Antoni Sawicki

The Use of the Integral Method in the Determination of the Mayr-Voronin and Cassie-Voronin Simplified Mathematical Models of Electric Arc with the Changeable Length of the Plasma Column

Abstract: The article discusses the possibility of extending the usability of well-known integral method formulas used to determine the parameters of the Mayr and Cassie mathematical models of fixed-length arc. Simulation tests discussed in the article involved the simplified variants of the Mayr-Voronin and Cassie-Voronin models of electric arc with dissipated power proportional to the volume of the elongated column. Results obtained in the tests proved the usability of the above-named formulas when calculating the parameters of modified arc models within a wide range of elongation rate changes.

Keywords: electric arc, Mayr model, Cassie model, Voronin model

DOI: 10.17729/ebis.2022.1/5

Introduction

Various electric arc burning conditions in welding machines and electrothermal equipment increase the usability of mathematical models of electromagnetic processes in power supply circuits of numerous industrial machines. The selection of an appropriate mathematical model makes it possible to improve the accuracy of simulation results. However, the criterion of such a selection is connected with diagnostic possibilities involving the easy determination of individual parameters of a given model. On one hand, applied models should be accurate, yet on the other hand, they should be relatively simple, where a small number of parameters makes it possible to quickly and explicitly determine related values. Taking into account various external factors affecting arc increases the complexity of models. Factors which most commonly influence arc are changes of the plasma column length. Some researchers try to take the aforesaid factors into consideration and make parameters of previously known simple mathematical models variable [1, 2].

This article discusses the Mayr-Voronin and Cassie-Voronin models having variable parameters dependent on the length and the cross-sectional area of the plasma column [3, 5]. In addition, the article presents dependences enabling the experimental determination of parameters of the simplest variants of the aforesaid models, i.e. the Mayr and Cassie models having constant parameters [6, 7]. The application of the known formulas of the integral method required the additional variation of the former. Measurement values of voltage and power were

dr hab. inż. Antoni Sawicki – SEP – Association of Polish Electrical Engineers/ (FSNT-SEP), Częstochowa Division

linked to the length of the column. Applied current excitation was characterised by sinusoidal waveform, constant frequency and various amplitude. The study also involved the application of various rates of arc length-related time changes and the obtainment of various ultimate values.

Mathematical models of arc with changes of plasma column length

If experimental current-voltage characteristics of electric arc have significant fragments of steeply falling segments of quasi-hyperbolic curves, such arc is usually regarded as low-current one. The primary feature of such arc is the exponential distribution of the enthalpy of the plasma column in relation to its conductance:

$$\frac{\sigma}{\sigma_{0M}} = \exp\left(\frac{q_V}{q_{0M}}\right) \quad (1)$$

where q_V – volumetric density of enthalpy, J/m³; σ – specific conductivity of the arc column, S/m; σ_{0M} and q_{0M} – coefficients of approximation of the plasma conductivity function. The power balance equation has the following form:

$$\frac{dQ}{dt} = P_{el} - P_{dis} = u_{col} i - P_{dis} \quad (2)$$

where Q – plasma enthalpy, J; P_{el} – electric power supplied to the column, W; P_{dis} – thermal power dissipated from the arc column, W; u_{col} – column voltage, V; $Q = q_V V = q_V lS$, $g = \sigma S / l$ – column electric conductance and S – column cross-sectional area.

Assuming that power dissipated from the column is proportional to the side area of the column:

$$P_{dis} = P_{ds}(l,S) = p_S l \sqrt{4\pi S} \quad (3)$$

it is possible to obtain the equation of the dynamic Mayr-Voronin model of arc with changing length $l(t)$ and cross-sectional area $S(t)$. The form of the equation [3, 5] is as follows:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{Ms}(S)}\left(\frac{u_{col} i}{P_{Ms}(l,S)} - 1\right) - \frac{1}{l}\frac{dl}{dt}\left(1 + \ln\frac{g\,l}{\sigma_{0M} S}\right) + \frac{1}{S}\frac{dS}{dt}\left(1 - \ln\frac{g\,l}{\sigma_{0M} S}\right) \quad (4)$$

where
– damping function in s:

$$\theta_{Ms}(S) = \frac{q_{0M}}{p_S}\sqrt{\frac{S}{4\pi}} \quad (5)$$

– function of dissipated power in W:

$$P_{Ms}(l,S) = p_S l \sqrt{4\pi S} \quad (6)$$

If, instead of condition (3), it is assumed that dissipated power is proportional to the volume of the column:

$$P_{dis} = P_{dv}(V) = P_{dv}(l,S) = p_V V = p_V lS \quad (7)$$

only other auxiliary function-related dependences can be obtained:
– damping variable in s:

$$\theta_{Mv} = \frac{q_{0M}}{p_V} = const \quad (8)$$

– function of dissipated power in W:

$$P_{Mv}(l,S) = p_V lS \quad (9)$$

where p_V – volumetric density of dissipated power, W/m³.

If it is assumed that changes of the column length are relatively slow ($dl/dt = 0$) and that the range of current value changes is not wide, then ($dS/dt \cong 0$) and the following equation:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{Ms}(S)}\left(\frac{u_{col} i}{P_{Ms}(l,S)} - 1\right) \quad (10)$$

being an equivalent of the Mayr model, is obtained:

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M} \quad (11)$$

with $P_{Ms}(l, S) = P_M = const$ and $\theta_{Ms}(S) = \theta_M =$ const.

If experimental current-voltage characteristics of electric arc have significant fragments of horizontal segments of curves, such arc is usually regarded as high-current one. The primary feature of such arc is the linear distribution of the enthalpy of the plasma column in relation to its conductance:

$$\sigma = \sigma_{0C} \cdot \frac{q_V}{q_{0C}} \tag{12}$$

where σ_{0C} and q_{0C} – coefficients of approximation of the plasma conductivity function.

Assuming that power dissipated from the column is proportional to the side area of the column (3), it is possible to obtain the equation of the dynamic Cassie-Voronin model of arc with changing length $l(t)$. The form of the equation is the following [5]:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{Cs}(S)}\left(\frac{u_{col}^2}{U_{Cs}^2(l,S)} - 1\right) - \frac{2}{l}\frac{dl}{dt} \tag{13}$$

where, using the dependence:

$$g = \frac{\sigma_{0C}}{q_{0C}}\frac{q_V S}{l} \tag{14}$$

it is possible to obtain:

– damping function in s:

$$\theta_{Cs}(S) = \frac{q_V}{p_S}\sqrt{\frac{S}{4\pi}} \tag{15}$$

– function of plasma column voltage in V²:

$$U_{Cs}^2(l,S) = E_{Cs}^2(S)\cdot l^2 \tag{16}$$

where E_{Cs} – intensity of the electric field in the arc column, V/m.

If, instead of condition (3), it is assumed that dissipated power is proportional to the volume of the column (9), then, instead of (15), a function independent of S is obtained:

$$\theta_{Cv} = \frac{q_V}{p_V} = const \tag{17}$$

In turn, the auxiliary function of arc column voltage adopts the following form:

$$U_{Cv}^2(l) = \frac{P_{dv}(l,S)}{g} = E_{Cv}^2 l^2 \tag{18}$$

where E_{Cv} – intensity of the electric field in the arc column, V/m.

If it is assumed that changes of the column length are relatively slow ($dl/dt = 0$) and the range of current value changes can be wide, the following equation:

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_{Cs}(S)}\left(\frac{u_{col}^2}{U_{Cs}^2(l,S)} - 1\right) \tag{19}$$

being an equivalent of the Cassie model, is obtained:

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2} \tag{20}$$

with $U_{Cs}^2(l,S) = U_C^2 = const$ and $\theta_{Cs}(S) = \theta_C = \text{const}$.

Determination of Mathematical Models of Electric Arc with Changes of the Arc Column Length and Sinusoidal Current Excitation

Methods discussed in [6] are used in the experimental determination of selected parameters of simplified mathematical models of arc with sinusoidal or rectangular current excitation (internal admittance of such a source is $Y_w = 0$ S) and relatively low disturbance both in the power supply source and in the arc column. Using the spectral or integral (mean and root-mean square) values of current, power and voltage waveforms related to the plasma column, it is possible to automatically determine parameters of mathematical models, which significantly eliminates the subjective selection of data necessary for analysis.

In the article, the determination of the parameters of the Mayr-Voronin and Cassie-Voronin models with changes of the arc plasma column involved the use of the integral method. Models used in the tests were characterised by dissipated power proportional to the volume of the column. The radius of free arc is, primarily, the function of quasi-statically changing current $r_c = f(I^{2/3})$ [8]. However, the detailed definition of the dependence concerning the cross-sectional area of the column is difficult as it depends not only on the value of current but also on the type and pressure of gas as well as external factors (washing around by the gas stream, squeezing with the cool walls of the duct, interaction with the laser beam, etc.). In addition, in dynamic states close to $i \cong 0$ A, plasma may not disappear but even increase its volume and, as a result, increase the damping function [9]. As regards the Mayr-Voronin model, it was assumed that the diameter of the plasma column was small and constant. In turn, as regards the Cassie-Voronin model, the adopted constant diameter of the plasma column was larger. The circuit was powered by a source having the sinusoidal current waveform $i = I_m \cos \omega t$. The information presented in the previous chapter and in publications [6, 7] was used to select equations (presented in Table 1) necessary to determine parameters of selected mathematical models of arc. As can be seen, changes of length do not affect values of time constants. However, the remaining parameters are characterised by the linear dependence on the aforesaid length.

Table 1 contains the following designations: P_{col} – column active power, U_{rms} – root-mean-square voltage, I_{rms} – root-mean-square current, E_{rms} – root-mean-square electric field intensity in the column, u_{col} – momentary voltage value, av – index informing about average value calculation, u_{col} – momentary value of column voltage, e_{col} – momentary value of electric field intensity in the column and p_{col} – linear density of power dissipated from the column. The values of electric power intensity and those of linear power density can be easily calculated knowing the measured length of arc $e_{col} = u_{col}/l$ and $E_{rms} = U_{rms}/l$, $E_C = U_{Cl}/l$, $p_{col} = P_{col}/l$. However, the above-named activities are not necessary in order to identify parameters of models.

If the values of near-electrode voltage drops of arc have already been determined using known experimental methods [10], the assessment is concerned with the possibility of determining parameters of the model of the column with various rates of column length changes. The circuit is powered by sinusoidal current having variable amplitude and a constant frequency of 50 Hz.

Figures 1 and 2 present the results concerning the simulation of linearly stretched arc burning in a circuit powered by sinusoidal current having an amplitude of 10 A. The selected parameters were $q_{0M} = 5\times10^5$ J/m³, $\sigma_{0M} = 800$ S/m, $S = 9\times10^{-5}$ m² and initial condition $G_0 = 2000$ S. The study involved taking into consideration various densities of dissipated power as well as various arc column elongation rates. The

Table 1. Formulas used in the determination of parameters of mathematical models of the electric arc column based on measurements performed in the circuit with the source of sinusoidal current

Parameters of the simplified arc model (dl/dt = 0)	**Parameters of the modified model of arc having the changeable column length (dl/d$t \neq 0$)**
Mayr model (11) $P_M = P_{col}$ $\theta_M = \dfrac{1}{2\omega\sqrt{\left(\dfrac{U_{rms} I_{rms}}{P_{col}}\right)^4 - 1}}$	Mayr-Voronin model (10) $P_{Ml}(l) = P_{col}(l) = p_M l$ $\theta_{Ml} = \dfrac{1}{2\omega\sqrt{\left(\dfrac{U_{rms}(l) I_{rms}}{P_{col}(l)}\right)^4 - 1}} = \dfrac{1}{2\omega\sqrt{\left(\dfrac{E_{rms} I_{rms}}{p_{col}}\right)^4 - 1}}$
Cassie model (20) $U_C = U_{rms} = \sqrt{\left(u_{col}^2\right)_{av}}$ $\theta_C = \dfrac{\dfrac{\left(u_{col}^4\right)_{av}}{U_C^4} - 1}{2\omega\sqrt{3 - 2\dfrac{\left(u_{col}^4\right)_{av}}{U_C^4}}}$	Cassie-Voronin model (19) $U_{Cl}(l) = \sqrt{\left(u_{col}^2(l)\right)_{av}} = \sqrt{\left(e_{col}^2 l^2\right)_{av}} = l\sqrt{\left(e_{col}^2\right)_{av}}$ $\theta_{Cl} = \dfrac{\dfrac{\left(u_{col}^4(l)\right)_{av}}{U_{Cl}^4(l)} - 1}{2\omega\sqrt{3 - 2\dfrac{\left(u_{col}^4(l)\right)_{av}}{U_{Cl}^4(l)}}} = \dfrac{\dfrac{\left(e_{col}^4\right)_{av}}{E_C^4} - 1}{2\omega\sqrt{3 - 2\dfrac{\left(e_{col}^4\right)_{av}}{E_C^4}}}$

time of elongation operation amounted to 1 s. As expected, it was observed that the transient state was followed by a linear increase in power P_M and the fast stabilisation of constant damping function values.

Figures 3 and 4 present the results concerning the simulation of linearly stretched arc burning in a circuit powered by sinusoidal current having an amplitude of 150 A. The selected parameters of the Cassie-Voronin model of arc were $q_V = 1{\times}10^4$ J/m^3, $q_{0C} = 16{\times}10^6$ J/m^3, $\sigma_{0C} = 800$ S/m, $S = 1{,}437{\times}10^{-3}$ m^2 and initial condition $G_0 = 2000$ S. The study involved taking into consideration various densities of dissipated power as well as various arc column elongation rates. As expected, it was observed that the transient state was followed by a linear increase in voltage U_C and the stabilisation of constant damping function values.

Concluding remarks

1. Well-known integral methods used in the determination parameters of the Mayr and Cassie mathematical models of electric arc with the constant length of the plasma column can also be applied to identify parameters of the Mayr-Voronin and Cassie-Voronin models of electric arc with the changeable length of the plasma column.

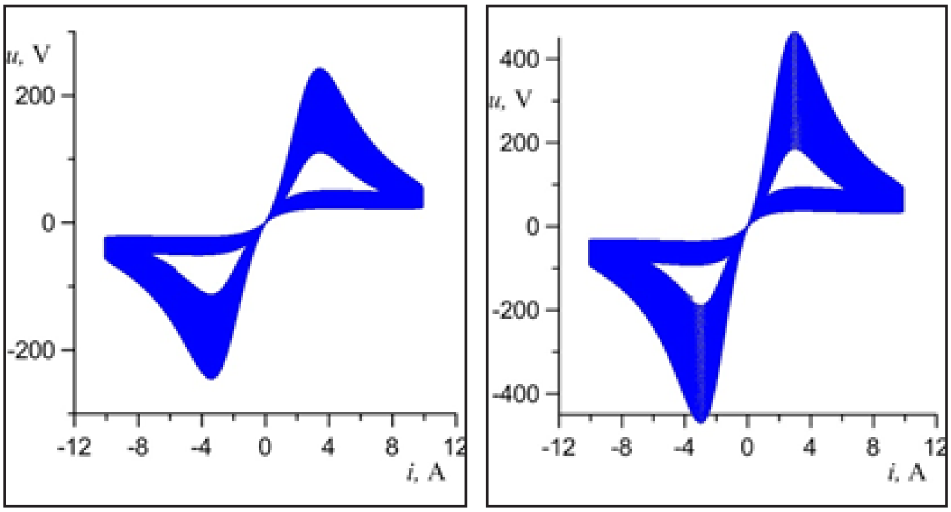

Fig. 1. Dynamic current-voltage characteristic of arc having the stretched plasma column and described using the Mayr-Voronin model: a) parameters ($p_V = 6{\times}10^8$ W/m^3, $l = (4 + 5t){\times}10^{-3}$, m) and b) parameters ($p_V = 7{\times}10^8$ W/m^3, $l = (5 + 8t){\times}10^{-3}$, m)

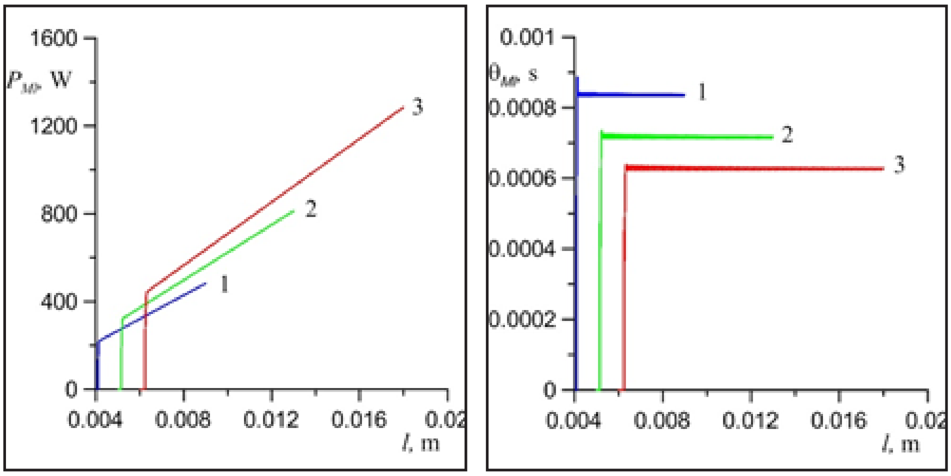

Fig. 2. Diagrams of changes of parameters of arc having the stretched plasma column and described using the Mayr-Voronin model: (1 - ($p_V = 6{\times}10^8$ W/m^3, $l = (4 + 5t){\times}10^{-3}$, m), 2 - ($p_V = 7{\times}10^8$ W/m^3, $l = (5 + 8t){\times}10^{-3}$, m), 3 – ($p_V = 8{\times}10^8$ W/m^3, $l = (6 + 12t){\times}10^{-3}$, m)): a) power P_{Ml} and b) damping function θ_{Ml}

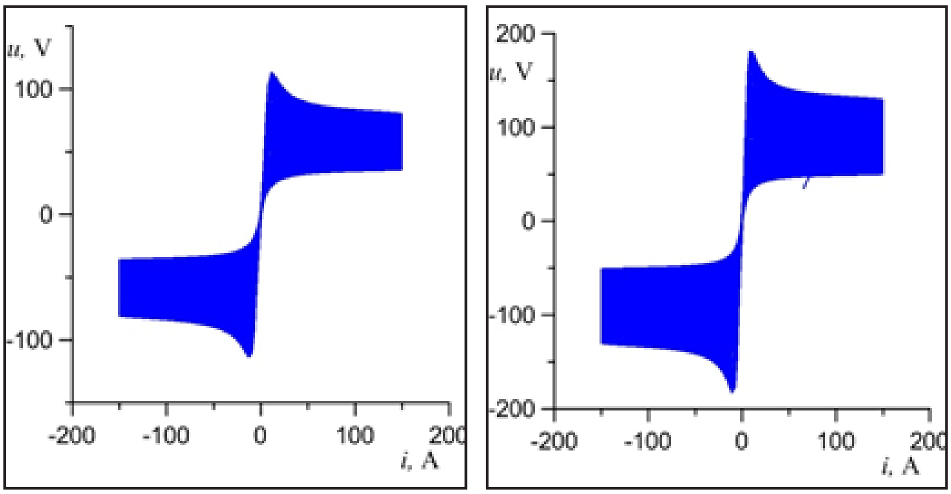

Fig. 3. Dynamic current-voltage characteristic of arc having the stretched plasma column and described using the Cassie-Voronin model a) parameters (p_V=40×10^6 Wm^{-3}, $l = (4 + 5t){\times}10^{-3}$, m) and b) parameters ($p_V = 50{\times}10^6$ Wm^{-3}, $l = (5 + 8t){\times}10^{-3}$, m)

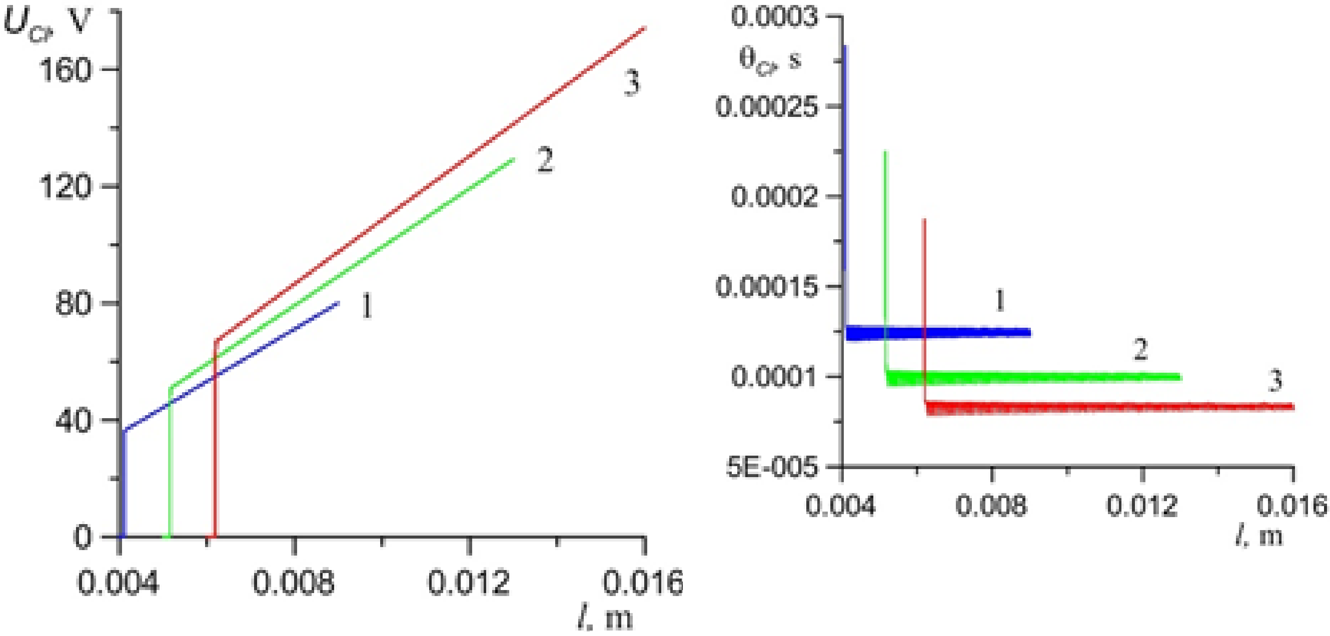

Fig. 4. Diagrams of changes of parameters of arc having the stretched plasma column and described using the Cassie-Voronin model: (1 - (p_V = 40×10^6 Wm^{-3}, l = (4 + 5t)×10^{-3}, m), 2 - (p_V= 50×10^6 Wm^{-3}, l = (5 + 8t)×10^{-3}, m), 3 – (p_V= 60×10^6 Wm^{-3}, l = (6 + 10t)×10^{-3}, m)): a) voltage U_{Cl} and b) damping function θ_{Cl}

2. Only the very short initial phase of simulations was characterised by significant changes and differences between calculated and expected results. The aforementioned situation was caused by transitory processes in the circuit with arc.

3. The selection of a mathematical model as well as its parameters and the pre-set accuracy of their determination are related to ultimate rates at which the effect of changes of the arc column length is properly taken into consideration. Excessive rates and elongation are also responsible for the instability of arc discharge and termination.

4. Changes of the plasma column length constitute the primary method enabling the control of arc power in many industrial machines. The possibility of identifying parameters of the mathematical model of such arc could facilitate the development and implementation of control algorithms.

References:

[1] Janowski T., Jaroszyński L., Stryczewska H.D.: Modification of the Mayr's electric arc model for gliding Arc Analysis. XXVI International Conference on Phenomena in Ionized Gases, Nagoya, Japan 2001/7/17, pp. 341–342

[2] Marciniak L.: Model of the arc earth-fault for medium voltage networks. Central European Journal of Engineering 2011, no. 2, pp. 168–173.

[3] Voronin A.A.: Povyšenie èffektivnosti kontaktno-dugogasitel`nyh sistem sil`notočnyh kommutacionnyh apparatov s udlinâûŝejsâ dugoj. Avtoreferat dis. k.t.n., Samara 2009.

[4] Voronin A.A., Kulakov P.A.: Mathematical model of the electric arc. Vestnik of Samara State Technical University. Technical Sciences Series, 2012, vol. 20, no. 4, pp. 155–162.

[5] Sawicki A.: Modified Voronin models of electric arc with disturbed geometric dimensions and increased energy dissipation. Archives of Electrical Engineering, 2021, vol. 70, no. 1, pp. 89–102 (doi: 10.24425/aee.2021.136054).

[6] Sawicki A., Haltof M.: Spectral and integral methods of determining parameters in selected electric arc models with a forced sinusoid current circuit. Archives of Electrical Engineering, 2016, vol. 65, no. 1, pp. 87–103 (doi: 10.1515/aee-2016-0007).

[7] Sawicki A., Haltof M.: Problems of Determining Parameters of Mathematical Electric Arc Models in Circuits with Bipolar Current Source. Electricity 2016, no. 1, pp. 25–34.
[8] Kopersak V.M.: The theory of welding processes-1. KPI, Kiev 2011.
[9] Kalasek V.: Measurements of time constants on cascade d.c. arc in nitrogen, TH-Report 71-E18, Eindhoven, 1971, pp. 1–30.
[10] Sawicki A., Haltof M.: Selected Methods Used in Experimental Determination of Near-Electrode Voltage Drops of Electric Arcs. Part 1: Direct Methods Used in Determination of Near-electrode Voltage Drops. Bulletin of the institute of Welding, 2015, no. 6, pp. 61–69.