a}{dt} = i^2 - i_\theta^2 \tag{8}$$

or in another form

$$\theta \frac{di_\theta^2}{dt} + 2\theta i_\theta \frac{\int_0^{i_\theta} \frac{\partial}{\partial p_a} U_{st}(i_\theta, p_a) di_\theta}{U_{st}(i_\theta, p_a)} \frac{dp_a}{dt} = i^2 - i_\theta^2 \tag{9}$$

The value of voltage on the arc column is determined using the static characteristic

$$u = \frac{U_{st}(i_\theta, p_a)}{i_\theta} i \tag{10}$$

Because of the fact that such values as voltage $U_{st}(i_\theta, p_a)$, conductance $g(i_\theta, p_a)$ and enthalpy $Q(i_\theta, p_a)$ are functions of state current i_θ and depend on parameter p_a, also the damping function does not have constant value $\theta(i_\theta, p_a) = g\frac{dQ}{di_\theta^2}$. The introduction of a requirements, according to which $\theta(i_\theta, p_a) = \theta = const$ should satisfy the condition concerning the value of enthalpy

$$Q(i_\theta, p_a) = 2\theta \int_0^{i_\theta} U_{st}(i_\theta, p_a) di_\theta \tag{11}$$

In practice, the obtainment of the aforesaid result can be difficult. For this reason, calculation results obtained using model (9) should be regarded as approximate.

The behaviour of arc during the elongation or shortening of the column depends on electric current and arc burning conditions. Low current favours the displacement of arc, leading to changes in dissipation power. If arc is free and horizontal or oblique, its natural elongation is favoured by upward convective currents of heated gas. Similarly, shapes and the arrangement of leads powering electrodes can extend arc columns as a result of blasts induced by the magnetic field generated by current flowing in the leads. In cases of long arcs, correlation $U(l)$ becomes non-linear [4]. In arc observed in publication [5], the length of which was restricted within the range of 3.8 cm to 8.9 cm, function $U(l) \propto l^{2/3}$ was obtained. When performing calculations of deflected arcs it is preferable to use the gradient of voltage calculated in relation to the axis of electrode displacement instead of the intensity of the electric field calculated along the column axis. The aforesaid gradient takes into consideration the distance between the electrodes instead of the actual column length. The correlation between the value of voltage and the length of arc can be nearly linear. The above-named property is typical of arcs characterised by appropriately high values of current. The columns of such arcs are washed around by a steady longitudinal gas flow. Electrodes and current supply leads are arranged coaxially with arc. Such an arc case can be treated as specific in relation to that discussed in the article.

Experimental tests concerning electric arcs revealed certain arch characteristics including, among other things, an effect related to the reduction of arc length

$$\lim_{l_c \to 0} U_a(l_c) = U_{AK} + \lim_{l_c \to 0} U_{col}(l_c) = U_{AK} \quad (12)$$

$$\lim_{l_a \to 0} U_a(l_a) = 0\ \text{V} \quad (13)$$

where l_c – arc column length; l_a – total arc length; U_a – resultant arc voltage. Condition (13) corresponds to a short circuit and arc termination. Formula (12) reveals that individual components of the sum in expressions concerning the drop of voltage in the column satisfy boundary conditions:

– within the low-current range

$$\lim_{l_c \to 0} \frac{P_M(l_c)}{I} = 0\ \text{V} \quad (14)$$

or

$$\lim_{l_c \to 0} U_s(l_c) = 0\ \text{V} \quad (15)$$

– within the high-current range

$$\lim_{l_c \to 0} U_C(l_c) = U_C(0) = 0\ \text{V} \quad (16)$$

$$\lim_{l_c \to 0} R_p(l_c) I = 0\ \text{V} \quad (17)$$

where $I > 0$ A.

The use of power sources of non-linear external characteristics for powering arcs of excessively elongated columns leads to discharge instability and arc termination.

Experimental tests of stream plasma torches revealed that the effect of gas flow on the current-voltage characteristics of arc is non-linear [4, 17] and depends on the value of gas mass stream, its direction in relation to the axis of arc and on the chemical composition of gas. In spite of the foregoing, approximations based on linear approximations are used frequently [1, 6].

Changes in the flowing gas mass stream $\dot{m}$ and washing around the arc column trigger a similar effect to that induced by length changes. An excessive flow (high rate of gas flow in the constrictor channel) can terminate arc. In particular, the transverse flow of gas is accompanied by the elongation of column and the deterioration of discharge stability. In turn, the decay of a forced gas flow is not accompanied by the termination of arc. For this reason, it is possible to suggest the following boundary conditions:

– within the low-current range

$$\lim_{\dot{m} \to 0} \frac{P_M(\dot{m})}{I} = \lim_{\dot{m} \to 0} \frac{P_{m0} + P_{m1} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{pm}}}{I} = \frac{P_{m0}}{I} > 0\ \text{V} \quad (18)$$

or

$$\lim_{\dot{m} \to 0} U_s(\dot{m}) = U_s(0) > 0\ \text{V} \quad (19)$$

– within the high-current range

$$\lim_{\dot{m} \to 0} U_C(\dot{m}) = U_C(0) > 0\ \text{V} \quad (20)$$

$$\lim_{\dot{m} \to 0} R_p(\dot{m}) I = \lim_{\dot{m} \to 0} \left(R_{m0} + R_{m1} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{rm}} \right) I = R_{m0} I > 0\ \text{V} \quad (21)$$

where P_{mo}, P_{m1}, R_{mo}, R_{m1} – constant coefficients of approximation; I > 0 A.

During the modelling of electric arc the value of pressure $p = 0$ Pa is not taken into consideration and arc will not burn under such conditions. Low pressure values ($p < 100$ Pa) correspond to typical glow discharge characterised by higher voltage and reduced current. The above-named discharge is spread in relatively large space, is characterised by lower heating efficiency, yet at the same time, by the increased efficiency of the plasma-chemical treatment of metal elements.

Experimental tests concerning free high-current arc burning in argon [18] revealed the dependence related to the intensity of electric field $E(p) \propto p^{2/3}$. Tests performed by Allum [19], concerned with arc in argon or helium, revealed that the exponent of pressure could be even lower and restricted within the range of 0.43 to 0.55. An even lower value of 0.4 was provided by Gueye [20] (in relation to tests concerning discharges in hydrogen). The above deliberations take into consideration the absolute values of pressure. Most welding machines are operated in a gas atmosphere having atmospheric or near-atmospheric pressure. For this reason, the following boundary values can be proposed:

– within the low-current range

$$\lim_{p\to p_0}\frac{P_M(p)}{I}=\lim_{p\to p_0}\frac{P_{p0}+P_{p1}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{pp}}}{I}=\frac{P_{p0}}{I}>0\ \text{V} \tag{22}$$

$$\lim_{p\to p_0}U_s(p)=U_s(p_0)>0\ \text{V} \tag{23}$$

– within the high-current range

$$\lim_{p\to p_0}U_C(p)=U_C(p_0)>0\ \text{V} \tag{24}$$

$$\lim_{p\to p_0}R_p(p)\cdot I=\lim_{p\to p_0}\left(R_{p0}+R_{p1}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{rp}}\right)\cdot I=R_{p0}I>0\ \text{V} \tag{25}$$

where p_0 – value of base pressure; P_{p0}, P_{p1}, R_{p0}, R_{p1} – constant coefficients of approximation corresponding to values of parameters under pressure $p = p_0$; $I > 0$ A. If modelling concerns arcs in compressed gases, value po is usually adopted as matching the value of atmospheric pressure ($p_0 \approx 10^5$ Pa). In turn, if modelling is concerned with arcs in subatmospheric pressure, it is favourable to adopt the value of base pressure as the lowest numeral among available data ($p_0 \geq 100$ Pa).

Table 3 contains expressions used in equation (9), describing arc within the low-current

Table 3. Modelling the effect of one-parameter disruptions used in equation (9), describing arc within the low-current range

Voltage components, their derivatives, power integrals	**Variable parameter p_a =**		
	length l_c	**gas mass stream $\dot{m}$**	**gas pressure p**
$P_M(p_a)=$	$P_l\cdot\left(\frac{l_c}{l_0}\right)^{n_{pl}}$	$P_{m0}+P_{m1}\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{pm}}$	$P_{p0}+P_{p1}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{pp}}$
$dP_M/dp_a=$	$n_{pl}\frac{P_l}{l_0}\cdot\left(\frac{l_c}{l_0}\right)^{n_{pl}-1}$	$n_{pm}\frac{P_{m1}}{\dot{m}_0}\cdot\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{pm}-1}$	$n_{pp}\frac{P_{p1}}{p_0}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{pp}-1}$
$\int_0^{i_\theta}\frac{\partial}{\partial p_a}\frac{P_M(i_\theta,p_a)}{i_\theta}di_\theta=$	$n_{pl}\frac{P_l}{l_0}\left(\frac{l_c}{l_0}\right)^{n_{pl}-1}\ln i_\theta$	$n_{pm}\frac{P_{m1}}{\dot{m}_0}\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{pm}-1}\ln i_\theta$	$n_{pp}\frac{P_{p1}}{p_0}\left(\frac{p-p_0}{p_0}\right)^{n_{pp}-1}\ln i_\theta$
$\int_0^{i_\theta}\frac{\partial}{\partial p_a}\frac{P_M(i_\theta,p_a)i_\theta}{i_\theta^2+I_M^2}di_\theta=$	$\frac{n_{pl}}{2}\frac{P_l}{l_0}\left(\frac{l_c}{l_0}\right)^{n_{pl}-1}\times$ $\times\ln\left(i_\theta^2+I_M^2\right)$	$\frac{n_{pm}}{2}\frac{P_{m1}}{\dot{m}_0}\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{pm}-1}\times$ $\times\ln\left(i_\theta^2+I_M^2\right)$	$\frac{n_{pp}}{2}\frac{P_{p1}}{p_0}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{pp}-1}\times$ $\times\ln\left(i_\theta^2+I_M^2\right)$
$U_s(p_a)=$	$U_{sl}\cdot\left(\frac{l_c}{l_0}\right)^{n_{sl}}$	$U_{sm0}+U_{sm1}\cdot\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{sm}}$	$U_{sp0}+U_{sp1}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{sp}}$
$dU_s/dp_a=$	$n_{sl}\frac{U_{sl}}{l_0}\cdot\left(\frac{l_c}{l_0}\right)^{n_{sl}-1}$	$n_{sm}\frac{U_{sm1}}{\dot{m}_0}\cdot\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{sm}-1}$	$n_{sp}\frac{U_{sp1}}{p_0}\cdot\left(\frac{p-p_0}{p_0}\right)^{n_{sp}-1}$
$\int_0^{i_\theta}\frac{\partial U_s(i_\theta,p_a)}{\partial p_a}\left(\frac{I_s}{i_\theta}\right)^n di_\theta=$	$n_{sl}\frac{U_{sl}I_s^n}{l_0}\left(\frac{l_c}{l_0}\right)^{n_{sl}-1}\times$ $\times\frac{i_\theta^{1-n}}{1-n}$	$n_{sm}\frac{U_{sm1}I_s^n}{\dot{m}_0}\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{sm}-1}\times$ $\times\frac{i_\theta^{1-n}}{1-n}$	$n_{sp}\frac{U_{sp1}I_s^n}{p_0}\left(\frac{p-p_0}{p_0}\right)^{n_{sp}-1}\times$ $\times\frac{i_\theta^{1-n}}{1-n}$
$\int_0^{i_\theta}\frac{\partial U_s(i_\theta,p_a)}{\partial p_a}\left(\frac{I_s i_\theta}{i_\theta^2+I_M^2}\right)^n di_\theta=$	$n_{sl}\frac{U_{sl}I_s^n}{l_0}\left(\frac{l_c}{l_0}\right)^{n_{sl}-1}\times$ $\times F_{iM}\left(i_\theta^2+I_M^2\right)$	$n_{sm}\frac{U_{sm1}I_s^n}{\dot{m}_0}\left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{sm}-1}\times$ $\times F_{iM}\left(i_\theta^2+I_M^2\right)$	$n_{sp}\frac{U_{sp1}I_s^n}{p_0}\left(\frac{p-p_0}{p_0}\right)^{n_{sp}-1}\times$ $\times F_{iM}\left(i_\theta^2+I_M^2\right)$

range, depending on a selected parameter, i.e. arc column length, gas mass stream or gas pressure. Parameter P_{m0} corresponds to gas flow decay $\dot{m} = 0$ kg/s, whereas parameter P_{p0} corresponds to arc burning in gas under pressure p_0. Values of exponents n_{pl}, n_{pm}, n_{pp}, n_{sl}, n_{sm} and n_{sp} should be higher than zero.

Integral function F_{iM} being part of the expressions in the last line of Table 3 has the following form

$$F_{iM}\left(i_\theta^2 + I_M^2\right) = \int_0^{i_\theta}\left(\frac{i_\theta}{i_\theta^2 + I_M^2}\right)^n di_\theta = \frac{i_\theta\left(\frac{i_\theta}{i_\theta^2 + I_M^2}\right)^n\left(\frac{i_\theta^2 + I_M^2}{I_M^2}\right)^n {}_2F_1\left(n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{i_\theta^2}{I_M}\right)}{n+1} \quad (26)$$

and contains a hypergeometric function having the following form

$$ {}_2F_1\left(n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{i_\theta^2}{I_M}\right) = 1 + \frac{n\frac{n+1}{2}}{1!\frac{n+3}{2}}\left(-\frac{i_\theta^2}{I_M}\right) + \frac{n(n+1)\frac{n+1}{2}\left(\frac{n+1}{2}+1\right)}{2!\frac{n+3}{2}\left(\frac{n+3}{2}+1\right)}\left(-\frac{i_\theta^2}{I_M}\right)^2 + \ldots \quad (27)$$

Table 4 contains expressions used in equation (9), describing arc within the high-current range, depending on a selected parameter, i.e. arc column length, gas mass stream or gas pressure. Parameters U_{m0} and R_{m0} correspond to gas flow decay $\dot{m} = 0$ kg/s. However, because of the necessity of obtaining finite values of integral expressions it is advisable (comfortable) to adopt $\dot{m} = 0$ kg/s. Parameters U_{p0} and R_{p0} corresponds to arc burning in gas under pressure p_0. Values of applied exponents n_{ul}, n_{um}, n_{up}, n_{rl}, n_{rm} and n_{rp} should be higher than zero.

Table 4. Modelling the effect of one-parameter disruptions used in equation (9), describing arc within the high-current range

Voltage components, their derivatives, power integrals	**Variable parameter p_a =**		
	length l_c	**gas mass stream $\dot{m}$**	**gas pressure p**
$U_C(p_a) =$	$U_l \cdot \left(\frac{l_c}{l_0}\right)^{n_{ul}}$	$U_{m0} + U_{m1} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{um}}$	$U_{p0} + U_{p1} \cdot \left(\frac{p - p_0}{p_0}\right)^{n_{up}}$
$dU_C / dp_a =$	$n_{ul}\frac{U_l}{l_0} \cdot \left(\frac{l_c}{l_0}\right)^{n_{ul}-1}$	$n_{um}\frac{U_{m1}}{\dot{m}_0} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{um}-1}$	$n_{up}\frac{U_{p1}}{p_0} \cdot \left(\frac{p - p_0}{p_0}\right)^{n_{up}-1}$
$\int_0^{i_\theta}\frac{\partial}{\partial p_a}U_c(i_\theta, p_a)di_\theta =$	$n_{ul}\frac{U_l}{l_0} \cdot \left(\frac{l_c}{l_0}\right)^{n_{ul}-1} i_\theta$	$n_{um}\frac{U_{m1}}{\dot{m}_0} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{um}-1} i_\theta$	$n_{up}\frac{U_{p1}}{p_0} \cdot \left(\frac{p - p_0}{p_0}\right)^{n_{up}-1} i_\theta$
$R_p(p_a) =$	$R_l \cdot \left(\frac{l_c}{l_0}\right)^{n_{rl}}$	$R_{m0} + R_{m1} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{rm}}$	$R_{p0} + R_{p1} \cdot \left(\frac{p - p_0}{p_0}\right)^{n_{rp}}$
$dR_p / dp_a =$	$n_{rl}\frac{R_l}{l_0} \cdot \left(\frac{l_c}{l_0}\right)^{n_{rl}-1}$	$n_{rm}\frac{R_{m1}}{\dot{m}_0} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{rm}-1}$	$n_{rp}\frac{R_{p1}}{p_0} \cdot \left(\frac{p - p_0}{p_0}\right)^{n_{rp}-1}$
$\int_0^{i_\theta}\frac{\partial}{\partial p_a}R_p(i_\theta, p_a)i_\theta di_\theta =$	$n_{rl}\frac{R_l}{l_0} \cdot \left(\frac{l_c}{l_0}\right)^{n_{rl}-1}\frac{i_\theta^2}{2}$	$n_{rm}\frac{R_{m1}}{\dot{m}_0} \cdot \left(\frac{\dot{m}}{\dot{m}_0}\right)^{n_{rm}-1}\frac{i_\theta^2}{2}$	$n_{rp}\frac{R_{p1}}{p_0} \cdot \left(\frac{p - p_0}{p_0}\right)^{n_{rp}-1}\frac{i_\theta^2}{2}$

Concluding remarks:

1. The above-presented extensive set of components of functions approximating static characteristics of a column of unspecified and determined ignition voltage can be useful when modelling arcs in circuits of technological devices, burning within wide ranges of current excitation.
2. The formulas concerning one-parameter disruptions can be used when modelling arcs with successively operating (one by one) controlling actions of various physical nature.
3. The forms of approximating functions adopted in the article can facilitate the development of methods enabling the experimental determination of parameters of arc mathematical models.
4. In relation to the approximations of static characteristics U(I, p) it is recommendable to adopt base value po below the lower range of pressure changes.

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