

Antoni Sawicki

Mathematical Differential and Integral Models in the Macromodelling of Electric Arc Using Voltage and Current Controlled Sources Part 2. Selected Mathematical Arc Macromodels with Explicitly Defined Current and Voltage Characteristics

Abstract: The article justifies the application of explicitly defined static current and voltage characteristics in mathematical models of dynamic electric arc. The study involved the use of the generalised function approximating the above-named characteristics to create the differential and integral forms of the Novikov-Schellhase, Pentegov and Mayr-Pentegov mathematical models. Macromodels of arc were developed using the differential form of mathematical models and controlled voltage sources as well as using the integral form of models and controlled current sources. The effectiveness of macromodels was verified by means of simulations of processes in circuits with electric arc.

Keywords: electric arc, Novikov-Schellhase model, Pentegov model, Mayr-Pentegov model

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Introduction

The introduction of static characteristics to dynamic models of electric arc is connected with the adoption of certain additional simplifying assumptions. If such a situation takes place when a mathematical model is nearly ready, the initial conditions of energy balance are at risk of deformation. As a result, the appropriately precise modelling of physical processes by mathematical models may be more difficult to demonstrate. A more favourable approach should involve the introduction of static characteristics as early as at the initial stage of the development of a mathematical model.

However, also in this case, as in any simplified model describing the extensive set of complex physical phenomena, it is necessary to apply many simplifying assumptions. The gradual abandonment of assumed simplifications may result from the Pareto principle, whereas the identification of parameters of models may result from optimisation procedures [1].

The application of static characteristics for the development of dynamic models of electric arc facilitates the following:

1. identification of certain parameters of dynamic models in the differential and integral form [2];

Antoni Sawicki, PhD (DSc) Habilitated Eng.; SEP – Stowarzyszenie Elektryków Polskich - Association of Polish Electrical Engineers (FSNT-NOT – Polish Federation of Engineering Associations – Central Technical Organisation), Częstochowa

2. taking into consideration disturbances in the form of interference and control effects in mathematical models;
3. control of changes in characteristic parameters of electric arc (e.g. ignition voltage, residual conductance) [2–4];
4. in certain cases, interpretation of physical processes in circuits with electric arc.

Engineering practice uses several well-known mathematical differential models of electric arc including current-voltage characteristics. The development of models in the integral form may enable the extension of technical applications of mathematical models with various forms of current-voltage characteristics. The above-named forms are frequently preferred when developing models of burning arc exposed to intense external disturbances leading to instability [5, 6].

In the absence of intentional activities aimed to disturb the plasma column, dynamic characteristics of arc are usually symmetric. In such a situation, the modelling of arc only requires the knowledge of the static characteristic in the first quarter of the coordinate system (I, U). The aforesaid approach makes it possible to simplify experimental tests and create macromodels of electric arc.

Differential and integral form of the Novikov-Schellhase model of electric arc

The Mayr modified model of arc [7] can be presented in the conductance form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_M} \left[\frac{P_{col}(t)}{P_{dis}(t)} - 1 \right] \quad (1),$$

where θ_M – corresponds to the time of thermal process relaxation; electric power supplied to arc amounts to

$$P_{col}(t) = u_{col} i = \frac{i^2}{g} \quad (2).$$

If the static current-voltage characteristic $U(I)$ is used, the power of arc dissipation is described by the following dependence

$$P_{dis}(t) \cong U(|i|) \cdot |i| = \frac{i^2}{G(i)} \quad (3),$$

where $G(I)$ – static characteristic of column conductance. By substituting formulas (2) and (3) to (1) it is possible to obtain the Mayr modified equations in the conductance form

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_M} \left[\frac{G(i)}{g} - 1 \right] \quad (4),$$

where

$$G(i) = \frac{i^2}{P_{dis}(i)} = \frac{i^2}{U(|i|, p) \cdot |i|} \quad (5),$$

and p designates the vector of the parameters of the characteristic. Based on the foregoing it is possible to express model (1) in the Novikov-Schellhase conductance form [8, 9]

$$\frac{1}{g} \frac{dg}{dt} = \frac{1}{\theta_{sh}} \left[\frac{|i|}{g \cdot U(|i|, p)} - 1 \right] = \frac{1}{\theta_{sh}} \left[\frac{|u|}{U(|i|, p)} - 1 \right] \quad (6).$$

Using equation (6) it is possible to obtain the macromodel of the arc column, where the voltage of the controlled voltage source is expressed by the following formula

$$u = \frac{i}{g} \quad (7).$$

The transformation of formula (6) enables the obtainment of the integral form of the model.

$$g = g_0 \exp \left[\frac{1}{\theta_{sh}} \int \left(\frac{|u|}{U(|i|, p)} - 1 \right) dt \right] \quad (8).$$

In the macromodel built on the basis of the aforesaid integral model, an appropriately directed controlled source of current constitutes the non-linear conductance of arc

$$i = ug \quad (9).$$

The differential and integral form of the Pentegov model of electric arc

The mathematical modelling of the dynamic characteristics of the electric arc column can be performed using the Pentegov model developed along with Sidoretz [10]. In the aforesaid model, hypothetical arc is considered instead of actual arc. In the hypothetical arc model the conductance of the arc column is defined as the

function of fictitious (virtual) state current $i\theta(t)$ described by differential equation

$$\theta \frac{di_{\theta}^2}{dt} + i_{\theta}^2 = i^2 \quad (10)$$

with defined time constant $\theta = \text{const}$. The Pentegov model represents a non-linear two-terminal network which is balanced in terms of energy, thermally inert of the 1st order, linear, stationary and electrically inertialess. In accordance with the adopted assumption, the current and voltage of the above-named model satisfy condition

$$\frac{i}{u} = \frac{i_{\theta}}{U(i_{\theta})} = g \quad (11),$$

where $U(I)$ – static current-voltage characteristic of arc. The Pentegov assumption imposes restrictions on the parameters of the mathematical model by the following dependence

$$Q(i_{\theta}, p) = 2\theta \int_0^{i_{\theta}} U(i_{\theta}, p) di_{\theta} \quad (12),$$

where Q – plasma enthalpy. In the general case, arc column voltage is expressed by dependence

$$u = \frac{U(i_{\theta}, p)}{i_{\theta}} i \quad (13).$$

In the macromodel of arc, the controlled voltage source, the value of which is expressed by formula (12), is directed oppositely to current and, in this manner, has the properties of the passive non-linear element.

The model expressed by equation (10) can also adopt the integral form

$$\begin{aligned} i_{\theta}^2 &= i_{\theta 0}^2 \exp\left(\frac{1}{\theta} \int \left(\frac{i^2}{i_{\theta}^2} - 1\right) dt\right) = \\ &= i_{\theta 0}^2 \exp\left(\frac{1}{\theta} \int \left(\frac{u^2 g^2}{i_{\theta}^2} - 1\right) dt\right) \end{aligned} \quad (14).$$

Similar to the Cassie equation and its modifications [11], the above presented equation can be on both sides provided with the extraction of roots. As a result, the equation becomes simplified and the value of denominator containing time constant increases by twice.

$$\begin{aligned} i_{\theta} &= i_{\theta 0} \exp\left(\frac{1}{2\theta} \int \left(\frac{i^2}{i_{\theta}^2} - 1\right) dt\right) = \\ &= i_{\theta 0} \exp\left(\frac{1}{2\theta} \int \left(\frac{u^2 g^2}{i_{\theta}^2} - 1\right) dt\right) \end{aligned} \quad (15)$$

The knowledge of state current i_{θ} makes it possible to determine the efficiency of i of the controlled current source having the properties of the passive non-linear element

$$i = \frac{i_{\theta}}{U(i_{\theta}, p)} u \quad (16).$$

The differential and integral form of the Mayr-Pentegov model of electric arc

More generalised in relation to (10) is the equation of the Mayr-Pentegov having form [12]

$$\theta(i_{\theta}, p) \frac{di_{\theta}^2}{dt} + i_{\theta}^2 = i^2 \quad (17),$$

containing the designation of the damping function

$$\theta(i_{\theta}, p) = Q_p \frac{dg}{di_{\theta}^2} \quad (18)$$

dependent on coefficient Q_p in the formula for enthalpy [13]. As previously, p represents the vector of parameters. In the macromodel of the arc column with the controlled voltage source (developed on the basis of the above-presented equation) it is possible to use a dependence analogous to (13)

$$u = \frac{i}{G(i_{\theta}^2, p)} = \frac{U(i_{\theta}, p)}{i_{\theta}} i \quad (19).$$

The integral form of the mathematical Mayr-Pentegov model with the variable damping function is the following

$$\begin{aligned} i_{\theta}^2 &= i_{\theta 0}^2 \exp\left(\int \frac{1}{\theta(i_{\theta}^2, p)} \left(\frac{i^2}{i_{\theta}^2} - 1\right) dt\right) = \\ &= i_{\theta 0}^2 \exp\left(\int \frac{1}{\theta(i_{\theta}^2, p)} \left(\frac{u^2 g^2}{i_{\theta}^2} - 1\right) dt\right) \end{aligned} \quad (20).$$

Similar to the Cassie equation and its modifications [11], the above presented equation can be simplified by providing it on both sides with the extraction of roots. The formula obtained as a result is the following

$$i_{\theta} = i_{\theta 0} \exp\left(\int \frac{1}{2\theta(i_{\theta}^2, p)}\left(\frac{i^2}{i_{\theta}^2} - 1\right) dt\right) =$$

$$= i_{\theta 0} \exp\left(\int \frac{1}{2\theta(i_{\theta}^2, p)}\left(\frac{u^2 g^2}{i_{\theta}^2} - 1\right) dt\right) \quad (21)$$

Based on the calculated value of state current and conductance using

$$i = G(i_{\theta}^2, p) \cdot u = \frac{ui_{\theta}}{U(i_{\theta}, p)} \quad (22)$$

it is possible to determine the efficiency of appropriately directed current source representing non-linear resistance.

Static characteristics of arc

The general form of the static current-voltage characteristic has the following form

$$U(I, p) = \frac{P_M I}{I^2 + I_M^2} + U_C + R_p I \quad (23),$$

where parameters $(P_M, I_M, U_C, R_p) \in p$ can be defined similar to those of the modified mathematical models of arc [11]: P_M – power of the modified Mayr model, W; I_M – current corresponding to ignition voltage in the modified Mayr model, A; U_C – voltage in the Cassie model, V; R_p – column resistance related to the rising characteristic, Ω .

To use the simplified Mayr-Pentegov model (17) it is necessary to calculate conductance

$$G(I^2, p) = \frac{I}{U(I, p)} =$$

$$= \frac{I^2}{\frac{P_M I^2}{I^2 + I_M^2} + U_C \sqrt{I^2} + R_p I^2} \quad (24),$$

the derivative of which is expressed by a formula without simplifications in relation to current I^2

$$\frac{dG(I^2, p)}{dI^2} = \quad (25).$$

$$= \frac{2P_M (I^2)^{3/2} + I_M^4 U_C + 2I_M^2 U_C I^2 + U_C (I^2)^2}{2\sqrt{I^2} \left(P_M \sqrt{I^2} + I_M^2 U_C + I_M^2 R_p \sqrt{I^2} + U_C I^2 + R_p (I^2)^{3/2} \right)^2}$$

Figures 1 and 2 present the diagrams of current-voltage characteristics expressed by formula (23) and corresponding diagrams

of conductance characteristics (24). The minimum values of conductance $G(I = 0) > 0$ S and, at the same time, the values of the components of conductance at this point (Fig. 3) reach a finite maximum. In accordance with formula (18), the foregoing provides the maximum value of the time constant. The above-named states were observed during the experimental tests of arc [14].

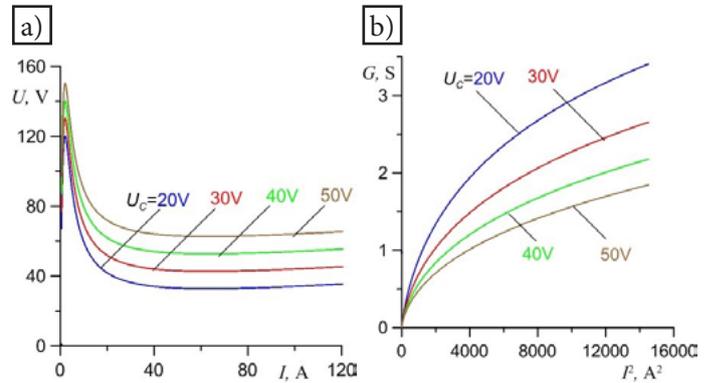


Fig. 1. Static characteristics of arc described by formulas (23) and (24) ($P_M = 400$ W, $I_M = 2$ A, $R_p = 0,1$ Ω): a) current-voltage characteristic $U(I, p)$; b) conductance characteristic $G(I^2, p)$

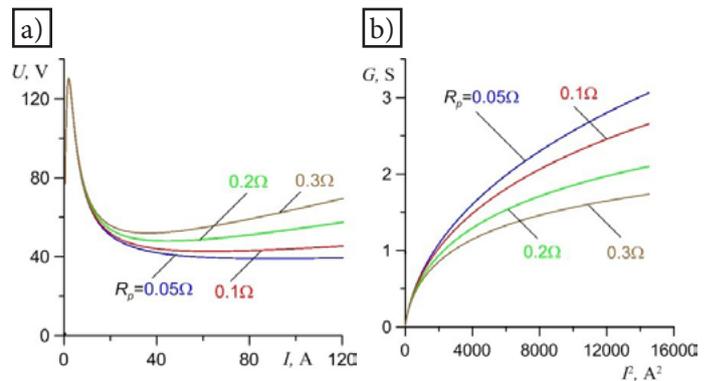


Fig. 2. Static characteristics of arc described by formulas (23) and (24) ($P_M = 400$ W, $I_M = 2$ A, $U_C = 30$ V): a) current-voltage characteristic $U(I, p)$; b) conductance characteristic $G(I^2, p)$

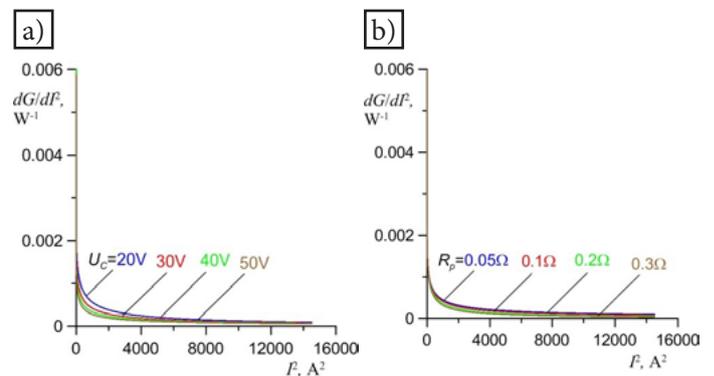


Fig. 3. Characteristics of the derivative of electric arc conductance $dG(I^2, p)/dI^2$ described by formula (25) ($P_M = 400$ W, $I_M = 2$ A): a) characteristic with resistance $R_p = 0,1$ Ω ; b) characteristic with voltage $U_C = 30$ V

Results of simulations of differential and integral models in the macromodelling of electric arc

The effectiveness of the differential and integral mathematical models and corresponding macromodels with controlled sources was verified through the simulation of processes in the electric circuit. Excitation was obtained using the source of current generating trapezoid bipolar waveform (amplitude $I_m = 160$ A, up-slope and down-slope of $96\ 000$ A/s) and a frequency of 50 Hz. The adopted simplification consisted in ignoring near-electrode voltage drops as they could be “roughly” treated as components of voltage U_C . Because of the restrictions related to the volume of work, the diagrams present families of dynamic characteristics. The blue colour and the red colour indicate the results obtained using the differential mathematical models, whereas the green colour and the brown colour indicate the results obtained using the mathematical models in the integral form. The use of the same parameters resulted in the overlapping of the results of the tests of the differential models with those of the integral ones. For this reason it was necessary to use varying parameters in order to present the effect of their changes on tendencies of dynamic current-voltage characteristics to undergo deformations. The macromodels utilising the integral form of mathematical models and controlled current sources were characterised by the higher stability of simulation processes in relation to low damping function (time constant) values.

Figure 4 presents the dynamic current-voltage characteristics of arc described using the Novikov-Schellhase model. The static characteristic used in the aforesaid model was expressed by formula (23). As expected, an increase in parameter U_C resulted in the deformation and shifting of the branches of the characteristics towards higher voltage. In turn, an increase in parameter R_p resulted in the deformation of the characteristics within the high-current range.

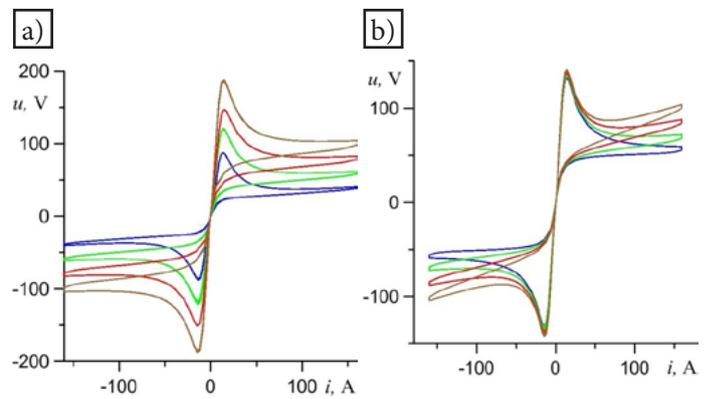


Fig. 4. Dynamic characteristics of electric arc ($P_M = 400$ W, $I_M = 2$ A, $q = 1 \cdot 10^{-4}$ s):

- a) characteristic described by formulas (6) and (7) ($R_p = 0.1 \Omega$, $U_C = 20$ V [blue line], 40 V [green line], 60 V [red line], 80 V [brown line]);
- b) characteristic described by formulas (8) and (9) ($U_C = 50$ V, $R_p = 0.02 \Omega$ [blue line], 0.1Ω [green line], 0.2Ω [red line], 0.3Ω [brown line])

Figure 5 presents the dynamic current-voltage characteristics of arc described by the Pentegov model (10)-(16). Also in the aforesaid case, the static characteristic was used (23). The tests involved an increase in parameters P_M and U_C , which, shifted the characteristics towards higher voltage. Afterwards, parameters P_M and R_p were increased, which resulted not only in shifting the characteristics towards higher voltage, but also in their deformation, particularly in the high-current range.

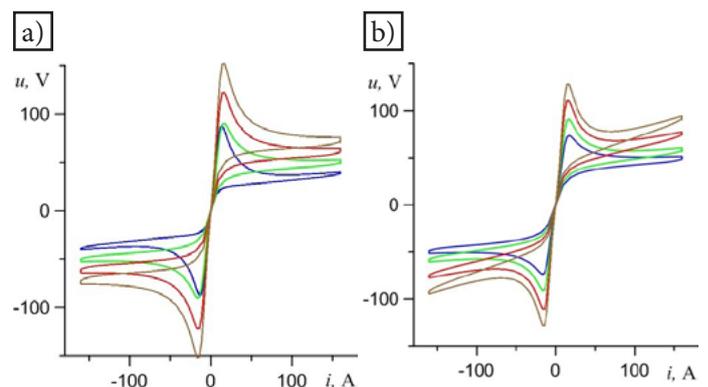


Fig. 5. Dynamic characteristics of electric arc ($I_M = 2$ A, $\theta = 1 \cdot 10^{-4}$ s):

- a) characteristic described by formulas (10) and (13) ($R_p = 0.05 \Omega$; $P_M = 150$ W, $U_C = 30$ V [blue line]; $P_M = 300$ W, $U_C = 40$ V [green line]; $P_M = 450$ W, $U_C = 50$ V [red line]; $P_M = 600$ W, $U_C = 60$ V [brown line]);
- b) characteristic described by formulas (15) and (16) ($U_C = 40$ V; $P_M = 150$ W, $R_p = 0.05 \Omega$ [blue line]; $P_M = 300$ W, $R_p = 0.1 \Omega$ [green line]; $P_M = 450$ W, $R_p = 0.2 \Omega$ [red line]; $P_M = 600$ W, $R_p = 0.3 \Omega$ [brown line])

Figure 6 presents the dynamic current-voltage characteristics of arc described by Mayr-Pentegov model (17)-(22). The case involved the use of the static characteristic, expressed by formulas (23) and (24) and the simultaneous changes of several parameters. Similar to the previous case, it was possible to notice the shifting and deformation of the shape of the dynamic characteristics.

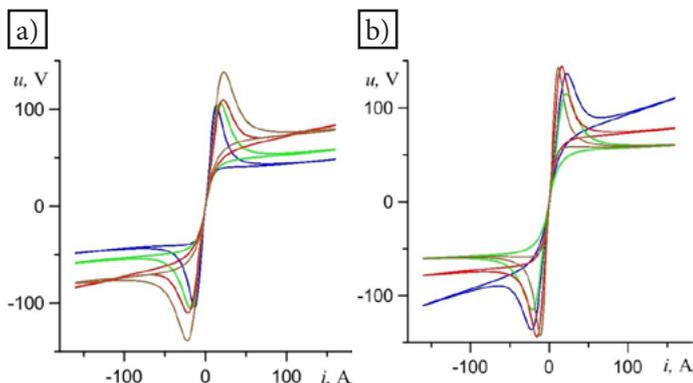


Fig. 6. Dynamic characteristics of electric arc ($I_M = 2 \text{ A}$, $q = 1 \cdot 10^{-4} \text{ s}$): a) characteristic described by formulas (17) and (19) ($P_M = 400 \text{ W}$, $U_C = 30 \text{ V}$, $R_p = 0.1 \Omega$, $Q_p = 0.1 \text{ J}$ [blue line]; $P_M = 400 \text{ W}$, $U_C = 40 \text{ V}$, $R_p = 0.1 \Omega$, $Q_p = 0.2 \text{ J}$ [green line]; $P_M = 300 \text{ W}$, $U_C = 50 \text{ V}$, $R_p = 0.2 \Omega$, $Q_p = 0.3 \text{ J}$ [red line]; $P_M = 500 \text{ W}$, $U_C = 60 \text{ V}$, $R_p = 0.1 \Omega$, $Q_p = 0.4 \text{ J}$ [brown line]); b) characteristic described by formulas (21) and (22) ($P_M = 400 \text{ W}$; $U_C = 60 \text{ V}$, $R_p = 0.3 \Omega$, $Q_p = 0.4 \text{ J}$ [blue line]; $U_C = 50 \text{ V}$, $R_p = 0.05 \Omega$, $Q_p = 0.3 \text{ J}$ [green line]; $U_C = 60 \text{ V}$, $R_p = 0.1 \Omega$, $Q_p = 0.2 \text{ J}$ [red line]; $U_C = 50 \text{ V}$, $R_p = 0.05 \Omega$, $Q_p = 0.1 \text{ J}$ [brown line])

Conclusions

The integral forms of the mathematical models of arc, involving the use of current-voltage characteristics, make it possible to extend the libraries of simulation programmes by including new macromodels characterised by at least the same effectiveness as the macromodels utilising differential models.

The shape of the static characteristic adopted in the tests was sufficiently universal so that (as was demonstrated by the simulation results) the macromodels developed using the aforesaid characteristic could be used in the modelling of states of operation of many electrotechnical devices.

The sequence of the consideration of mathematical models used in the article was

characterised by the gradual weakening of adopted simplifying conditions. As a result, the final Mayr-Pentegov model may most closely represent actual processes occurring in circuits with electric arc.

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