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Modelling of Electric Processes in Circuits of Arc Plasma Torches

Part 2. Dynamic States in Circuits with Single-Phase Plasma Torches

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Abstract: The article discusses modified formulas identifying static current-voltage characteristics of arc in AC plasma torches. The modified formulas, making it possible take into account preset values of discharge ignition voltage, were used in the Mayr-Pentegov universal mathematical model. As a result, it was possible to simulate the operation of additional ionising and stabilising systems, commonly used in AC plasma torches. The article also presents results of numerical simulations concerning processes taking place in electric circuits with single-phase plasma torches of various operating parameters.

Keywords: electric arc, arc plasma torch, Mayr-Pentegov model

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Introduction

The knowledge of static current-voltage characteristics of electric arc enables the application of selected universal mathematical models in the simulation of processes taking place in arc plasma torches. Macromodels based on the aforesaid universal mathematical models are used to simulate processes taking place in DC or AC circuits. Requirements concerning the conformity of static experimental characteristics with model characteristics depend on a given (selected) range of current changes [1, 2]. Differences are primarily concerned with a relatively narrow range of current changes in areas where current passes through the zero value. The good knowledge of ignition voltage enables the more precise modelling of dynamic characteristics of arc in plasma torches. The selection of an approximating function affects the analytical form of an expression describing a damping function applied in the Mayr-Pentegov model [3, 4].

Modifications of static characteristics of arc in AC plasma torches

The appropriate modification of characteristic (3), discussed in [5], enables the obtainment of a dependence concerning the generalised current-voltage characteristics of linear plasma torch arc with specific ignition voltage:

$$U = \frac{A_u l^{\gamma} p^{\delta}}{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}} \left(\frac{I_s I}{I^2 + I_M^2} \right)^{-2\alpha} \tag{1}$$

The coordinates of the extreme point have the following values:

$$I = I_M \qquad U = U_M = \frac{A_u l^{\gamma} p^{\delta}}{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}} \left(\frac{I_s}{2 I_M} \right)^{-2\alpha} \tag{2}$$

where, as mentioned before, $\alpha < 0$.

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The first stage of numerical tests involved the determination of families of static current-voltage characteristics of arc plasma torches. Tables 1 and 2 presented in [5] provide information concerning values of parameters related to functions approximating measurement data. Figures 1 and 2 present diagrams corresponding to various sets of parameters and two approximation cases, i.e. with defined and unspecified values of ignition voltage. The first case corresponds to the truncation of characteristics to values of voltage presentable graphically. The diagrams obtained in the aforesaid manner indicate the effectiveness of applied modifications. The difference between adopted data and those presented in Tables 1 and 2 in [5] results from the existence of many various designs of plasma torches, their operability with various gas mixtures as well as from the graphic presentation of interesting fragments of characteristics families.

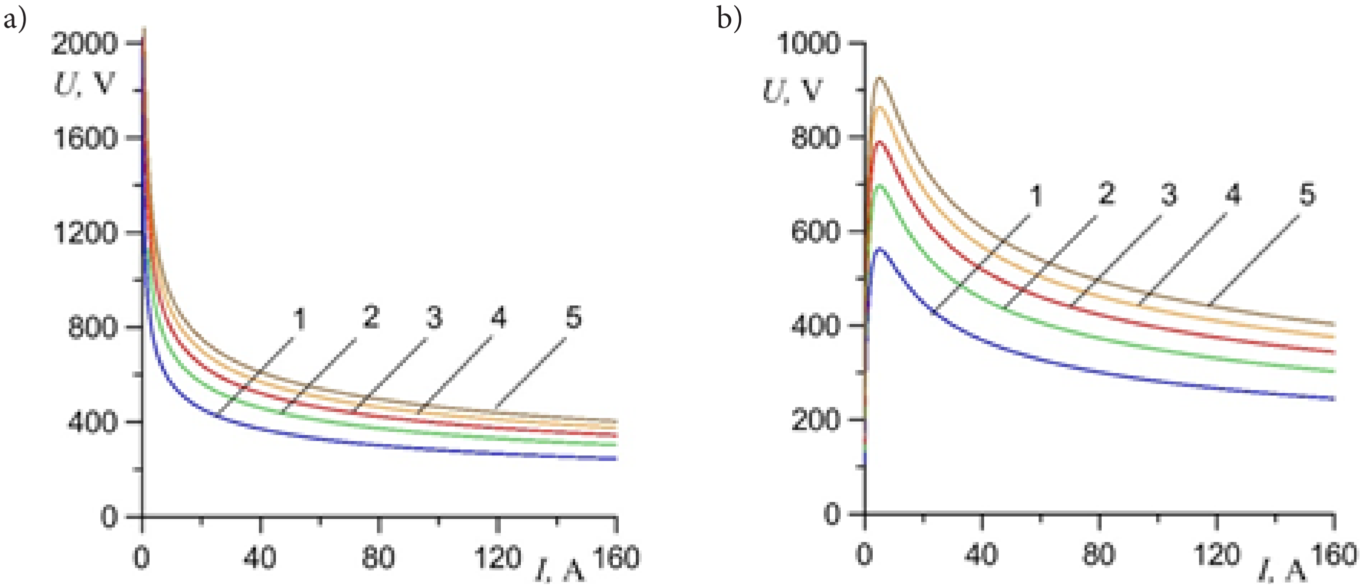

Fig. 1. Families of static current-voltage characteristics of single-phase plasma torch arc: a) characteristics with the unspecified value of ignition voltage and b) characteristics with the defined value of ignition voltage (1 - $\dot{m}$ = 2·10^{-3} kg/s, 2- $\dot{m}$ = 4·10^{-3} kg/s, 3- $\dot{m}$ = 6·10^{-3} kg/s, 4 - $\dot{m}$ = 8·10^{-3} kg/s, 5- $\dot{m}$ = 10·10^{-3} kg/s, d = 5·10^{-3} m, p = ·10^{5} Pa, l = 0.1 m, I_s = 1 A, I_M = 5 A, A_u = 2100 V, α = -0.15, β = 0.16, γ = 1·10^{-3} and δ = 0.2)

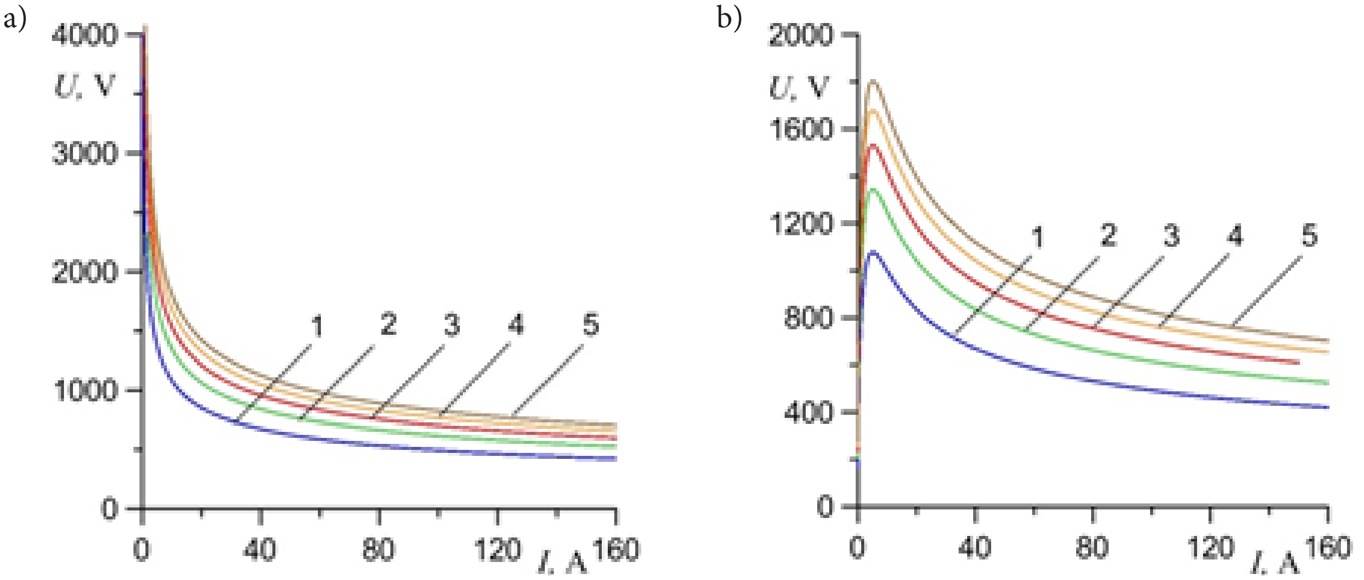

Fig. 2. Families of static current-voltage characteristics of single-phase plasma torch arc: a) characteristics with the unspecified value of ignition voltage and b) characteristics with the defined value of ignition voltage (1 - $\dot{m}$ = 2·10^{-3} kg/s, 2- $\dot{m}$ = 4·10^{-3} kg/s, 3- $\dot{m}$ = 6·10^{-3} kg/s, 4 - $\dot{m}$ = 8·10^{-3} kg/s, 5- $\dot{m}$ = 10·10^{-3} kg/s, d = 5·10^{-3} m, p = ·10^{5} Pa, l = 0.1 m, I_s = 1 A, I_M = 5 A, A_u = 3000 V, α = -0.17, β = 0.16, γ = 0.1 and δ = 0.25)

Mayr-Pentegov mathematical model of electric arc in calculations of AC plasma torches

Because of very short times related to the relaxation of physical process in plasma (in comparison with periods of applied current excitation), the value of averaged experimental dynamic characteristics is their relation with static current-voltage characteristics. The aforesaid property was used to build mathematical models utilising modified static characteristics taking into account finite values of discharge ignition voltage.

The modelling of electric arc in AC plasma torches usually involves the use of well–known models of free arc. The stabilisation of the position and the length of the plasma column by the stream of gas and the magnetic field is suitable for the aforesaid models significantly more than intense disturbances commonly present in most electrical devices with free arc.

Because of numerous design and operation-related properties of plasma torches, i.e. a large number of designs, the use of alternating and direct current, device start-up processes and the use of gas flows characterised by various parameters, mathematical models used in simulations of arc should take into account various shapes of static and dynamic characteristics. Such wide approximation-related possibilities are characteristic of hybrid mathematical models [6], the Pentegov-Sidorec model [7] and the Mayr-Pentegov model [4]. Because of the possible use of any static current-voltage characteristics, the Pentegov-Sidorec and the Mayr-Pentegov models are very comfortable. The aforesaid characteristics could constitute families of curves, depending on various physical parameters (pressure, gas mass stream, column length, etc.).

The Mayr-Pentegov model constitutes a system of two basic equations, one of which is differential, whereas the other one is algebraic. The variant of the model with the non-linear first-order differential equation describes the dynamic of changes of state current $i_\theta(t)$, corresponding to changes of plasma temperature

$$\theta(i_\theta, p_a)\frac{di_\theta^2}{dt} + i_\theta^2 = i^2 \tag{3}$$

and containing the designation of the damping function

$$\theta(i_\theta, p_a) = Q_p \frac{dg}{di_\theta^2} \tag{4}$$

The damping function depends not only on state current i_θ , but also on the vector of parameters p_a. The indications used in the above-named equations include i – excitation current, g – arc column conductance, Q_p – sub-tangent to the diagram of function $g(Q)$ in relation to approximations obtained using the exponential function and Q – plasma enthalpy. The assumption that $\theta(i_{\theta,} p)$ = const. leads to the obtainment of the simpler Pentegov-Sidorec mathematical model [7].

Equation (3) can be expressed in the following form:

$$i_\theta^2 = \int_0^t \frac{1}{\theta(i_\theta, p_a)}\left(i^2 - i_\theta^2\right)d\tau + i_{\theta 0}^2 \tag{5}$$

The second, algebraic, equation describes momentary changes of voltage u on the column

$$u = \frac{U(i_\theta, p_a)}{i_\theta} i \tag{6}$$

where $U(I,\ p_a)$ – static current-voltage characteristic.

As regards the description of dynamic characteristics of arc and in comparison with the Pentegov model, the Mayr-Pentegov model is more precise as it is characterised by the attenuation of reduction assumptions. However, the Mayr-Pentegov model requires the identification of the static conductance derivative in relation to squared current (4).

In terms of linear single-phase plasma torch arc having current-voltage characteristic described by formula (3) in [5], static conductance is expressed by the following formula:

$$G(I)=\frac{I}{U}=\frac{I}{\dfrac{A_u l^{\gamma} p^{\delta}}{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}}\left(\dfrac{1}{I}\right)^{-2\alpha}}=\frac{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta} I^{1-2\alpha}}{A_u l^{\gamma} p^{\delta}} \tag{7}$$

but it can also be expressed in the following manner:

$$G(I^2)=\frac{I}{U}=\frac{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}\left(I^2\right)^{\frac{1-2\alpha}{2}}}{A_u l^{\gamma} p^{\delta}} \tag{8}$$

The derivative of conductance in relation to squared current can be determined using the following formula

$$\frac{dG(I^2)}{dI^2}=\frac{(0{,}5-\alpha)\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}\left(I^2\right)^{\frac{-1-2\alpha}{2}}}{A_u l^{\gamma} p^{\delta}} \tag{9}$$

Taking into account the finite value of ignition voltage improves the stability of numerical calculations concerning power supply systems with electric arc [4].

In order to use the Mayr-Pentegov model to calculate dynamic states of plasma torches (using modified static current-voltage characteristics) it is also necessary to determine derivatives of conductance in relation to squared current. As regards general correlation described by formula (9) in [5], the formula expressing the modified function of static conductance is the following:

$$G(I^2)=\frac{I}{U}=\frac{\sqrt{I^2}}{\dfrac{A_u l^{\gamma} p^{\delta}}{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}}\left(\dfrac{I_s\sqrt{I^2}}{I^2+I_M^2}\right)^{-2\alpha}} \tag{10}$$

where the derivative of conductance in relation to squared current can be expressed as follows:

$$\frac{dG(I^2)}{dI^2}=\frac{\left(\dfrac{I}{I^2+I_M^2}\right)^{1+2\alpha}\left[(2\alpha+1)I_M^2-(2\alpha-1)I^2\right]}{2I_s^{-2\alpha}\dfrac{A_u l^{\gamma} p^{\delta}}{\dot{m}^{\alpha-\beta} d^{\alpha+\beta+\gamma-\delta}}I^2} \tag{11}$$

The second stage of numerical tests involved the identification of derivatives of conductance in relation to squared current and corresponding to various sets of parameters and two approximation cases, i.e. with defined and unspecified, values of ignition voltage. The first case corresponds to the truncation of characteristics to values of voltage presentable graphically. The results of related calculations are presented in Figures 6 and 7. As can be seen, the effect of the value of ignition voltage on the shape of the damping function characteristics is not significant.

Because of the fact that the effect of arc ignition is only present within the low-current range, differences between characteristics with defined and those with unspecified ignition

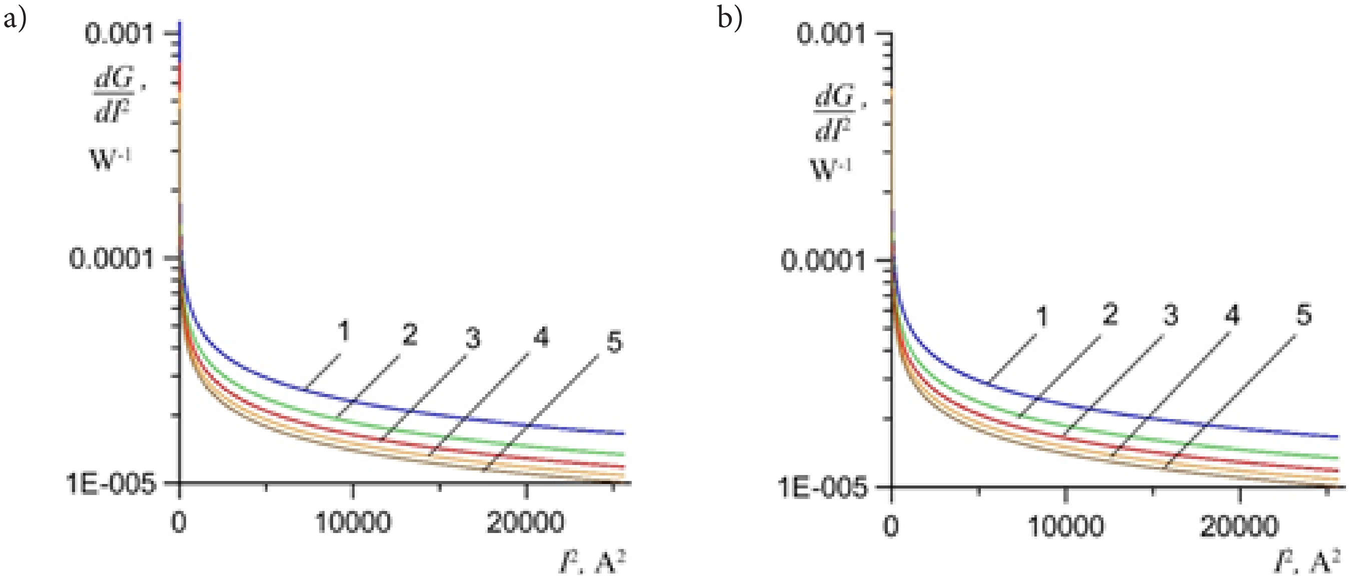

Fig. 3. Families of characteristics of conductance derivative dG/dI^2 of static characteristics of single-phase plasma torch arc: a) characteristics with the unspecified value of ignition voltage and b) characteristics with the defined value of ignition voltage (1 - $\dot{m} = 2\cdot10^{-3}$ kg/s, 2- $\dot{m} = 4\cdot10^{-3}$ kg/s, 3- $\dot{m} = 6\cdot10^{-3}$ kg/s, 4 - $\dot{m} = 8\cdot10^{-3}$ kg/s, 5- $\dot{m} = 10\cdot10^{-3}$ kg/s, $d = 5\cdot10^{-3}$ m, $p = \cdot10^{5}$ Pa, $l = 0.1$ m, $I_s = 1$ A, $I_M = 5$ A, $A_u = 21000$ V, $\alpha = -0.15$, $\beta = 0.16$, $\gamma = 1\cdot10^{-3}$ and $\delta = 0.2$)

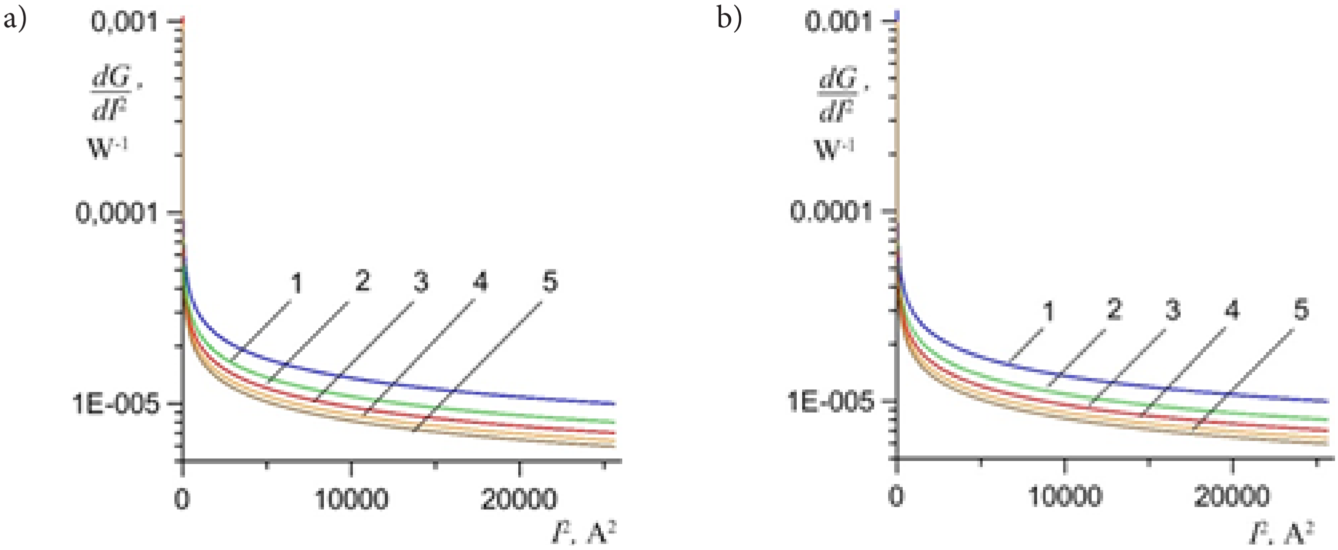

Fig. 4. Families of characteristics of conductance derivative dG/dI2 of static characteristics of single-phase plasma torch arc: a) characteristics with the unspecified value of ignition voltage and b) characteristics with the defined value of ignition voltage (1 - $\dot{m}$ = $2\cdot10^{-3}$ kg/s, 2- $\dot{m}$ = $4\cdot10^{-3}$ kg/s, 3- $\dot{m}$ = $6\cdot10^{-3}$ kg/s, 4 - $\dot{m}$ = $8\cdot10^{-3}$ kg/s, 5- $\dot{m}$ = $10\cdot10^{-3}$ kg/s, $d = 5\cdot10^{-3}$ m, $p = \cdot10^5$ Pa, $l = 0.1$ m, $I_s = 1$ A, $I_M = 5$ A, $A_u = 3000$ V, $\alpha = -0.17$, $\beta = 0.16$, $\gamma = 0.1$ and $\delta = 0.25$)

voltage (presented in Figures 3 and 4) are not visible easily. For this reason, data from Figure 3 (shown using the logarithmic scale) are presented in Figure 5 also on the x-axis. As a result, in Figure 5b it is possible to observe the non-linearity of the characteristics. Consequently, the same values of conductance derivatives (as in Fig. 5a) can be observed in relation to significantly higher values of state current.

Simulation of dynamic processes in AC plasma torches

Most mathematical models of high-current electric arc (e.g. Cassie-Mason or Lowke models) connect the constant value of the damping function (the so-called time constant) [8] with the cross-sectional area of the plasma column

$$\theta \propto \rho c_p \frac{r_a^2}{\lambda} \tag{12}$$

where ρ – gas density, c_p – specific heat of gas

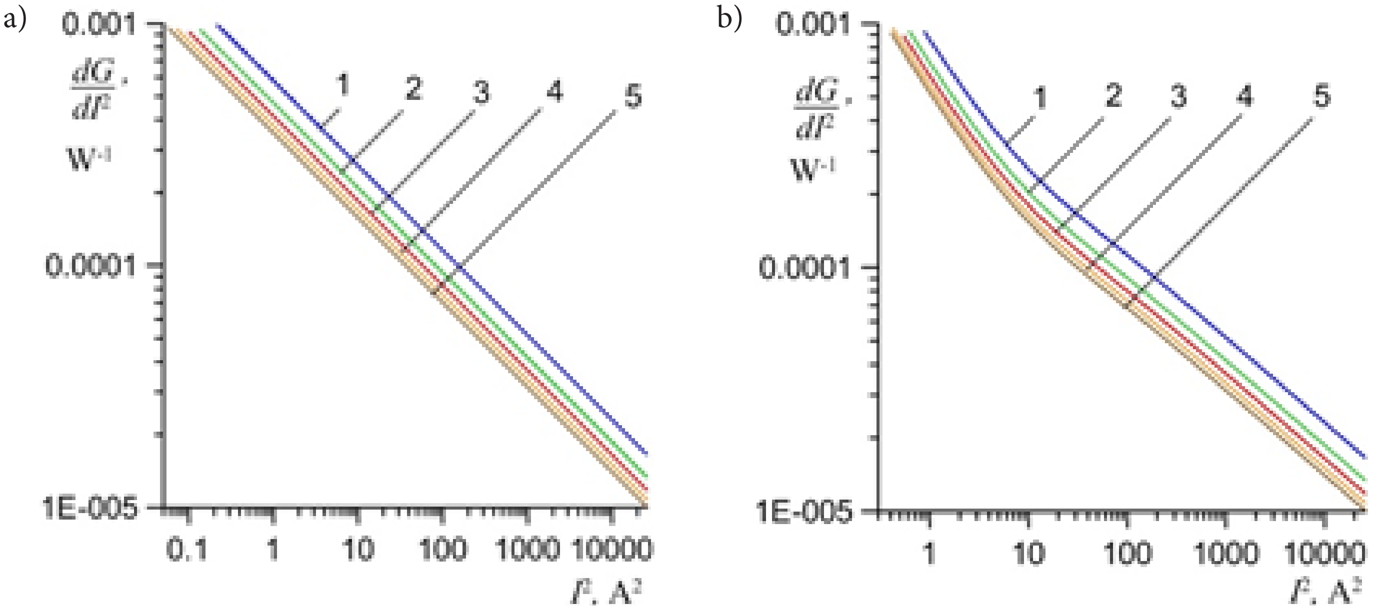

Fig. 5. Families of characteristics of conductance derivative dG/dI^2 of static characteristics of single-phase plasma torch arc: a) characteristics with the unspecified value of ignition voltage and b) characteristics with the defined value of ignition voltage (data as in the caption under Figure 6)

having pressure p, λ – gas heat conduction coefficient and r_a – plasma column radius. A similar relationship with the area of cross-section is present in the damping function of the Voronin model [9]. The area of the column cross-section depends not only on current (in welding arc $r_a = f(I^{2/3})$), but also on the type of plasma-forming gas and its pressure, the temperature of gas in the discharge area, the flow rate and the direction of shielding gas washing around the plasma column, the diameter of the plasma torch discharge channel as well as the amplitude and frequency of magnetic field effect [10]. The experimental tests of arc [11] revealed a significant increase in the damping function, which could be attributed to the attenuated pressure exerted on plasma by its own magnetic field and by an increase in the length of radius r_a. The foregoing led to the conclusion that the damping function was strongly non-linear and even had its local minimum. The above-presented behaviour roughly corresponded to the shape of the diagram of function (4), taking into account specific static current-voltage characteristics.

Presently, there are several (spectral and integral [12]) methods enabling the experimental determination of the time constant of electric arc characterised by preset and simplified static current-voltage characteristics and powered by sinusoidal current. It is possible to precisely determine the time constant of the Mayr mathematical model solely on the basis of static current-voltage characteristics and characteristics of root-mean-square values. The best corresponding model of static characteristics described by power functions (2) in [5] is Zarudi's mathematical model [7]. However, in terms of the above-named model, no simple analytical expressions for the time constant are available. For this reason, it was recognised

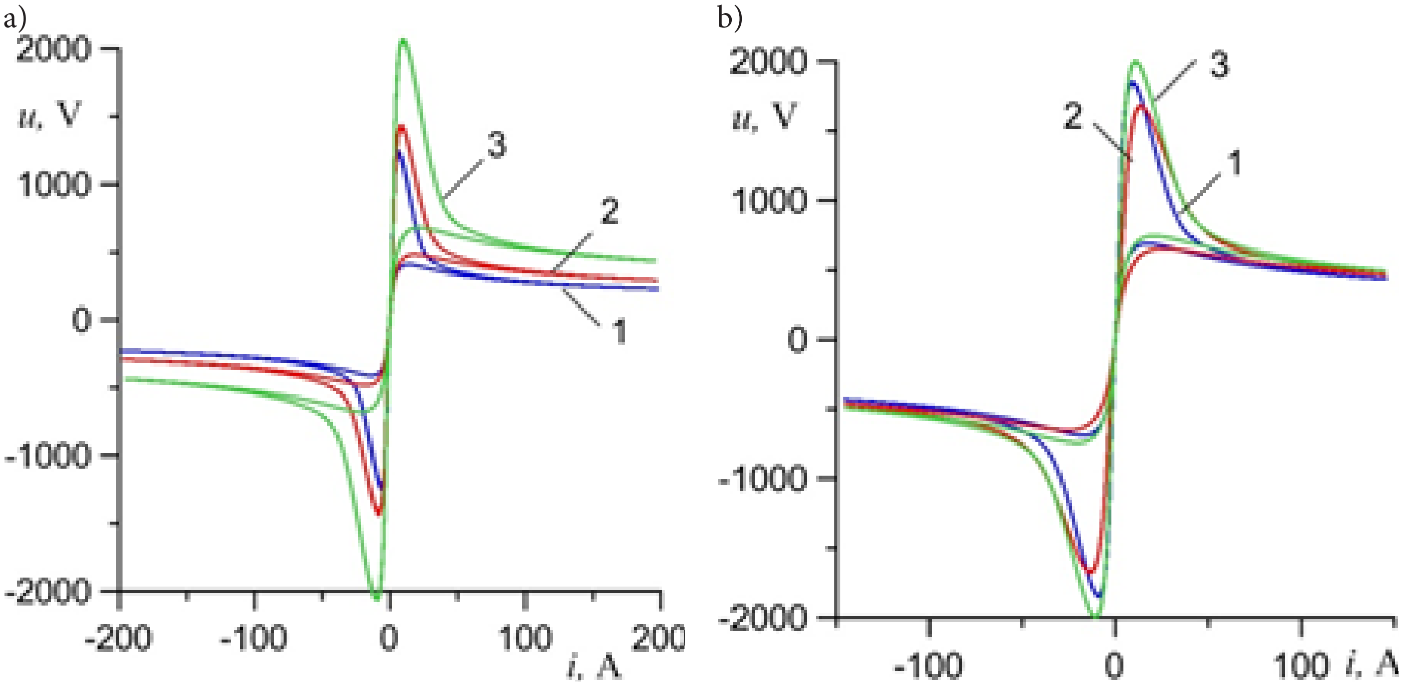

Fig. 6. Dynamic current-voltage characteristics of single-phase plasma torch arc: a) characteristics with the unspecified value of ignition voltage: 1 - ($\dot{m} = 2{\cdot}10^{-3}$ kg/s, $d = 5{\cdot}10^{-3}$ m, $p = {\cdot}10^5$ Pa, $l = 0.1$ m, $A_u = 2100$ V, $\alpha = -0.15$, $\beta = 0.18$, $\gamma = 0$, $\delta = 0.2$, $Q_p = 0.5$ J, $(i_\theta)_{\min} = 2.65$ A), 2 - ($\dot{m} = 4{\cdot}10^{-3}$ kg/s, $d = 5{\cdot}10^{-3}$ m, $p = {\cdot}10^5$ Pa, $l = 0.1$ m, $A_u = 2100$ V, $\alpha = -0.15$, $\beta = 0.16$, $\gamma = 0.01$, $\delta = 0.2$, $Q_p = 1$ J, $(i_\theta)_{\min} = 3.55$ A), 3 - ($\dot{m} = 6{\cdot}10^{-3}$ kg/s, $d = 5{\cdot}10^{-3}$ m, $p = {\cdot}10^5$ Pa, $l = 0.1$ m, $A_u = 2100$ V, $\alpha = -0.15$, $\beta = 0.16$, $\gamma = 0{,}1$, $\delta = 0.2$, $Q_p = 2$ J and $(i_\theta)_{\min} = 3.95$ A),
b) characteristics with the defined value of ignition voltage: 1 - ($\dot{m} = 2{\cdot}10^{-3}$ kg/s, $d = 5{\cdot}10^{-3}$ m, $p = {\cdot}10^5$ Pa, $l = 0.1$ m, $I_s = 1$ A, $I_M = 4$ A, $A_u = 3000$ V, $\alpha = -0.17$, $\beta = 0.15$, $\gamma = 0.1$,$\delta = 0.25$, $Q_p = 2$ J, $(i_\theta)_{\min} = 3.1$ A), 2 - ($\dot{m} = 2.5{\cdot}10^{-3}$ kg/s, $d = 5{\cdot}10^{-3}$ m, $p = {\cdot}10^5$ Pa, $l = 0.1$ m, $I_s = 1$ A, $I_M = 6$ A, $A_u = 3000$ V, $\alpha = -0.17$, $\beta = 0.15$, $\gamma = 0.01$, $\delta = 0.25$, $Q_p = 4$ J, $(i_\theta)_{\min} = 5.5$ A), 3 - ($\dot{m} = 3{\cdot}10^{-3}$ kg/s, $d = 5{\cdot}10^{-3}$ m, $p = {\cdot}10^5$ Pa, $l = 0.1$ m, $I_s = 1$ A, $I_M = 4.5$ A, $A_u = 3000$ V, $\alpha = -0.17$, $\beta = 0.15$, $\gamma = 0{,}1$, $\delta = 0.25$, $Q_p = 3$ J and $(i_\theta)_{\min} = 3.8$ A)

that the most rational approximation of inertia was the damping function present in the Mayr-Pentegov model. To a certain extent, the quasi-hyperbolic shape of the aforesaid function corresponded to experimental data [11].

The application of approximations of current-voltage characteristics with reduced ignition voltage could also be justified when using discharge ionisers and the Novikov-Schellhase mathematical model of arc [13]. In turn, as regards the use of the Pentegov-Sidorec or Mayr-Pentegov models, it is beneficial if the following condition is satisfied

$$0 < \left(i_\theta\right)_{\min} < I_M$$

where $(i_\theta)_{\min}$ - minimum value of state current.

The second stage of the tests involved the creation of two macromodels of single-phase plasma torch arc using formulas (5), (6), (9) and (11). Afterwards, the aforesaid macromodels were used in power supply circuits with single-phase power sources. The models generated sinusoidal waveforms having an amplitude of 200 A and a frequency of 50 Hz. The results of the simulation are presented in Figure 6.

Concluding remarks

1. Simplified approximations of static current-voltage characteristics of arc burning in various plasma torches (operating with various gases) enable the creation of modified static characteristics, which, due to an additional parameter (ignition voltage) could make it easier to take into account the effect of ionising and stabilising systems.
2. It is possible to create universal macromodels of arcs in plasma torches with additional igniting and stabilising systems. The usability of such macromodels results from the improved simulation of processes in power supply systems of various single-phase plasma torches.
3. Because of the very similar form of dependences approximating static current-voltage characteristics of DC plasma torch arc as well as single and three-phase plasma torch arc, the methodology discussed in the article can be used to calculate dynamic states of welding and electrothermal devices having various power and dedicated to various applications.

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