

Antoni Sawicki

# The Mayr-Pentegov Model of Electric Arc with Selected Static Current-Voltage Characteristics

---

**Abstract:** The article discusses the basic properties of the Mayr-Pentegov mathematical model of electric arc. The study involved the selection of a set of generalised functions enabling the approximation of static voltage-current characteristics. The functions were used to determine the derivatives of the conductance function in relation to the squared current (used to calculate the non-linear damping function). In addition, the article presents the families of static characteristics dependent on parameters of approximating functions. As a result of simulations of processes in the circuit with the electric arc model, the families of dynamic current-voltage characteristics were obtained. The application of the wide ranges of parameter changes demonstrated the usability of the developed model utilising various approximations of static characteristics.

**Keywords:** electric arc, Pentegov model, Mayr-Pentegov model

**DOI:** [10.17729/ebis.2020.3/6](https://doi.org/10.17729/ebis.2020.3/6)

---

## Introduction

The variety of electric arc burning conditions in electrical and electrotechnical equipment necessitates the application of various mathematical models. The aforesaid conditions (e.g. the value of current as well as its waveform and frequency) and the properties of a given model affect the possibly easy experimental determination of parameters, the accuracy of the mapping of static and dynamic characteristics, the effectiveness of control system operation, the complexity of the design and the proper functioning of arc imitators etc. Simple mathematical models are characterised by a small number of parameters and, consequently, the ease of their identification. At the same time, by adopting many simplifying

assumptions, the above-named models are characterised by narrow ranges of applicability and lower approximation accuracy. In turn, complex mathematical models are characterised by a large number of parameters and, consequently, greater difficulty related to their identification. On the other hand, such models may be characterised by wide ranges of applicability and higher approximation accuracy. For this reason, it is worth developing theories about such complex models, where the determination of parameters can be performed in stages and under various physical conditions. This article presents a significantly generalised model, the complexity of which is connected with curves of static characteristics. However, it is known that most of the parameters of the

---

dr hab. inż. Antoni Sawicki (PhD (DSc) Habilitated Eng.) – SEP – Stowarzyszenie Elektryków Polskich (Association of Polish Electrical Engineers), Częstochowa Division

aforsaid characteristics are determined very easily, using direct current or pulsed current of rectangular waveform.

The article discusses a set of generalised and, at the same time, multi-variant functions used for approximating the static current-voltage characteristics of arc. Because of the introduction of a new damping function in the Mayr-Pentegov model of arc it was necessary to define modified conductance-squared current static characteristics. The correctness of obtained formulas was verified using numerical calculations. The results were presented as families of static and dynamic current-voltage characteristics.

### Assumptions of the Mayr-Pentegov mathematical model of electric arc

The Pentegov mathematical model of electric arc constitutes the generalisation of known and commonly used Mayr, Cassie and Zaru-di linear mathematical models [1]. Similar to the above-named simple models, in terms of the Pentegov model the initial point is the energy balance equation. In addition, many simplifying assumptions are used in relation to the Pentegov model. The types of the aforsaid assumptions affect the current range of the applicability of the models (their stability) and the accuracy of the approximation of measurement data. A subsequent stage related to the development of theoretical tests performed in order to reduce the aforsaid limitations and significantly extend approximating possibilities is the Mayr-Pentegov non-linear [2, 3].

The Mayr-Pentegov model utilises the assumption concerning the exponential change of conductance triggered by changes in the enthalpy of thermal plasma  $Q$

$$\exp\left(\frac{Q}{Q_p}\right) = \frac{g}{G_p} \quad (1)$$

where  $Q_p$  – subtangent of the diagram of function  $g(Q)$ ,  $G_p$  – point of the intersection of the diagram with the y-axis  $g(0) = G_p$ . After the

differentiation of the above-presented expression the following proportion is obtained

$$\frac{dQ}{Q_p} = \frac{dg}{g} \quad (2)$$

Similar to the Pentegov model, in the Mayr-Pentegov model takes into consideration hypothetical arc (instead of actual one), where the conductance of the arc column is defined as the function of fictitious state current  $i_\theta(t)$ , changing along with specified non-linear damping function  $\theta(i_\theta, p)$ . Again, similar to the Pentegov model [4], arc in the circuit is modelled by means of a two-terminal network, which is balanced in terms of energy, first-order thermally inert, non-linear, stationary and electrically inertialess. In accordance with related assumptions, the current and voltage of the model satisfy the condition

$$\frac{i}{u} = \frac{i_\theta}{U} = g \quad (3)$$

where  $(I)$  – static current-voltage characteristic of arc. Based on the equation of power balance in the column

$$\frac{dQ}{dt} + Ui_\theta = ui \quad (4)$$

where  $dQ/dt$  – derivative of the changes in the internal energy of plasma,  $ui$  – supplied electric energy;  $Ui_\theta$  – electric power dissipated from the column) it is possible to obtain a 1<sup>st</sup> order non-linear differential equation describing the dynamics of changes in state current  $i_\theta(t)$ , corresponding to changes in plasma temperature

$$\theta(i_\theta, p) \frac{di_\theta^2}{dt} + i_\theta^2 = i^2 \quad (5)$$

where the damping function is designated as follows

$$\theta(i_\theta, p) = Q_p \frac{dg}{di_\theta^2} \quad (6)$$

The above-named function depends not only on state current but also on the vector of

parameters  $p$ . Parameters used in the formulas for the approximation of static characteristic  $U(I, p)$  and constant parameter  $Q_p$  are determined on the basis of experimental tests of arc.

Formulas (3), (5) and (6) can be used to create a macromodel of arc in a simulation software programme. Then, non-linear resistance is mapped by a controlled voltage source having the following value

$$u = \frac{U(i_\theta)}{i_\theta} i \quad (7)$$

and the arrow directed oppositely to current flow [5].

In cases of certain macromodels of arc [6, 7] it is more convenient to apply the theoretically equipollent integral form of the Mayr-Pentegov model. In such a situation, equation (5) can be expressed in the following form

$$\begin{aligned} i_\theta &= i_{\theta 0} \exp\left(\int_0^t \frac{1}{2\theta(i_\theta^2, p)} \left(\frac{i^2}{i_\theta^2} - 1\right) d\tau\right) = \\ &= i_{\theta 0} \exp\left(\int_0^t \frac{1}{2\theta(i_\theta^2, p)} \left(\frac{u^2 g^2}{i_\theta^2} - 1\right) d\tau\right) \end{aligned} \quad (8)$$

Then, the non-linear resistance is mapped by the controlled current source having the following value

$$i = G(i_\theta^2, p) \cdot u = \frac{u i_\theta}{U(i_\theta, p)} \quad (9)$$

and directed oppositely to existing voltage.

Similar to selected versions of the Pentegov model [8, 9], in the case under discussion the determination of parameters of the Mayr-Pentegov model may be performed in two stages, depending on the type of current excitation and the existence of developed identification methods. In the second case, the families of the static characteristics of arc  $U(I, p)$  are determined separately. Afterwards, constant parameter  $Q_p$  and the remaining parameters connected with the dynamics of processes in arc (e.g. residual conductance) are identified [10].

## Selected functions approximating the static current-voltage characteristics of electric arc

Further deliberations involved the most general formulas used for the approximation of the static current-voltage characteristics of electric arc. After assuming that values of certain parameters amounted to zero it was possible to obtain special cases of approximation frequently applied in computational practice. Because of the fact that in some computational models the above-named characteristics are used in the modelling of AC arc, sometimes it is worth taking into consideration a specific value of ignition voltage, resulting from physical processes occurring in the plasma column during momentary current decay [11–13]. The moderation of the passage of the voltage diagram across the origin of coordinates ( $i, u$ ) can also be obtained through the reduction of voltage on the arc column.

Based on the approach proposed by H. Aytona, the formula for the approximation of changes of voltage on the arc column can be expressed in the following form

$$U(I, p) = \frac{P_M}{I} + U_C + R_p I \quad (10)$$

where  $p$  represents the vector of parameters, out of which  $P_M$  – corresponds to the power of arc, particularly significant within the low-current range, similar to the Mayr model,  $U_C$  – corresponds to voltage on the arc column, similar to the Cassie model,  $R_p$  – corresponds to the resistance of arc, particularly significant within the high-current range. As can be seen, function (10) can be used with the assumption of the indeterminacy of ignition voltage and the quasi-hyperbolic reduction of voltage along with an increase in current. For the use of the Mayr-Pentegov model [3], the function of static conductance adopts the following form:

$$\begin{aligned} G(I^2, p) &= \frac{I}{U(I, p)} = \\ &= \frac{I^2}{P_M + U_C I + R_p I^2} = \\ &= \frac{I^2}{P_M + U_C \sqrt{I^2} + R_p I^2} \end{aligned} \quad (11)$$

whereas its derivative adopts the following form

$$\frac{dG(I^2, p)}{dI^2} = \frac{2P_M + U_C I}{2(P_M + U_C I + R_p I^2)^2} \quad (12)$$

A more general form of the formula for the approximation of changes in voltage on the arc column can be obtained using an approach proposed by Nottingham, i.e.:

$$U(I, p) = U_0 \left( \frac{I_0}{I} \right)^n + U_C + R_p I \quad (13)$$

where  $U_0, n, U_C, R_p$  – constant approximation parameters. In the above-named case, the value of ignition voltage also remains indefinite. Similar to the previous case, it is possible to determine the static conductance of the arc column

$$G(I^2, p) = \frac{I^{n+1}}{U_0 I_0^n + U_C I^n + R_p I^{n+1}} = \frac{(I^2)^{(n+1)/2}}{U_0 I_0^n + U_C (I^2)^{n/2} + R_p (I^2)^{(n+1)/2}} \quad (14)$$

The derivative of the aforesaid conductance is the following

$$\frac{dG(I^2, p)}{dI^2} = \frac{I^{n-1} (U_0 I_0^n (n+1) + U_C I^n)}{2(U_0 I_0^n + I^n (U_C + R_p I))^2} \quad (15)$$

Discussed below are cases of the approximation of static current-voltage characteristics including the determinacy of ignition voltage [10]. Based on the modification of approximating function (10) it is possible to obtain the following formula

$$U(I, p) = \frac{P_M I}{I^2 + I_M^2} + U_C + R_p I \quad (16)$$

where, approximately, the value of current  $I_M$  – corresponds to the abscissa of the extreme point on the static characteristic. The aforesaid current is related to the value of residual conductance [10]. The static conductance of the column can be calculated using the following formula:

$$G(I^2, p) = \frac{I(I^2 + I_M^2)}{P_M I + U_C (I^2 + I_M^2) + R_p (I^2 + I_M^2) I} = \frac{\sqrt{I^2 (I^2 + I_M^2)}}{P_M \sqrt{I^2} + U_C (I^2 + I_M^2) + R_p (I^2 + I_M^2) \sqrt{I^2}} \quad (17)$$

The derivative of the above-named static conductance is expressed by the following formula

$$\frac{dG}{dI^2} = \frac{2P_M I^3 + U_C (I^2 + I_M^2)^2}{2I (I(P_M + R_p (I^2 + I_M^2)) + U_C (I^2 + I_M^2))^2} \quad (18)$$

In turn, after the modification of formula (13), aimed to identify the finite value of ignition voltage on the static characteristic, the following formula is obtained:

$$U(I, p) = U_0 \left( \frac{I_0 I}{I^2 + I_M^2} \right)^n + U_C + R_p I \quad (19)$$

The formula corresponds to static conductance

$$G(I^2, p) = \frac{I(I^2 + I_M^2)^n}{U_0 (I_0 I)^n + U_C (I^2 + I_M^2)^n + R_p (I^2 + I_M^2)^n I} = \frac{\sqrt{I^2 (I^2 + I_M^2)^n}}{U_0 I_0^n (I^2)^{n/2} + U_C (I^2 + I_M^2)^n + R_p (I^2 + I_M^2)^n \sqrt{I^2}} \quad (20)$$

The derivative of the aforesaid conductance is expressed by the following formula

$$\frac{dG(I^2, p)}{dI^2} = \frac{(I^2 + I_M^2)^{n-1} (U_0 I_0^n I^n (I_M^2 (1-n) + I^2 (1+n)) + U_C (I^2 + I_M^2)^{n+1})}{2I (U_0 I_0^n I^n + (U_C + R_p I) (I^2 + I_M^2)^n)^2} \quad (21)$$

In the approximations of characteristics of long (high-voltage) arc, voltage component  $U_C$  may reach high values. At the same time, the above-named component is lower than the value of ignition voltage, decisive for the course of electric processes. For this reason, in order to obtain more accurate approximation, in some cases it is necessary to introduce the reduction of component  $U_C$ , particularly if ignition voltage with corresponding residual conductance is taken into consideration. In the general case

of the determinacy of ignition voltage and voltage reduction it is possible to use the static current-voltage characteristic

$$U(I, p) = \frac{P_M I}{I^2 + I_M^2} + \frac{U_C I}{I + I_W} + R_p I \quad (22)$$

As a result, it is possible to determine the conductance of the arc column

$$G(I^2, p) = \frac{(I^2 + I_M^2)(I + I_W)}{P_M(I + I_W) + U_C(I^2 + I_M^2) + R_p(I^2 + I_M^2)(I + I_W)} = \frac{(I^2 + I_M^2)(\sqrt{I^2 + I_W})}{P_M(\sqrt{I^2 + I_W}) + U_C(I^2 + I_M^2) + R_p(I^2 + I_M^2)(\sqrt{I^2 + I_W})} \quad (23)$$

The conductance derivative is calculated using the following formula

$$\frac{dG}{dI^2} = \frac{2P_M I(I + I_W)^2 + U_C(I^2 + I_M^2)^2}{2I((I + I_W)(P_M + R_p(I^2 + I_M^2)) + U_C(I^2 + I_M^2))^2} \quad (24)$$

Similar to formulas (13) and (19), after taking into account the possible power change of the voltage component within the low-current range, it is possible to approximate data using the following formula

$$U(I, p) = U_0 \left( \frac{I_0 I}{I^2 + I_M^2} \right)^n + \frac{U_C I}{I + I_W} + R_p I \quad (25)$$

The formula can be used to determine the correlation for the conductance of the column

$$G(I^2, p) = \frac{(I^2 + I_M^2)^n (I + I_W)}{U_0 I_0^n I^{n-1} (I + I_W) + U_C (I^2 + I_M^2)^n + R_p (I^2 + I_M^2)^n (I + I_W)} = \frac{(I^2 + I_M^2)^n (\sqrt{I^2 + I_W})}{U_0 I_0^n (I^2)^{(n-1)/2} (\sqrt{I^2 + I_W}) + U_C (I^2 + I_M^2)^n + R_p (I^2 + I_M^2)^n (\sqrt{I^2 + I_W})} \quad (26)$$

The derivative of the expression has the following form

$$\frac{dG}{dI^2} = \frac{(I^2 + I_M^2)^{n-1} (U_0 I_0^n (I + I_W)^2 I^{n+1} (I_M^2 (1-n) + I^2 (1+n)) + U_C I^3 (I^2 + I_M^2)^{n+1})}{2I^2 (U_0 I_0^n (I + I_W) I^n + I (I^2 + I_M^2)^n (U_C + R_p (I + I_W)))^2} \quad (27)$$

As can be seen in the above-presented correlations, taking into consideration a larger number of possible physical effects, mapped by the

static characteristics of the arc column, increases the complexity of expressions concerning the conductance derivative. Because of the Author's suggestion that AC arc should be modelled using the Pentegov model or the Mayr-Pentegov model (with state current  $i_\theta > 0$ ), the obtained expressions are correct within wide ranges of changes of current exciting physical processes in the column. The most general formulas are numbered (25)–(27).

Such parameters of static characteristics as  $I_M$  and  $I_W$  are connected with dynamic processes when current passes the zero value. As a result, their determination should take place during AC powering [9].

### Families of static characteristics of electric arc defined by the parameters of approximating functions

The above-presented formulas for static current-voltage characteristics of arc correspond to formulas for the static characteristics of conductance and conductance derivative. Because of the limited volume of the article, only selected parameters of function  $U(I, p)$  were subjected to gradual changes. The related results are presented as the family of characteristics.

Figures 1a and 1c present the families of the static current-voltage characteristics  $U(I, p)$  of arc with unspecified ignition voltage and the quasi-hyperbolic dependence of voltage on current within the low-current range (10). The above-named families correspond to the families of conductance derivative characteristics  $dG(I^2, p)/dI^2$  presented in Figures 1b and 1d. Changes in parameter  $P_M$  lead to the deformation of voltage curves  $U$  within the

low-current range, whereas changes in parameter  $U_C$  trigger deformations (shifts) of characteristics within the entire current range. In both cases, increases in the above-named parameters correspond to increases in voltage. The effect of changes in the

aforsaid parameters on the deformations of conductance derivative curves is significantly lower. An increase in  $P_M$  slightly increases the conductance derivative. In turn, an increase in

voltage  $U_c$  decreases the conductance derivative. Near the zero value of current, the functions of the conductance derivative may adopt relatively high values, preventing arc termination.

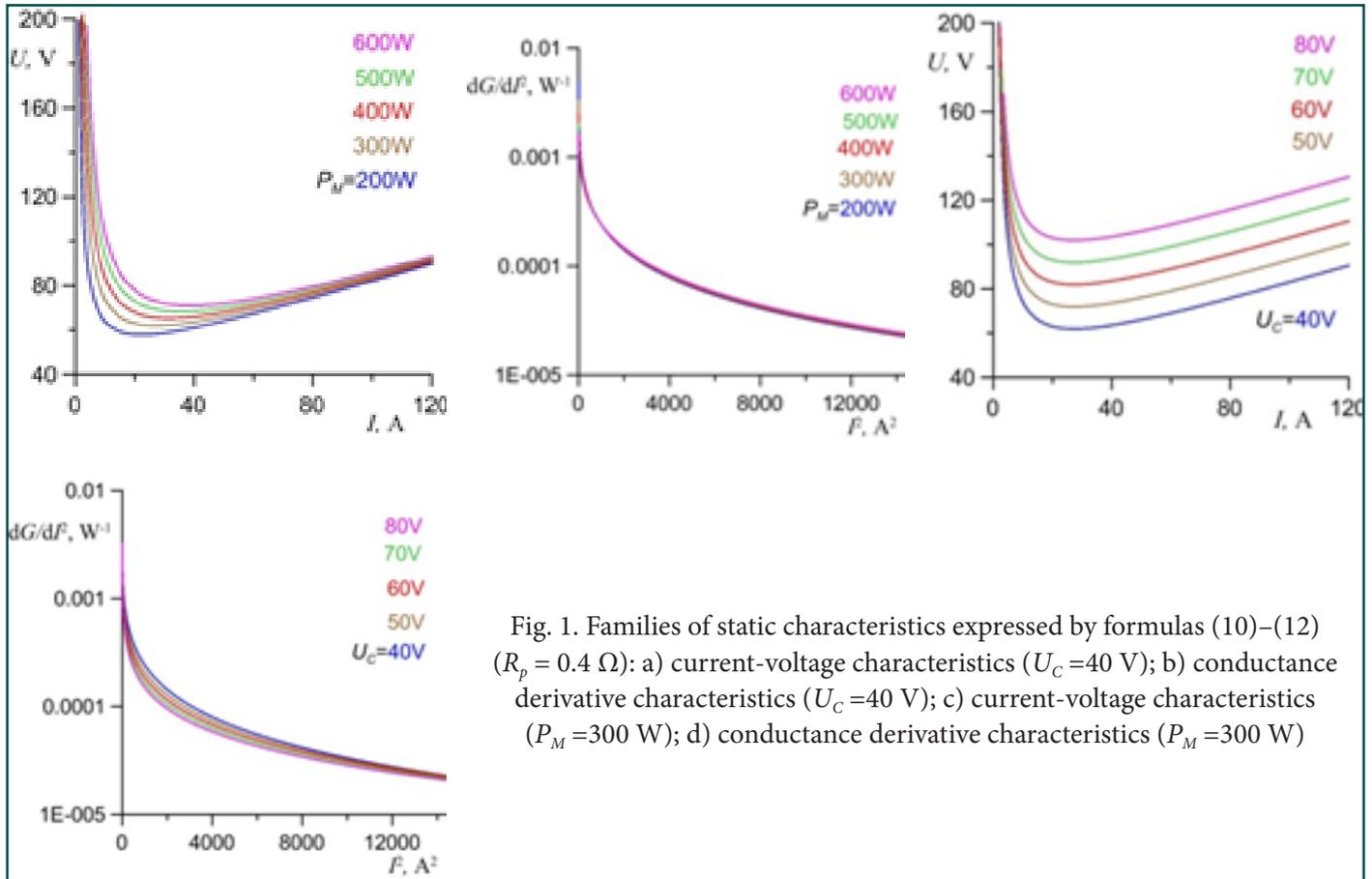


Fig. 1. Families of static characteristics expressed by formulas (10)–(12) ( $R_p = 0.4 \Omega$ ): a) current-voltage characteristics ( $U_c = 40 \text{ V}$ ); b) conductance derivative characteristics ( $U_c = 40 \text{ V}$ ); c) current-voltage characteristics ( $P_M = 300 \text{ W}$ ); d) conductance derivative characteristics ( $P_M = 300 \text{ W}$ )

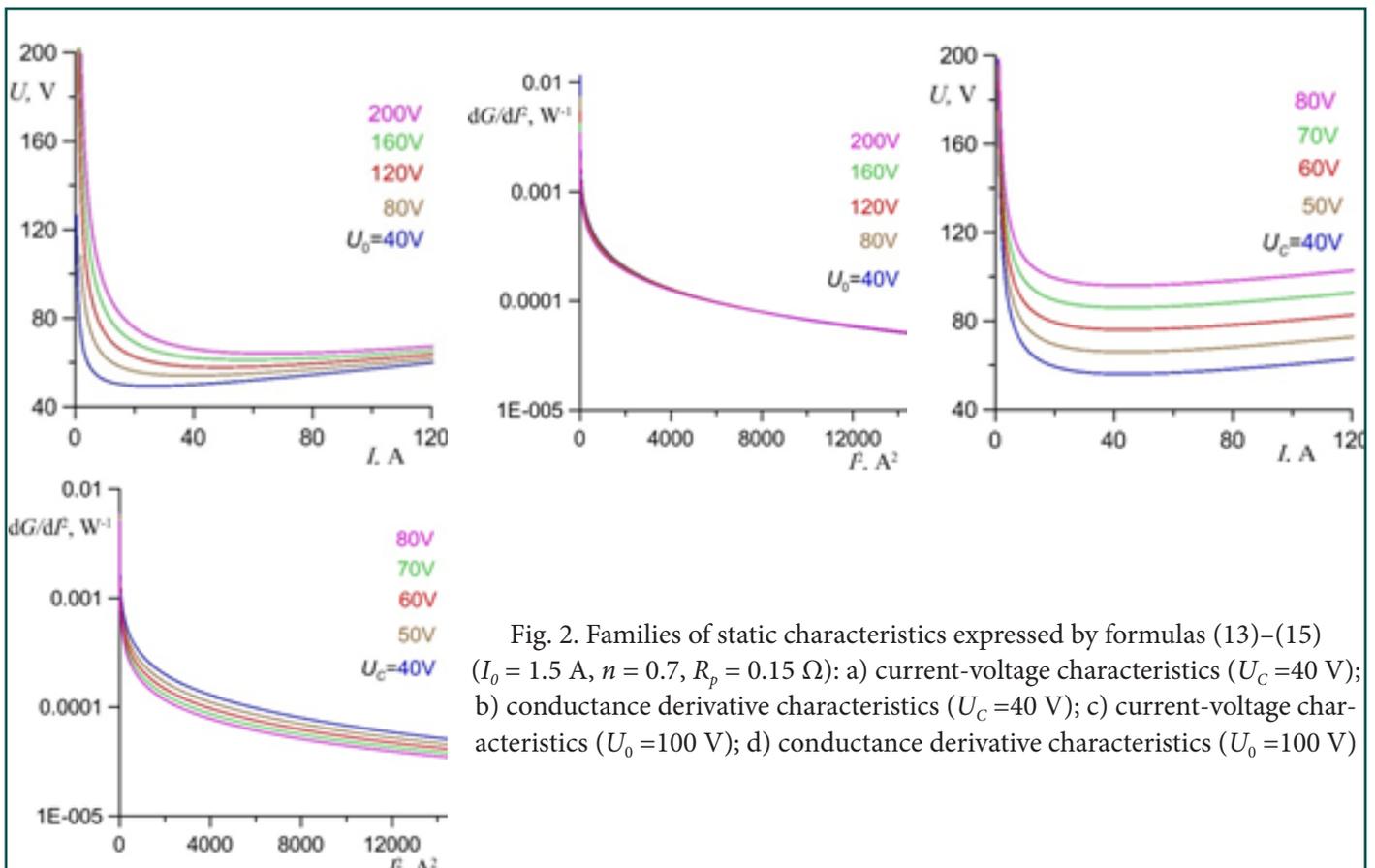


Fig. 2. Families of static characteristics expressed by formulas (13)–(15) ( $I_0 = 1.5 \text{ A}$ ,  $n = 0.7$ ,  $R_p = 0.15 \Omega$ ): a) current-voltage characteristics ( $U_c = 40 \text{ V}$ ); b) conductance derivative characteristics ( $U_c = 40 \text{ V}$ ); c) current-voltage characteristics ( $U_0 = 100 \text{ V}$ ); d) conductance derivative characteristics ( $U_0 = 100 \text{ V}$ )

Figures 2a and 2c present the families of the static current-voltage characteristics  $U(I, p)$  of arc with unspecified ignition voltage and the power dependence of voltage on current within

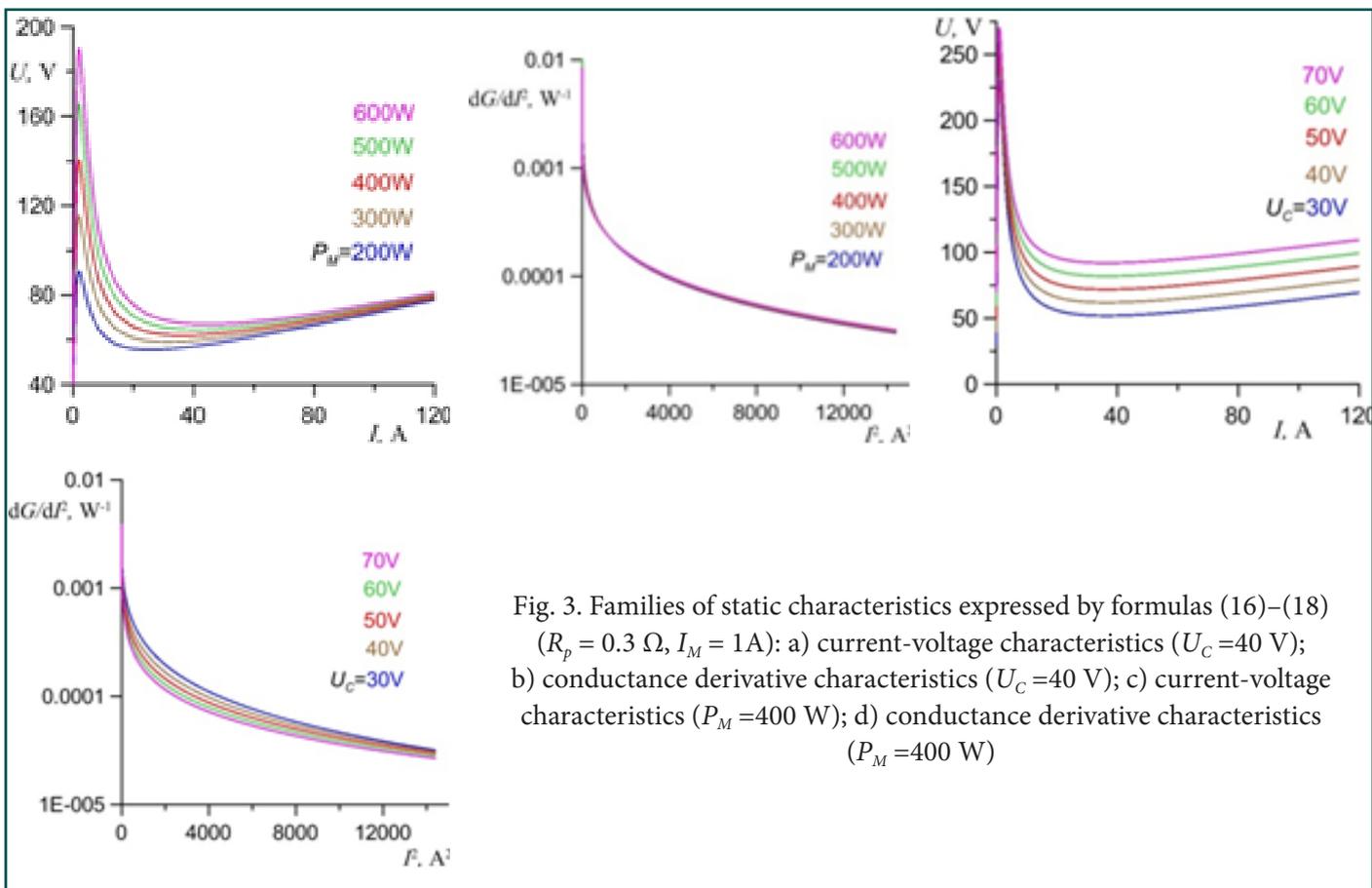


Fig. 3. Families of static characteristics expressed by formulas (16)–(18) ( $R_p = 0.3 \Omega, I_M = 1A$ ): a) current-voltage characteristics ( $U_C = 40 V$ ); b) conductance derivative characteristics ( $U_C = 40 V$ ); c) current-voltage characteristics ( $P_M = 400 W$ ); d) conductance derivative characteristics ( $P_M = 400 W$ )

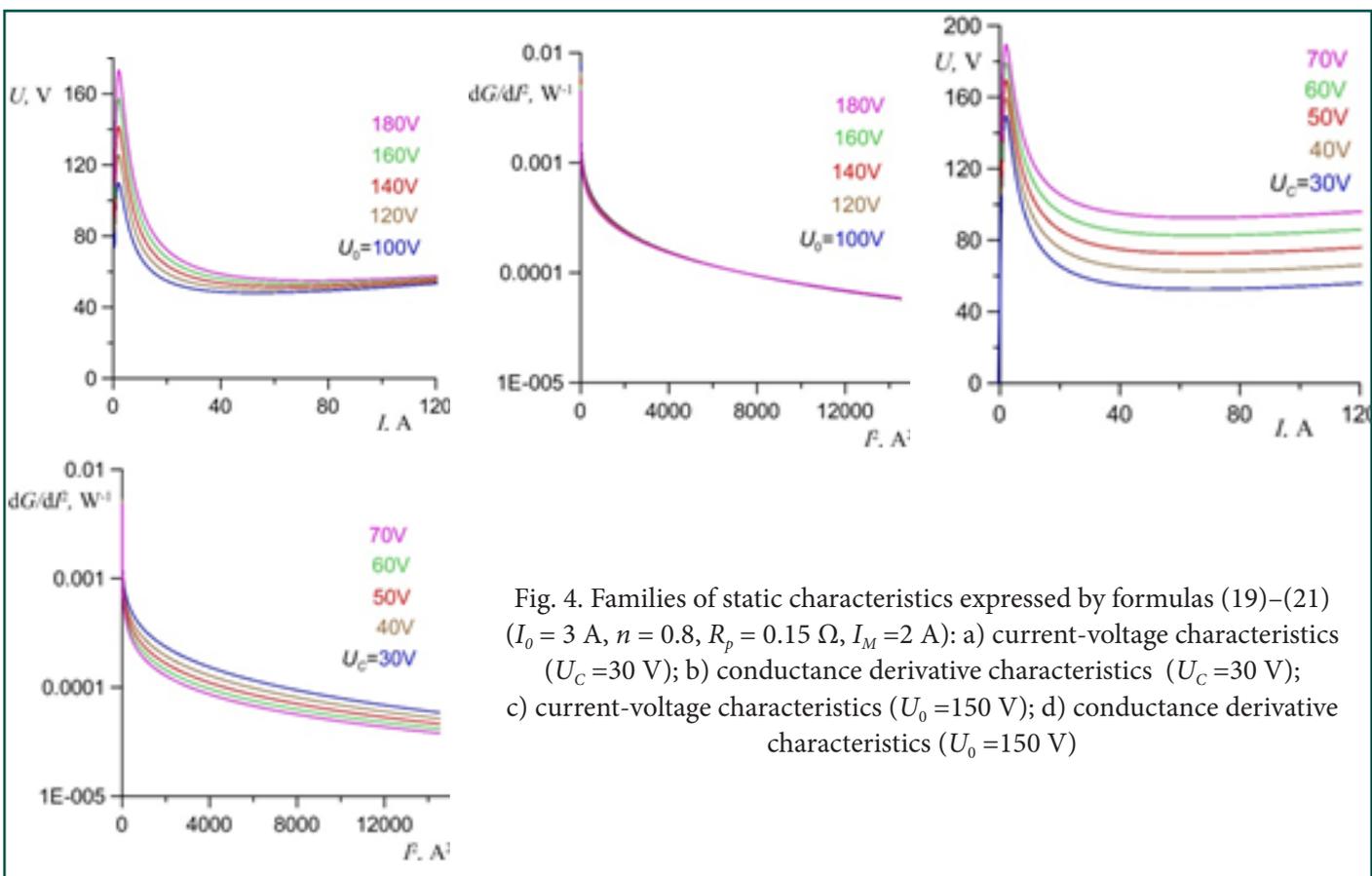


Fig. 4. Families of static characteristics expressed by formulas (19)–(21) ( $I_0 = 3 A, n = 0.8, R_p = 0.15 \Omega, I_M = 2 A$ ): a) current-voltage characteristics ( $U_C = 30 V$ ); b) conductance derivative characteristics ( $U_C = 30 V$ ); c) current-voltage characteristics ( $U_0 = 150 V$ ); d) conductance derivative characteristics ( $U_0 = 150 V$ )

the low-current range. The above-named families correspond to the families of conductance derivative characteristics  $dG(I^2, p)/dI^2$  presented in Figures 2b and 2d. Changes in parameter  $U_0$  and changes in parameter  $U_C$  lead to

similar deformations of characteristics. However, in comparison with the previous case, the approximating possibilities of these dependences are higher.

Figures 3a and 3c present the families of the

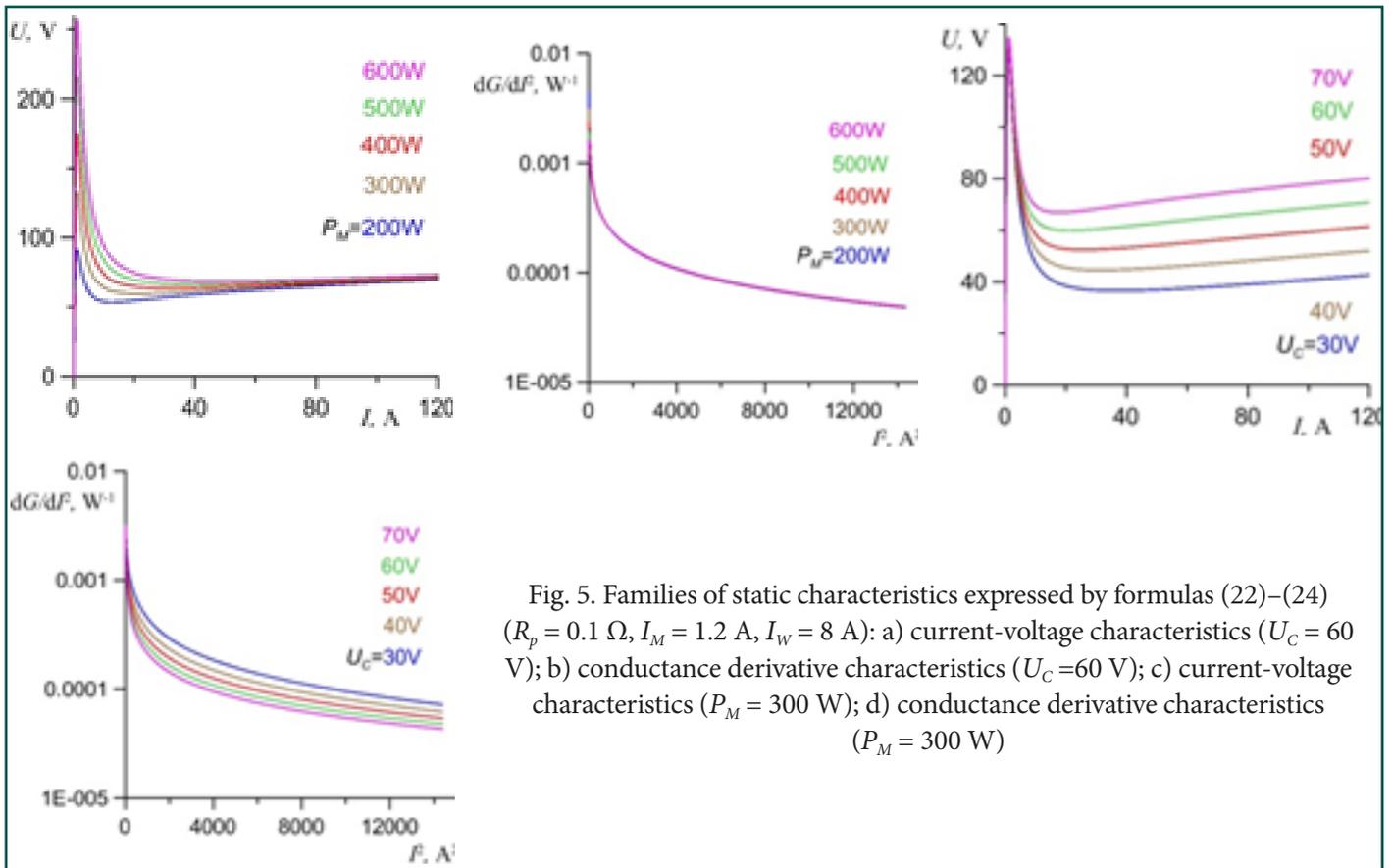


Fig. 5. Families of static characteristics expressed by formulas (22)–(24) ( $R_p = 0.1 \Omega$ ,  $I_M = 1.2 \text{ A}$ ,  $I_W = 8 \text{ A}$ ): a) current-voltage characteristics ( $U_C = 60 \text{ V}$ ); b) conductance derivative characteristics ( $U_C = 60 \text{ V}$ ); c) current-voltage characteristics ( $P_M = 300 \text{ W}$ ); d) conductance derivative characteristics ( $P_M = 300 \text{ W}$ )

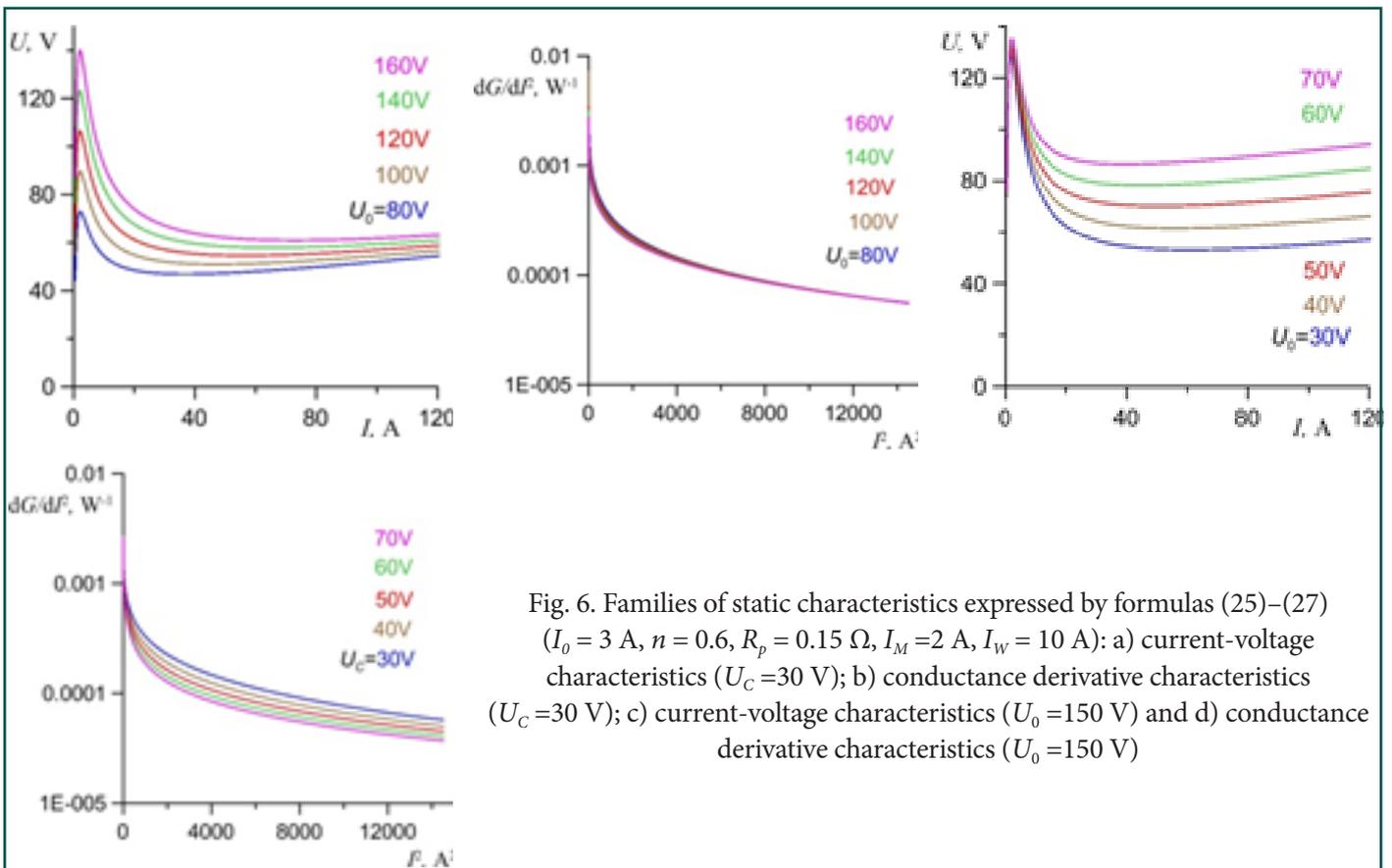


Fig. 6. Families of static characteristics expressed by formulas (25)–(27) ( $I_0 = 3 \text{ A}$ ,  $n = 0.6$ ,  $R_p = 0.15 \Omega$ ,  $I_M = 2 \text{ A}$ ,  $I_W = 10 \text{ A}$ ): a) current-voltage characteristics ( $U_C = 30 \text{ V}$ ); b) conductance derivative characteristics ( $U_C = 30 \text{ V}$ ); c) current-voltage characteristics ( $U_0 = 150 \text{ V}$ ) and d) conductance derivative characteristics ( $U_0 = 150 \text{ V}$ )

static current-voltage characteristics of arc with unspecified ignition voltage and the quasi-hyperbolic dependence of voltage  $U(I, p)$  on current within the low-current range. The above-named families correspond to the families of conductance derivative characteristics with their convergence towards the non-zero (positive) value of residual conductance. The diagrams of the conductance derivative (Fig. 3b and 3d) are similar to the previous ones. Also in this case, an increase in power  $P_M$  slightly increases the conductance derivative, whereas an increase in voltage  $U_C$  do decreases the conductance derivative.

Figures 4a and 4c present the families of the static current-voltage characteristics  $U(I, p)$  of arc with unspecified ignition voltage and the power dependence of voltage on current within the low-current range (19). The above-named families correspond to the families of conductance derivative characteristics  $dG(I^2, p)/dI^2$  presented in Figures 4c and 4d. In relation to Figure 3, changes in parameter  $U_0$  and changes in parameter  $U_C$  lead to similar deformations of characteristics. However, in comparison with the previous case, the approximating possibilities of these dependences are higher.

Figures 5a and 5b present the families of the static current-voltage characteristics of arc with simultaneously defined and reduced ignition voltage and the quasi-hyperbolic dependence of voltage  $U(I, p)$  within the low-current range (22). The above-named families correspond to the families of conductance derivative characteristics with their convergence towards the non-zero (positive) value of residual conductance. The families of conductance derivative characteristics  $dG(I^2, p)/dI^2$  are presented in Figures 5c and 5d. In relation

to Figure 3, changes in parameter  $P_M$  and changes in parameter  $U_C$  lead to similar deformations of characteristics. However, the approximating possibilities of these dependences are higher.

Figures 6a and 6c present the families of the static current-voltage characteristics of arc with defined ignition voltage and the power dependence of voltage on current within the low-current range (25). The above-named families correspond to the families of conductance derivative characteristics  $dG(I^2, p)/dI^2$  presented in Figures 6b and 6d. In relation to Figure 4, changes in parameter  $U_0$  and changes in parameter  $U_C$  lead to similar deformations of characteristics. However, the approximating possibilities of these dependences are higher.

The significant (sharp) growth of conductance derivative  $dG(I^2, p)/dI^2$  during the reduction of current near the zero value may not have a strictly physical justification. However, during the modelling of burning arc, the state of current is always  $i_\theta > 0$  A, which eliminates the effect of such conductance derivative values on dynamic current-voltage characteristics. In the aforesaid simulations, the minimum value of current amounted to approximately 4 A.

### Results of simulations of dynamic processes in the circuit with the Mayr-Pentegov model of electric arc

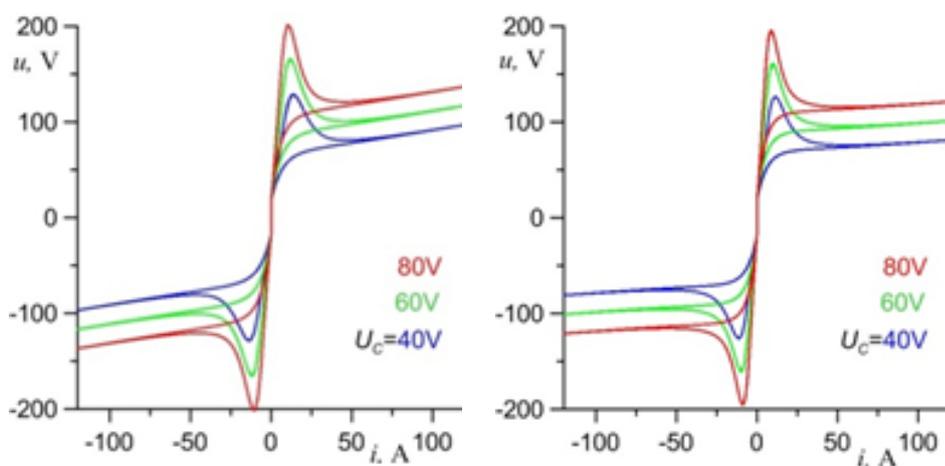


Fig. 7. Families of the dynamic characteristics of arc utilising static current-voltage characteristics with unspecified ignition voltage: a) with characteristics expressed by formula (10) ( $P_M = 300$  W,  $R_p = 0.4$   $\Omega$ ,  $Q_p = 0.1$  J) and b) with characteristics expressed by formula (13) ( $U_0 = 100$  V,  $I_0 = 1.5$  A,  $n = 0.7$ ,  $R_p = 0.15$   $\Omega$ ,  $Q_p = 0.2$  J)

The correctness of the above-presented mathematical dependences was verified by creating macromodels of arc and performing simulations of processes in the electric circuit. To this end it was necessary to use the source of sinusoidal current having amplitude  $I_M = 120$  A and frequency  $f = 50$  Hz, ensuring the stable burning of arc. The adopted constant sum of near-electrode voltage drops was  $U_{AK} = 18$  V. The parameters of the models were adjusted to obtain the high probability of characteristics.

Figure 7 presents families of the dynamic current-voltage characteristics of the arc model utilising static characteristics with unspecified ignition voltage values. The cases subjected to analysis involved the quasi-hyperbolic voltage change within the low-current range (10) and the power voltage change within the low-current range (13).

Figure 8 presents families of the dynamic current-voltage characteristics of the arc model utilising static characteristics with defined ignition voltage values. The cases subjected to analysis involved the quasi-hyperbolic voltage change within the low-current range (16) and the power voltage change within the low-current range (19).

Figure 9 presents families of the dynamic current-voltage characteristics of the arc model utilising static characteristics with defined and reduced ignition voltage values. The cases subjected to analysis involved the quasi-hyperbolic voltage change within the low-current range (22) and the power voltage change within the low-current range (25).

The selection of a function approximating static current-voltage characteristics for the version of the created Mayr-Pentegov mathematical model depends on many factors including the accuracy of obtained results, ease of determining the parameters of the model, allowed time of calculations, the operational stability of the computational programme etc.

### Concluding remarks

1. The above-presented set of generalised analytical functions useful in the approximation of data derived from arc-related experimental tests enables the rational selection of static current-voltage characteristics taking

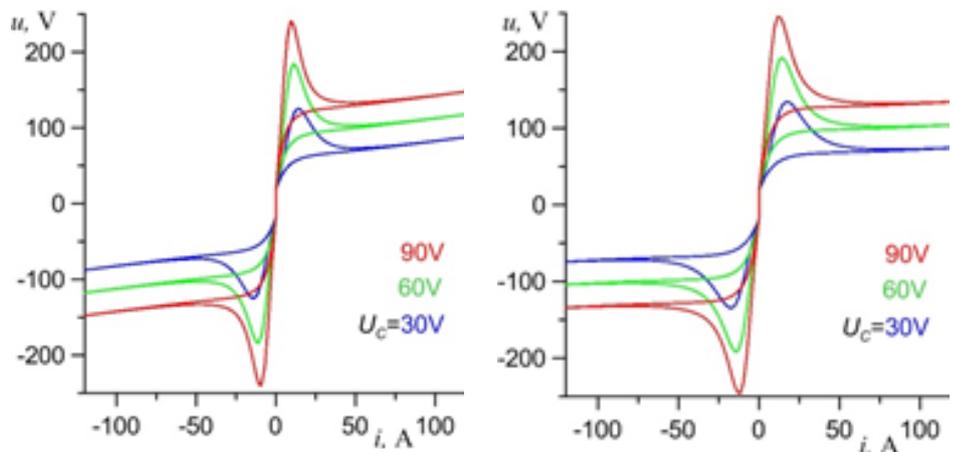


Fig. 8. Families of the dynamic characteristics of arc utilising static current-voltage characteristics with defined ignition voltage: a) with characteristics expressed by formula (16) ( $P_M = 400$  W,  $I_M = 1$  A,  $R_p = 0.3$   $\Omega$ ,  $Q_p = 0.3$  J) and b) with characteristics expressed by formula (19) ( $U_0 = 150$  V,  $I_0 = 3$  A,  $n = 0.8$ ,  $I_M = 2$  A,  $R_p = 0.15$   $\Omega$ ,  $Q_p = 0.5$  J)

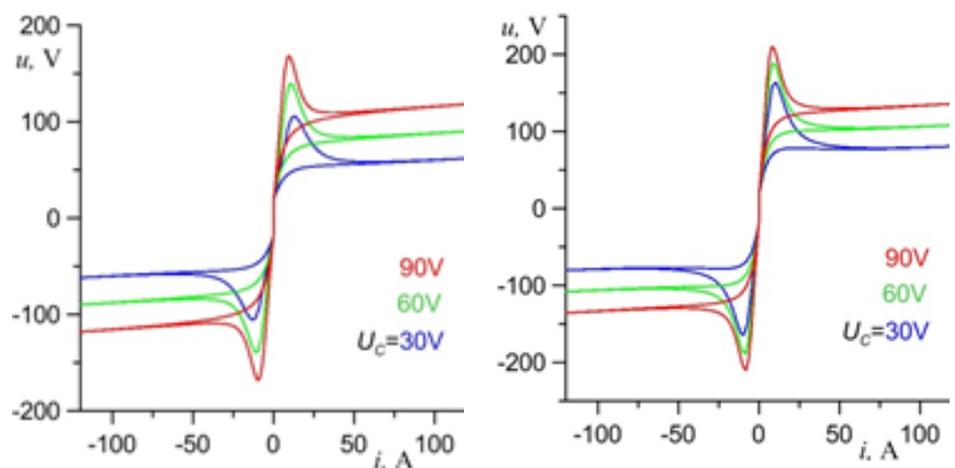


Fig. 9. Families of the dynamic characteristics of arc utilising static current-voltage characteristics with defined and reduced ignition voltage: a) with characteristics expressed by formula (22) ( $P_M = 400$  W,  $I_M = 1.2$  A,  $I_w = 8$  A,  $R_p = 0.1$   $\Omega$ ,  $Q_p = 0.2$  J) and b) with characteristics expressed by formula (25) ( $U_0 = 150$  V,  $I_0 = 3$  A,  $n = 0.6$ ,  $I_M = 2$  A,  $I_w = 10$  A,  $R_p = 0.15$   $\Omega$ ,  $Q_p = 0.2$  J)

into account the simplicity of the description of primary electric phenomena (finite ignition voltage value, non-zero residual conductance value).

2. In spite of the highly generalised forms of the functions describing static current-voltage characteristics it was possible to obtain compact analytical expressions for arc conductance derivatives.
3. Because of the simple notation of the Mayr-Pentegov model in the differential and integral forms it is possible to easily create macromodels using controlled current and voltage sources.

## References:

- [1] Pientiegov I. V., Sidoriec V. N.: Sravnitelnyj analiz modelej dinamiczeskoj svarcznoj dugi. *Avtomat. Svarka* 1989, vol. 30, no. 2, pp. 33–36 (in Russian). Pentegov I. V., Sydorets V. N.: Comparative analysis of models of dynamic welding arc. *The Paton Welding Journal*, 2015, no. 12, pp. 45–48.
- [2] Sawicki A.: The universal Mayr-Pentegov model of the electric arc. *Przeгляд Elektrotechniczny (Electrical Review)* 2019, vol. 94, no. 12, pp. 208–211, doi:10.15199/48.2019.12.47.
- [3] Sawicki A.: Mathematical Differential and Integral Models in the Macromodelling of Electric Arc Using Voltage and Current Controlled Sources Part 2. Selected Mathematical Arc Macromodels with Explicitly Defined Current and Voltage Characteristics. *Biuletyn Instytutu Spawalnictwa*, 2020, no. 1, pp. 41–47.
- [4] Pientiegov I. V.: Matematyčeskaja model stolba dinamiczeskoj elektryčeskoj dugi. *Avtomatyčeskaja Svarka*, 1976, vol. 279, no. 6, pp. 8–12.
- [5] Sawicki A.: Mathematical Differential and Integral Models in the Macromodelling of Electric Arc Using Voltage and Current Controlled Sources Part 1. Variants of Electric Arc Macromodels Obtained from Various Forms of Differential or Integral Equations. *Biuletyn Instytutu Spawalnictwa*, 2019, no. 6, pp. 59–66.
- [6] Jaroszyński L., Stryczewska H. D.: Analiza numeryczna urządzeń wyładowczych na przykładzie reaktora plazmowego ze ślizgającym się wyładowaniem łukowym. *Prace Naukowe Instytutu Podstaw Elektrotechniki i Elektrotechnologii Politechniki Wrocławskiej. Konferencje, Wrocław 2000*, vol. 37, no. 12, pp. 331–335.
- [7] Jaroszyński L., Stryczewska H. D.: Computer simulation of the electric discharge in glidarc plasma reactor. *Conference: 3rd International Conference: Electromagnetic devices and processes in environment protection ELMECO-3*, June 2000.
- [8] Sawicki A., Haltof M.: Spectral and integral methods of determining parameters in selected electric arc models with a forced sinusoid current circuit. *Archives of Electrical Engineering* 2016, vol. 65, no. 1, pp. 87–103, doi: 10.1515/aee-2016-0007)
- [9] Sawicki A., Haltof M.: Problemy opriedelenija paramietrov matiematiczeskich modelej elektryčeskich dug v cepiach s istočznikami toka. *Elektryčestvo*, 2016, no. 1, pp. 25–34.
- [10] Sawicki A.: Classical and Modified Mathematical Models of Electric Arc. *Biuletyn Instytutu Spawalnictwa*, 2019, no. 4, pp. 67–73.
- [11] Marciniak L.: Model of the arc earth-fault for medium voltage networks. *Central European Journal of Engineering*, 2011, no. 2, pp. 168–173.
- [12] Marciniak L.: Implementacje modeli łuku ziemnozwarciowego w programach PSCAD i Matlab/Simulink. *Przeгляд Elektrotechniczny*, 2012, no. 9a, pp. 126–129.
- [13] Sawicki A.: Static Characteristics of Defined Ignition Voltage Used in the Modelling of Arc within a Wide Range of Current Excitation. *Biuletyn Instytutu Spawalnictwa*, 2019, no. 1, pp.45–52 doi: 10.17729/ebis.2019.1/5)