Michał Böhm

Determination of the Fatigue Service Life of Welded Joints Using the Spectral Method Defined in the Frequency Domain

Abstract: The paper discusses presently experienced problems concerned with forecasting the fatigue life of welded joints in terms of the spectral method defined in the frequency domain. In addition, the article presents the primary assumptions of the spectral method and describes the issue related to the recognition of the mean stress value and loads above the material yield point.

Keywords: welded joints, fatigue durability, spectra method, frequency domain

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Introduction

For many years, the determination of the fatigue service life of welded joints has constituted one of the crucial issues related to the operation of welded structures. The quality of calculated results concerning the fatigue service life of joints is strictly connected with engineer's own experience and structure-related information obtainable without performing laboratory tests. It is assumed that calculations are performed for the so-called constant amplitude cycles. Such cases require the assumption of an appropriate hypothesis related to the accumulation of fatigue failures and the determination of a forecast joint operation time expressed through the number of cycles or hours. However, the above-presented situation rarely reflects actual operating conditions, where loads are of changeable amplitude nature and frequently exceed a previously assumed amplitude limit adopted for calculations. In terms of changeable amplitude loads it is necessary to use cycle counting methods, appropriately arranging values of amplitudes and their effect on fatigue service life. To this end, it is necessary to know the history of the load or the forecast time spectrum of the load. The aforesaid data can be obtained by performing extensometric measurements, measurements involving the use of acceleration sensors or by applying video methods. However, the above-named methods are extremely time-consuming and, consequently, costly as they require the performance of very long measurements affecting the accuracy of calculated results. Regrettably, even a long measurement cannot provide a representative mean value, therefore it is necessary to perform several series of measurements at various spots of the structure (multiplying the costs of such an action). Naturally, a question that arises is whether there is a cheaper method enabling the obtainment of statistically relevant values concerning the stochastic history of the load affecting a given structure. A method which does make it possible to take into consideration the random nature of the load and

dr inż. Michał Böhm (PhD (DSc) Eng.) – Opole University of Technology, Faculty of Mechanical Engineering; Department of Mechanics and Machine Design

satisfy the condition related to the certain averaging of the process is the spectral method of fatigue service life determination. Importantly, the aforesaid method is significantly faster than the cycle counting method as a time required for calculating a single cycle constitutes between 1/10 and 1/20 of a time required to calculate time domain. The aforesaid method enables the obtainment of the so-called power spectral density (PSD) corresponding to the infinite number of courses in time. An exemplary graph presenting the power spectral density of the course of load is presented in Figure 1.

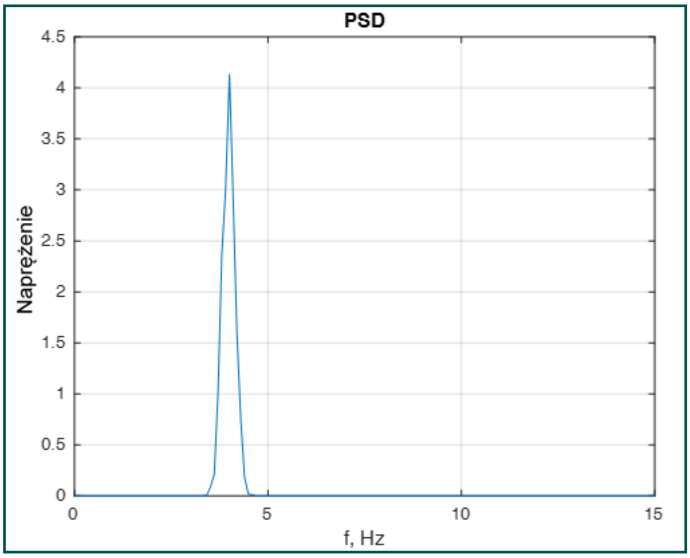

Fig. 1. Exemplary power spectral density related to the load curve

In relation to this method, the distribution of load amplitudes is of stationary and Gaussian nature, which is additionally consistent with the theory of stochastic processes expressed, among others, by J. S. Bendat and A. G. Piersol [1]. However, in reality it is not always possible to encounter such ideal random processes. Usually, various processes are non-stationary and non-Gaussian. Therefore, also the classical approach involving the use of the spectral method will require the correction of calculations by making allowances for disturbances in relation to ideal conditions. Recent years have seen the significant development of the aforesaid method in works by C. Bracessie et al. [2], A. Niesłony and M. Böhm [3,4]. However, the above-named phenomena are connected with external loads, not including welding process-triggered loads. Because of its high temperature and shrinkage in the heat affected zone, the welding process may generate additional stresses within the weld. The above-named stresses are defined in the process of determining fatigue service life using models taking into consideration the mean stress value such as, for instance the Goodman model or the Gerber model. This article discusses the limitations of the classical method used for the determination of the fatigue service life of welded joints in the frequency domain and presents currently used solutions making it possible to identify the effect of the mean stress value as well as the phenomenon of the non-linearity and non-gaussivity of the process.

Spectral-method based determination of fatigue service life

In terms of fatigue service life determined using the spectral method, the averaging of the load is related to the stochastic nature of the process. The above named-method consists in the generation of the power spectral density (PSD) of the load for a given mathematical model or by using a time course defined as a signal. The power spectral density of a random signal describes the general frequency structure of a process by means of the spectral density of the root-mean-square value of a physical signal under consideration. Using the narrow-range band-pass filter and averaging the signal at the filter output, the above-named value can be identified in relation to the range from f to f+Δf [1]:

$$\Psi_x(f,\Delta f) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t,f,\Delta f)\,dt, \qquad (1)$$

where Ψ_x – root-mean-square value of course $x(t)$, T – time of observation, $x(t, f, \Delta f)$ – component of function $x(t)$ within the frequency range from f to $f + \Delta f$.

For low values of Δf, formula (1) adopts the function of one sided PSD and adopts the following form:

$$G_x(f) = \lim_{\Delta f \to 0} \frac{\Psi_x(f,\Delta f)}{\Delta f} = \Psi_x =$$
$$= \lim_{\Delta f \to 0} \frac{1}{\Delta f}\left[\lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t,f,\Delta f)\,dt\right] \quad (2)$$

Afterwards, it is necessary to determine the so-called spectral moments, used to identify the distribution of probability density in relation to a given model

$$\xi_k = \int_0^\infty G_x(f)\, f^k df \quad (3)$$

for k=0, 1, 2, 3.

The application of the appropriate distribution of stress amplitude probability density is strictly connected with a type of an element to which loads will be subjected. There are many examples of such distributions of probability density related to a specific industrial sector [5–8]. However, there are also widely applicable distributions suitable for stationary Gaussian processes. An example of the aforesaid distributions include the Rayleigh distribution recommended to be used with the spectral moments by J. S. Bendat [9]:

$$p(\sigma_a) = \frac{\sigma_a}{\xi_0} exp\left(\frac{-\sigma_a{}^2}{2\xi_0}\right) \quad (4)$$

The model developed by T. Dirlik [10] was the first one to take into consideration the so-called tail for low values of stress amplitudes:

$$p(\sigma) = \frac{1}{2\sqrt{\xi_0}}\left[\frac{K_1}{K_4} e^{\frac{-Z}{K_4}} + \frac{K_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + K_3 Z e^{\frac{-Z^2}{2}}\right] \quad (5)$$

where K, R and Z are parameters determined using spectral moments and where C and m- are constants of the Basquin curve, b- tempering function dependent on power spectral density. In turn, ν_p is determined using the following dependence:

$$\nu_p = \frac{1}{2\pi}\sqrt{\frac{\xi_4}{\xi_2}} \quad (6)$$

Finally, values obtained using the above-presented calculations are entered into the ultimate formula for the determination of fatigue service life:

$$T_{cal} = \frac{1}{M^+ \int_0^\infty \frac{p(\sigma_a)}{N(\sigma_a)} d\sigma_a} \quad (7)$$

where $p(\sigma_a)$ – distribution of stress amplitude probability density, $N(\sigma_a)$ – number of cycles determined using curves S-N in relation to a given amplitude of stress σ_a, M^+ expected number of peaks at a time unit determined using the following dependence:

$$M^+ = \sqrt{\frac{\xi_4}{\xi_2}} \quad (8)$$

Internal stresses in terms of the spectral method

In relation to welded joints, an important fact concerning the spectral method is the fact that it does not directly take into consideration mean stresses, known to play an enormous role as far as welded joints are concerned. Frequently, when determining fatigue service life, internal stresses are taken into consideration as a mean value. Usually, such stresses are present in the area of the heat affected zone, which, as a result of previously adjusted welding process parameters, may be characterised by compressive or tensile internal stresses. One of the first works concerned with this phenomenon was by D. P. Kihl and S. Sarkani [11], who, using crosswise specimens (Fig. 2) joined with a fillet weld, undertook to determine fatigue service life applying the spectral method and taking the mean value into consideration:

$$N_{cal} = \left(1 - \frac{\sigma_m}{R_m}\right)^{-m} \cdot \frac{2^{\frac{m}{A}} \cdot \left(\sqrt{\xi_0}\right)^m \cdot A}{\Gamma\left(1 - \frac{m}{2}\right)} \quad (9)$$

where N_{cal} – number of cycles preceding fatigue crack initiation, R_m – tensile strength, A and m – constants determined using the Wöhler diagram for a constant amplitude load.

The above-presented approach enabled the determination of a number of cycles using information obtained from power spectral density and the distribution-related gamma function.

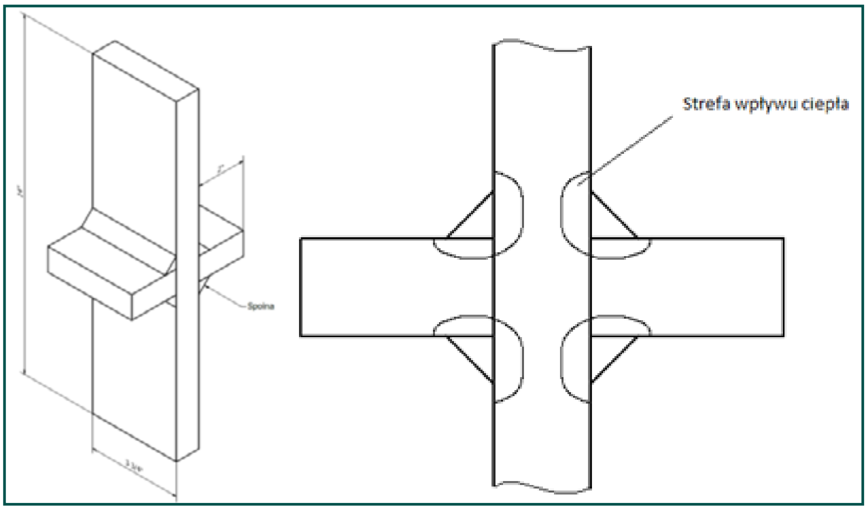

Fig. 2. Crosswise specimen used in fatigue tests performed by D. P. Kihl and involving the non-zero mean value of stress

However, the aforesaid approach was also limited by the fact that it was not universal. The publication by M. Böhm [12] presents a universal algorithm enabling the determination of fatigue service life defined in the frequency domain. The algorithm is based on the transformation of power spectral density through information concerning the mean values of stress:

$$G_{xT}(f) = [K(\sigma_m, P)]^2 G_x(f) \tag{10}$$

where K can be determined using one of the classical models, e.g. by W. Z. Gerber, [13] or any appropriately defined model:

$$K_{Ge} = \frac{1}{1-\left(\frac{\sigma_m}{R_m}\right)^2} \tag{11}$$

After the transformation of power spectral density (taking the mean value into consideration), the entire principal procedure of the determination of fatigue service life does not differ from that presented in this publication. Experimental tests by D. P. Kihl were performed for several levels of values of stress ratio $R=\sigma_{min}/\sigma_{max}$. Figure 3 presents the comparison of fatigue service life determined using the spectral method in relation to the approach adopted by D. P. Kihl as well as fatigue service life determined using the solution proposed by M. Böhm for R = 0.66 in relation to several classical models used to determine fatigue service life and based on publication [12].

Phenomena influencing the non-stationarity of the load affecting the welded joint

In addition to internal stresses, another important factor affecting the determination of fatigue service life is the maintaining of the stationarity of the load affecting the welded joint. The idealised load state can often lead to significant damage and accidents accompanying the use of a welded structure. The assumption of stationarity during operation as well as the distribution of amplitudes according to K. Gauss may result in ignoring certain phenomena, such as momentary overloads, which could exceed the material yield point. The presence of overload in a given structure affects the stationarity of the former, which should unequivocally exclude the application of the spectral method to determine fatigue service life. In such a case, the aforesaid loads should be analysed as conditionally non-stationary, with a separated part responsible for overload and power spectral density intensified using a coefficient taking this aspect into consideration. The first solution

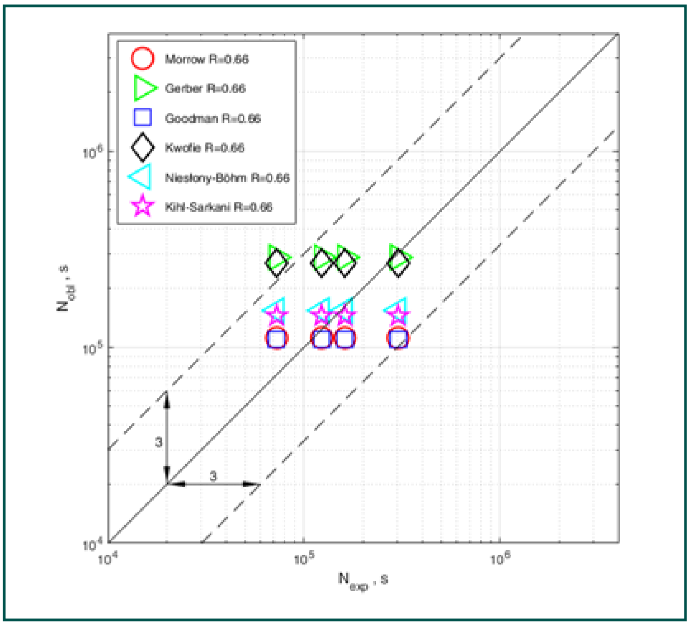

Fig. 3. Comparison of results of experimental and computed fatigue service life in relation to selected models (discussed in scientific reference publications) and stress ratio R = 0.66 [12]

of this type was proposed by M. Böhm and M. Kowalski [4], taking into consideration the peak factor for the value of a spectral kurtosis determined from the so-called kurtogram (Fig. 4) :

$$S_K = \frac{\xi_4}{\xi_2^2} - 2 \tag{12}$$

The spectral kurtosis diagram is used to determine peak factor CF, where the very idea of the peak factor can be illustrated as presented in Figure 5:

$$C_F = \frac{|S_{Kpeak}|}{S_{Krms}} \tag{13}$$

Afterwards, the intensification of power spectral density is performed using the following formula:

$$G_{xT}(f) = C_F \cdot G_x(f) \tag{14}$$

The above-presented deviation from stationarity can be used for calculations applying the classical path used for the determination of fatigue service life by means of the spectral method.

Concluding remarks

Because of the complex nature of the fatigue process in relation to welded joints, all methods used to determine fatigue service life will be closer to reality if more factors affecting it are taken into consideration. The methods presented in the study do not take into consideration structural properties such as the size of grains or various phases but are concerned with the use of mechanical properties of materials. As mentioned in the Introduction, the determination of fatigue service life based on the spectral method is significantly faster in comparison with methods defined in the time domain. In spite of the unquestionable advantages of the spectral methods it is necessary to know the limitations of the method along with corrections which must be applied because of the effect of the mean value. In addition, it is also

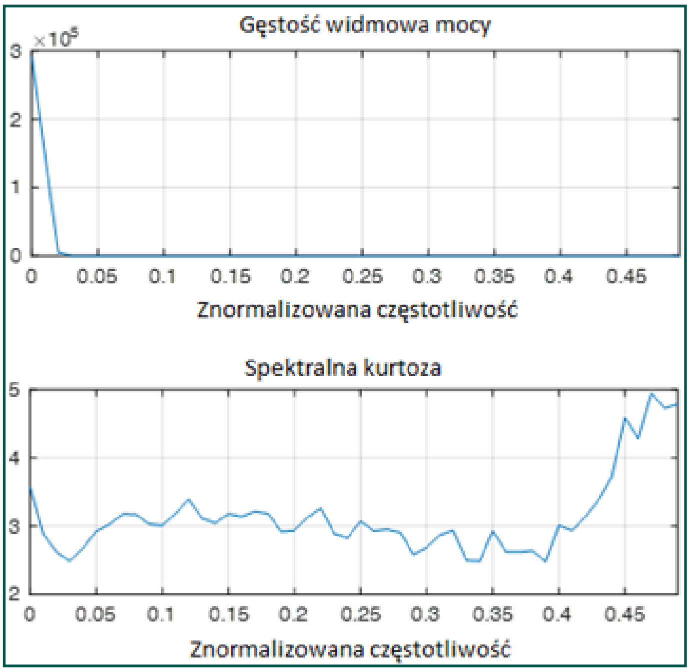

Fig. 4. Power spectral density and spectral kurtosis for normalised frequency and stationarity disturbed by the weld overload

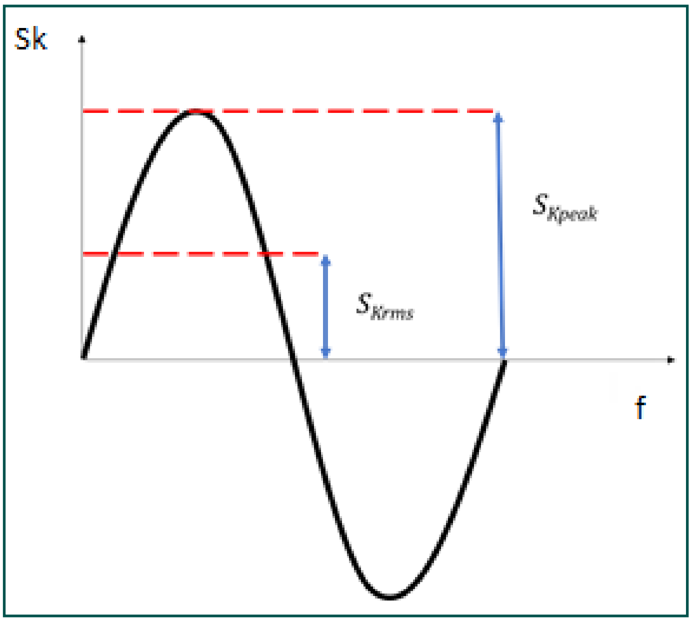

Fig. 5. Ideogram of the adjustment of the peak factor for the value of spectral kurtosis

necessary to be familiar with the phenomena of overload and stationarity disturbances potentially leading to the exceeding of the yield point of a given material. Because of the nature of the research, the study presents issues connected with presently developed solutions aimed to minimise errors resulting from lacking corrections. In spite of the fact that some works are at the simulation stage, they can be used by design engineers considering the application of the spectral method to determine the fatigue service life of welded joints.

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